Simulation based stress tests of banks’ regulatory capital adequacy

Samu Peura a,*, Esa Jokivuolle b,c

a Sampo plc, P.O. Box 1025, FIN-00075 Sampo, Finland
b Department of Accounting and Finance, Helsinki School of Economics, P.O. Box 1210, FIN-00101 Helsinki, Finland
c Bank of Finland, P.O. Box 160, FIN-00101 Helsinki, Finland

Received 6 February 2003; accepted 13 May 2003
Available online 23 December 2003

Abstract

Banks’ holding of reasonable capital buffers in excess of minimum requirements could alleviate the procyclicality problem potentially exacerbated by the rating-sensitive capital charges of Basel II. Determining the sufficient buffer size is an important risk management task for banks, which the Basel Committee suggests should be approached via stress testing. We present here a simulation-based approach to stress testing of regulatory capital adequacy where rating transitions are conditioned on business-cycle phase, and which takes into account business-cycle dynamics. Our approach is an extension of a typical credit portfolio analysis in that we simulate actual bank capital and minimum capital requirements simultaneously. Actual bank capital (absent mark-to-market accounting) is driven by bank income and default losses, whereas capital requirements within Basel II are driven by rating transitions. The joint dynamics of these determine the necessary capital buffers, given a confidence level for regulatory capital adequacy chosen by bank management. We provide a tentative calibration of this confidence level to data on actual bank capital ratios, which enables a ceteris-paribus extrapolation of bank capital under the current regime to bank capital under Basel II.

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JEL classification: G21; G32
Keywords: Basel II; Pillar 2; Bank capital; Stress tests; Procyclicality

*Corresponding author.
E-mail addresses: samu.peura@sampo.fi (S. Peura), esa.jokivuolle@hkkk.fi (E. Jokivuolle).
1. Introduction

The macroeconomic consequences of rating sensitive capital requirements have been debated actively during the consultation process on the new Basel Capital Accord. The critics argue that new requirements may amplify the natural procyclicality in banking in that they force banks to significantly cut back lending in recessions (see the views expressed e.g. by Danielsson et al., 2001; Erwin and Wilde, 2001). Therefore risk sensitive capital requirements are thought to trade off greater efficiency in capital allocation across banks against macroeconomic stability.

The effects of the new capital regulation on macroeconomy are likely to depend on the extent to which individual banks find it optimal to hedge against the increased volatility of the minimum capital requirement. In particular, several authors have suggested that the problem of procyclicality resulting from risk sensitive capital requirements be remedied through adjustment of banks’ capital buffers (e.g. Borio et al., 2001; Lowe, 2002). It appears that this argumentation has also been adopted by the Basel Committee itself. The idea is that under ‘normal’ business conditions banks should hold capital over minimum requirements, while this extra capital would be consumed during severe downturns through credit losses and through increases in minimum capital requirements. If the capital buffers were sufficient to outlast a downturn, lending would not have to be severely cut down, and hence there would be no credit crunch accelerating the downturn.

Ideally the question of banks’ optimal reaction to Basel II and of the size of the required capital buffers should be analyzed based on an optimizing model of bank behaviour. There is by now a lot of theoretical research which shows that banks optimally hold buffer stocks of capital to protect against the adverse consequences of running out of capital (Estrella, 2001; Furfine, 2001; Hojgaard and Taksar, 1999; Milne and Robertson, 1996; Milne and Whalley, 2001; Peura and Keppo, 2003). This research utilizes stylized models where banks with illiquid portfolios optimize their capital levels subject to minimum capital or liquidity constraints. The research shows that the precautionary capital stocks are the larger the more severe are the financial constraints and the more illiquid or costly to hedge are bank assets. Some of these models have also been calibrated to data on actual bank returns, in an attempt to find out whether the models can explain observed bank capital ratios (Peura and Keppo, 2003). This raises the question of whether the same models could be used

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1 Basel Committee (2002a) states that ‘to help address potential concerns about the cyclicality of the IRB approaches, the Committee agreed that meaningfully conservative credit risk stress testing by banks should be a requirement under the IRB approaches as a means of ensuring that banks hold a sufficient capital buffer under Pillar 2 of the new Accord’.

2 Furfine (2001) also calibrates his model to bank data, but his goal is not to explain the level of bank capital holdings. He presents evidence that banks reacted to the current Basel Accord by increasing their capital ratios, which suggests that banks’ holdings of buffer capital are not purely ‘economic capital’ (in the standard sense of the term), but a genuine response to minimum capital requirements.
to forecast banks’ capital holding behaviour under the minimum capital requirements of Basel II. Unfortunately, the existing models in the literature do not appear very suitable for this task, because it is not at all clear how to represent the asymmetric return distributions of typical banking portfolios, or the essential structure of the current versus the proposed new minimum capital regulation, in these stylized theories.

In this paper, we approach the question of sufficient capital buffers and banks’ reaction to Basel II from a single bank’s risk management viewpoint. That is, we identify how much capital a bank needs (in a risk sensitive capital regime) in order to be protected, at a desired statistical confidence level, against shocks to its actual capital and to its minimum capital requirement. Following Jokivuolle and Peura (2001), the required capital buffer in our approach is therefore a solution to a value-at-risk type criterion associated with regulatory capital adequacy. The model is calibrated to actual bank capital ratios under the current Basel regime through the confidence level of the value-at-risk criterion. Using the calibrated value of the confidence level, we calculate the implied capital ratios that the model generates under the internal ratings based (IRB) approach of Basel II.

Our value-at-risk approach is not a fully optimizing model of bank behaviour, but allows us to maintain realism in the description of minimum capital rules and bank portfolios. We believe that specifically because of these properties, our model can also be a useful planning tool for bank practitioners.

Our model of bank capital dynamics is an extension of a credit risk model of the CreditMetrics\textsuperscript{TM}-type (J.P. Morgan, 1997). We jointly simulate a bank’s actual book capital and minimum capital requirement. Actual capital (in the absence of mark-to-market accounting) is driven by bank income and credit losses, whereas capital requirements are driven by ratings transitions. A credit portfolio model which generates rating changes and defaults, and incorporates the minimum capital formulas, is therefore suitable for keeping track of the evolution of both the actual and the minimum capital. Another aspect in which we extend a typical credit portfolio model is that we employ an underlying conditioning variable which represents the state of the business cycle. The business cycle variable follows a two state Markov process, and the ratings transition probabilities in our model are conditioned on the state of the business cycle. Moreover, our multi-period simulation is in quarterly increments. The quarterly simulation interval is non-standard in credit risk contexts, but is well grounded since most banks report their capital adequacy to their regulators quarterly. We use the conditional transition matrices as well as the transition probabilities for the business cycle variable reported by Bangia et al. (2002). Our portfolio data on average bank portfolios in the US is based on Gordy (2000).

Basel Committee (2002a) has suggested using stress tests to identify the size of the capital buffers that banks would need under the new Basel regime. The suggestion contained no details on how stress testing should be done. While a typical stress test would be a deterministic move in the portfolio ratings distribution (corresponding to some historical period of credit distress, as e.g. in Carpenter et al. (2001), Catarineu-Rabell et al. (2002) or Erwin and Wilde (2001)) we suggest stress
testing of credit risk through varying the key parameters in a stochastic simulation.\footnote{Bangia et al. (2002) and Carey (2002) stress test banks’ economic capital through varying key parameters in a stochastic simulation.} In particular, we formulate stress tests within our simulation framework around the transition probabilities of the business cycle variable. These control for the expected duration of recessions, and hence our stress tests have intuitive interpretations in terms of average recession lengths that our multi-period simulations correspond to.

The paper is organized as follows. Section 2 contains empirical evidence on bank capital ratios under the current Basel regime. Section 3 presents our capital simulation framework, extending the work presented in Jokivuolle and Peura (2001). Section 4 presents our multi-period ratings transition model with an underlying business cycle variable driven by a two state Markov process, and discusses the parameterization of the model. Section 5 contains our main results on the behaviour of bank capital buffers. Section 6 concludes.

2. Bank capital ratios under the current Basel regime

We provide in this section a brief summary on bank capital ratios under the current Basel regime.\footnote{For an analysis of bank capital dynamics, see e.g. Ayuso et al. (2002), which is based on Spanish panel data and finds evidence of procyclicality in bank capital ratios.} We use Bankscope data, and limit ourselves to large banks in G10 countries, defined as those banks which on average have Tier-1 capital in excess of 3 billion Euros over the period 1997–2001.\footnote{We have restricted ourselves to this group of banks because our bank portfolios, taken from Gordy (2000), are averages over large US banks.} This sample contains 128 banks (also included in this set are banks which do not have data on all the 5 years). Summary statistics of the bank level time-averages are presented in Table 1.

We note that the median total capital ratio across the G10 banks is 11.2%. In the US, the median total capital ratio is 11.9%, while in Japan and Europe the median ratio is below 11%. The median Tier-1 capital ratio across the G10 banks is 7.3%. Again, median Tier-1 capital ratio in the US is higher than the G10 average, but in Japan it is considerably lower than the G10 average. The lower capital ratios of Japanese banks may be explained by the ongoing banking crises in the country, which has resulted in capital ratios falling below their optimal or target values. That Japanese banks indeed have lower quality portfolios than US or European banks is evident from the loan loss provision statistics in Table 1. These reveal that the median annual loan loss provision, as a percentage of the loan portfolio, over the 1997–2001 period has equalled 1.9% in Japan, compared to 0.52% and 0.53% in the US and Europe, respectively. Hence the difference between capital levels of US and Japanese banks could be explained by the fact that US and European banks are close to their target capitalizations, while Japanese banks are below their target capitalization levels.
Table 1
Data on large G10 banks’ capital and portfolio 1997–2001

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>US</th>
<th>Europe</th>
<th>Japan</th>
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<tbody>
<tr>
<td>Total capital ratio (%)</td>
<td>Median 11.2</td>
<td>11.9</td>
<td>10.8</td>
<td>10.9</td>
</tr>
<tr>
<td></td>
<td>St. dev. 2.4</td>
<td>2.2</td>
<td>2.9</td>
<td>1.3</td>
</tr>
<tr>
<td>Tier-1 capital ratio (%)</td>
<td>Median 7.3</td>
<td>8.6</td>
<td>7.4</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>St. dev. 2.8</td>
<td>2.5</td>
<td>3.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Loan portfolio (bnEur)</td>
<td>Median 87</td>
<td>33</td>
<td>91</td>
<td>117</td>
</tr>
<tr>
<td></td>
<td>St. dev. 116</td>
<td>92</td>
<td>74</td>
<td>168</td>
</tr>
<tr>
<td>Loan portfolio/total assets (%)</td>
<td>Median 56</td>
<td>62</td>
<td>51</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>St. dev. 15</td>
<td>19</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>Loan loss provision/loans (%)</td>
<td>Median 0.67</td>
<td>0.52</td>
<td>0.53</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>St. dev. 0.91</td>
<td>0.53</td>
<td>0.35</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Number of banks 128 33 57 33

Source: Bankscope. Medians and standard deviations apply to bank level time-series averages over the sample period 1997–2001. Includes banks in G10 countries with average Tier-1 capital over 3bnEur over the sample period.

Table 1 indicates that the median large bank holds over 11% of capital, which is over 3% points in excess of the minimum requirement. There is some variation in capital ratios between banks, but no banks are close to the 8% minimum, and very few banks even have capital ratios under 10%. Fig. 1 shows the distribution of the capital ratios of the US banks in our sample. This sample contains 33 banks, and

Fig. 1. Distribution of large US banks' average total capital ratios 1997–2001.
here we observe that only one bank has a total capital ratio less than 10%. Moreover, over 45% of the banks have total capital ratios between 11% and 12%. The distribution of capital ratios is somewhat skewed to the right, but there are only few banks with capital ratios over 13%. We think of these facts as supporting two stylized views on bank capital holdings. First, all banks appear to prefer to hold considerable buffer capital, say at least 2% points, in excess of their minimum requirements. Second, the majority of large banks hold between 3% and 5% points of buffer capital.

The parameter data and the portfolios on which our numerical analyses are based are from the US. Therefore we base our calibrations in Section 5 on the median capital ratio of large US banks shown in Table 1, 11.9%. In Section 5 we will also discuss the extent to which the variation in the observed bank capital ratios might be explained by our model.

3. Framework for stress testing of capital requirements

In this section we extend the framework for capital adequacy simulations presented in Jokivuolle and Peura (2001). As discussed in the introduction, our solution to determining required buffer capital is a probabilistic value-at-risk type criterion which measures the sufficiency of actual bank capital against minimum capital requirements over a defined horizon. Our framework is an extension of a typical credit portfolio model, in that we simulate actual bank capital and minimum capital requirements simultaneously. Within Basel II these are both stochastic, and their joint dynamics determines the required initial capital, or equivalently, the required capital buffer, given a confidence level for capital adequacy chosen by bank management.

3.1. Analysis based on book capital

We develop the VaR criterion here from an accounting identity governing bank capital dynamics. Our definition of bank capital is the book equity which is eligible in calculations of bank capital adequacy. 6 We simulate shocks to this measure of book capital and to book capital requirements, rather than to the market value of bank equity. We find that it is imperative to perform an analysis of regulatory capital adequacy in book value terms, since banks’ regulatory capital requirements are imposed on an accounting measure of capital. Moreover, banking books are not marked to market under the current accounting standards, 7 so that changes in the market valuation of bank loans do not feed into banks’ income and book capital dynamics.

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6 Own funds eligible as Tier-1 capital include share capital, reserve funds and premium funds. Own funds eligible as Tier-2 capital include revaluation reserves and subordinated capital. The bank capital that we simulate should be interpreted as the total of own funds (Tier-1 plus Tier-2) eligible in capital adequacy calculations.

7 Most of the content of banking books will not be marked to market even after the introduction of the new IAS rules.
Actual credit losses are deducted from book equity, however, so that credit losses need to be simulated in order to reproduce a bank’s book capital dynamics. We also find that our approach is mainly consistent with general bank risk management practice, since value-at-risk analyses on illiquid bank loan portfolios are regularly performed on a nominal value basis, rather than in a mark-to-market mode.

3.2. Bank capital dynamics and required capital buffers

We imagine a bank with assets consisting of illiquid corporate loans. We let $E_t$ be time $t$ bank capital (ex time $t$ dividends) and $R_t$ be the bank’s regulatory capital charge at time $t$. The regulatory capital requirement that must hold at each point in time is

$$E_t \geq R_t. \tag{1}$$

We show in the following how the periodic regulatory capital requirement (1), when applied to a bank operating subject to certain capital market imperfections, leads banks to hold capital buffers over the minimum regulatory requirement. Our analysis is based on a simplified bank capital dynamics. We let $I_t$ be the bank’s profit before credit losses during period $t$, $L_t$ be the bank’s credit losses during period $t$, $D_t$ be the dividends paid out of the bank capital at time $t$, and $S_t$ be the issues of new equity at time $t$. The bank’s capital dynamics then satisfies (this is not the only possible decomposition of capital dynamics, but useful for our purposes)

$$E_{t+1} = E_t + I_{t+1} - L_{t+1} - D_{t+1} + S_{t+1}. \tag{2}$$

The bank operates subject to capital market imperfections which are likely to hold as strict constraints only in severe macroeconomic downturns, but such scenarios matter the most in our capital adequacy calculations, so that it is with little loss of generality that we assume the imperfections to hold in all scenarios. In particular, we assume that sales of the existing portfolio, as well as new issues of equity, are ruled out. Moreover, in conditions where capital is scarce, dividends are likely to be withdrawn and new assets are not likely to be bought, in order to minimize the burden on capital. Therefore we assume that both the $D$ and the $S$ terms in (2) will be zero, and that the capital dynamics in (2) corresponds to the initial (time 0) portfolio, so that the capital dynamics that we simulate is given by

$$E_{t+1} = E_t + I_{t+1} - L_{t+1}. \tag{3}$$

Rolling the difference equation (3) forward gives us time $t$ capital as

$$E_t = E_0 + \sum_{s=1}^{t} I_s - \sum_{s=1}^{t} L_s. \tag{4}$$

We define the bank’s time $t$ capital buffer $B_t$ as

$$B_t = E_t - R_t. \tag{5}$$

Substituting (4) into (5), and applying the inequality (1), gives us an expression of the regulatory capital requirement at time $t$:
Here the capital buffer at time $t$ is expressed in terms of the initial capital buffer, the inflows and outflows of capital between time 0 and time $t$, as well as the change in the regulatory capital charge $R$ between time $t$ and time 0. In particular, $R_t$ here is the capital charge associated with the bank’s initial portfolio, less the capital relief due to any expirations and defaults of assets up to time $t$, evaluated based on the time $t$ ratings of the assets in the portfolio.

The constraint (6) states that the bank’s initial capital buffer must cover for two stochastic elements, the cumulative net profit (profit before credit losses less credit losses) and the cumulative change in the minimum capital requirement. Given a fixed initial portfolio with maturity $T$, condition (6) is to be monitored at each time $t$ between time 0 and the maturity of the initial portfolio $T$. Because holding of buffer capital will carry an opportunity cost, requiring (6) to hold in all possible states of the world is likely to be uneconomical to the bank. Therefore we introduce a value-at-risk type probabilistic regulatory capital requirement,

$$P \left[ \min_{0 \leq t \leq T} B_t \geq 0 \right] \geq \alpha, \quad (7)$$

where $B_t$ is given by (6), and $\alpha$ is a confidence level associated with regulatory capital adequacy, such as 99%. Eq. (7) is a constraint on the initial capital buffer $B_0$ and because $B_t$ is increasing in $B_0$ we expect there to be a minimum value for $B_0$ which satisfies (7). The required initial capital buffer $\hat{B}_0$ is therefore a solution to

$$\hat{B}_0 = \inf \left\{ B_0 : P \left[ \min_{0 \leq t \leq T} B_t \geq 0 \right] \geq \alpha \right\}. \quad (8)$$

An additional constraint for capital adequacy, commonly referred to as ‘economic capital constraint’, is formulated in terms of the probability that the bank runs completely out of capital over the relevant planning horizon, i.e. becomes insolvent. Denoting the confidence level associated with solvency by $\beta$, the economic capital constraint (using our notation) becomes

$$P \left[ \min_{0 \leq t \leq T} E_t \geq 0 \right] \geq \beta \iff P \left[ \min_{0 \leq t \leq T} (B_t + R_t) \geq 0 \right] \geq \beta, \quad (9)$$

where the equivalence follows from applying the definition of the buffer (5). The minimum initial capital buffer satisfying the economic capital constraint (9) can be formulated analogously to (8) as

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8 One opportunity cost associated with equity is the tax advantage of debt. In case of banks, ‘excess equity’ may also have implications on liquidity creation. In particular, Diamond and Rajan (2000) proposed a theory of optimal bank capital structure which is based on a trade-off between the effects on liquidity creation and financial distress.
where $B_t$ is given by (6). Because $R_t \geq 0$, comparison of (8) and (10) shows that when $\alpha = \beta$, (10) never yields a higher initial capital buffer than (8). Therefore the new regulatory capital constraint (8) makes the standard economic capital constraint redundant when the confidence levels used in the criteria are equal. We would however expect the confidence level applied to regulatory capital adequacy, $\alpha$, to be lower than the confidence applied to solvency, $\beta$. Therefore the capital buffer solving (8) may not in all cases dominate the capital buffer solving (10), but for typical values of $\alpha$ and $\beta$ as well as for typical bank portfolios this will be the case. 9 We illustrate the regulatory and the economic capital constraints (8) and (10) in Fig. 2.

Assuming that (8) determines the capital buffer, initial bank capital will be $E_0 = R_0 + B_0$, and bank capital ratio can be expressed as

$$\left(1 + \frac{\widehat{B}_0}{R_0}\right) 8\%.$$  

\[\text{(11)}\]

3.3. Determination of $\alpha$

In our framework $\alpha$ is an exogenous choice parameter. In a fully optimizing model of the bank, this confidence level would be endogenously determined from

\[\text{Numerical simulations on our model reported in Section 5 confirm this.}\]
the trade-offs that influence the bank’s choice of capital (see e.g. Estrella, 2001; Milne and Whalley, 2001; or Peura and Keppo, 2003). $z$ would be influenced by factors such as the costs and penalties associated with a violation of the regulatory capital requirement (1), the capital market frictions that affect the recapitalization of the bank, the sensitivity of the bank’s funding cost to the amount of capital held by the bank, and the availability of growth options to the bank. Also regulator’s concerns regarding the viability of bank capitalization would be reflected in $z$ through the Pillar 2 of the New Basel Accord.

Within our reduced form framework, the parameter $z$ may be calibrated based on information on actual bank capital ratios. Given a bank’s portfolio and its capital ratio, there is an implied value of $z$ which makes the capital ratio solved from the model equal to the observed capital ratio. We perform this type of calibration of $z$ in Section 5.

4. Rating transition model with business cycle dynamics

The model of the previous section implies that in order to solve the capital buffer from (8), the dynamics of a bank’s capital buffer must be simulated according to (6). The dynamics in (6) depends on the model of rating transitions used. Rating transitions determine the evolution of the minimum capital charge. Defaults, which determine the evolution of actual bank capital, are just special cases of rating transitions. This section presents the rating transition model on which our simulations are based on, describes the parameterization of this model, and discusses the business cycle scenarios that we use in our simulations.

4.1. Rating, income, and credit loss dynamics

Our model of rating transitions is a one-factor version of the CreditMetrics framework (J.P. Morgan, 1997), extended with an underlying conditioning variable which is interpreted as business cycle state. The one-factor CreditMetrics model is based on the Mertonian model of default and has been carefully described e.g. in Gordy (2000). The CreditMetrics model takes the transition probability matrix of ratings as given, which in our model is determined by the business cycle state. In particular, we assume that the business cycle variable is a two-state, time homogenous, Markov Chain, whose possible states are referred to as ‘expansion’ and ‘recession’. The transition matrix associated with the recession state is expected to display higher volatility in rating changes, and higher default probabilities, than is the transition matrix associated with the expansion state. Models of ratings dynamics of this type have been suggested by Bangia et al. (2002). Fig. 3 illustrates the rating transition model.

10 Nickell et al. (2000) and Amato and Furfine (2003) have empirically studied the effects of the business cycle on credit ratings.
Next we will explain how the evolution of ratings in our model determines the minimum capital requirement, bank income, and credit losses. We denote by $P_u$ the unconditional rating transition matrix, and we denote by $p_u(k)$ the unconditional default probability corresponding to rating $k$. Now we can express the bank level variables $R_t$, $I_t$, $L_t$ defined in Section 3 in terms of the following sums over obligors in the bank’s portfolio:

$$R_t = \sum_{i=1}^{#} f(p_u(k_{i,t})) N_i(1 - D_{i,t}),$$  \hspace{1cm} (12)

$$I_t = \sum_{i=1}^{#} \theta \gamma p_u(k_{i,t}) N_i(1 - D_{i,t}),$$  \hspace{1cm} (13)

$$L_t = \sum_{i=1}^{#} \gamma N_i(D_{i,t} - D_{i,t-1}).$$  \hspace{1cm} (14)

Here $f(\cdot)$ is the capital charge formula, $^1_1$ which takes as an argument the (unconditional) default probability, $k_{i,t}$ is the rating of obligor $i$ at time $t$, $N_i$ is the nominal exposure of obligor $i$, $\theta$ is a parameter which indicates the ratio of the nominal loan margin to the expected loss rate (the unconditional default probability times the

$^1_1$ The IRB capital charge formula from October 2002 is described in Basel Committee (2002b). The capital charge also contains a component for operational risks which is not present in our analysis. As far as the operational risk charge has low volatility relative to the credit risks charge, its presence will not influence the size of the required capital buffer significantly.
LGD percentage) in the portfolio, $\gamma$ is the LGD to nominal exposure ratio, $D_{t,i}$ is a stochastic process which equals 1 if obligor $i$ is in default at time $t$ and 0 otherwise, and $#$ is the number of obligors in the bank’s portfolio at time 0. Formula (13) implies that the bank earns income as a fixed multiple of its unconditional expected loss rate ($\bar{\gamma} p^u N$).\footnote{The parameterization for bank income in (13) implies that risk premium on credit risk is proportional to the amount of risk, as measured by expected loss, $\theta - 1$ being the proportionality factor.}

We assume that the underlying asset value correlations, which together with the transition probabilities determine the rating transition correlations, do not depend on the state of the business cycle. Consistent with industry standards and with the IRB capital charge formula,\footnote{In the October 2002 version of the IRB rules, the asset correlation on which the IRB capital charge is based on depends on the default probability of the counterparty, varying between 12% and 24%.} we use 20% asset correlation across all counterparties.

We perform multi-period simulations of rating changes in quarterly time increments. Credit portfolio models are typically implemented as one-period simulations with an annual horizon, but we find the quarterly time interval justified because banks in most countries report their capital adequacy to their regulators quarterly. Both the rating transition probabilities and the regime transition probabilities that we use are quarterly probabilities estimated based on US data. The conditional transition matrices for the expansion and the recession states we use are from Table 5.1 in Bangia et al. (2002), which are based on Standard and Poor’s data on US corporate ratings over the period 1981–1998. The regime switching matrix for the underlying business cycle variable that we use is from Table 5.2 in Bangia et al. (2002). We represent this matrix in the upper part of Table 2. The regime switching probabilities have been estimated from quarterly data on US business cycles over 1959–1998, as classified by the NBER. The stationary distribution of the business cycle state implied by this transition matrix is (79%, 21%), implying that roughly every fifth quarter is classified as recession. However, following a quarter that has been classified as recession, over 40% of the times the next quarter will be a recession, resulting in an expected recession length of 1.74 quarters. In other words the business cycle state is slightly positively autocorrelated, and our framework enables us to study the consequences of this autocorrelation on bank capital adequacy in recessions.

4.2. Business cycle scenarios

Here we introduce the business cycle scenarios on which our capital adequacy calculations are based on. Our multi-period model with embedded business cycle dynamics allows us to calculate bank capital requirements under various assumptions concerning the initial business cycle state as well as the duration of recessions.

Our unconditional simulations assume that the initial (time 0) business cycle state is selected randomly from the stationary distribution of the business cycle variable, following which the business cycle transitions obey the historical regime switching matrix in the upper part of Table 2. Our stress scenarios, on the contrary, always assume that the initial business cycle state is a recession. In the basic scenario termed
recession’, the business cycle variable evolves from the initial state according to the historical switching probabilities in the upper part of Table 2. In this scenario the expected length of a recession is 1.74 quarters. In our second scenario termed ‘long recession’ we assume that the expected length of a recession is 4 quarters. We achieve this by changing the probability of remaining in the recession state \( q_{rr} \) in terms of the notation in Fig. 3) appropriately. From the theory of first-order Markov chains, the expected duration of a recession is given by \( 1/(1 - q_{rr}) \). Then a targeted expected duration of recessions, denoted \( d \), is obtained by setting \( q_{rr} = 1 - 1/d \). The transition matrix generated in this manner is presented in the middle part of Table 2. In our third scenario termed ‘prolonged recession’, we assume that the expected length of a recession is 8 quarters, and again obtain this by changing the probability of remaining in the recession state. The resulting transition matrix is shown in the lower part of Table 2. Table 2 also shows the first-order autocorrelations of the business cycle variable in the three scenarios, which are 27%, 60% and 72%, respectively.

### Table 2

<table>
<thead>
<tr>
<th>Regime transition matrices</th>
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<tbody>
<tr>
<td><strong>Initial state</strong></td>
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<td>-----------------</td>
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<tr>
<td><strong>Recession’ scenario: Expected recession length 1.74 quarters</strong></td>
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<td>Expansion</td>
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<td>Recession</td>
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<td><strong>Long recession’ scenario: Expected recession length 4 quarters</strong></td>
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<td><strong>Prolonged recession’ scenario: Expected recession length 8 quarters</strong></td>
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‘recession’, the business cycle variable evolves from the initial state according to the historical switching probabilities in the upper part of Table 2. In this scenario the expected length of a recession is 1.74 quarters. In our second scenario termed ‘long recession’ we assume that the expected length of a recession is 4 quarters. We achieve this by changing the probability of remaining in the recession state \( q_{rr} \) in terms of the notation in Fig. 3) appropriately. From the theory of first-order Markov chains, the expected duration of a recession is given by \( 1/(1 - q_{rr}) \). Then a targeted expected duration of recessions, denoted \( d \), is obtained by setting \( q_{rr} = 1 - 1/d \). The transition matrix generated in this manner is presented in the middle part of Table 2. In our third scenario termed ‘prolonged recession’, we assume that the expected length of a recession is 8 quarters, and again obtain this by changing the probability of remaining in the recession state. The resulting transition matrix is shown in the lower part of Table 2. Table 2 also shows the first-order autocorrelations of the business cycle variable in the three scenarios, which are 27%, 60% and 72%, respectively.

### 4.3. Average bank portfolios

We take representative portfolios of US banks from a Federal Reserve Board survey as reported by Gordy (2000). The portfolios are reported in Table 3. Our results are based on two different quality distributions, referred to as ‘average quality’ and ‘high quality’. The table also shows the annual unconditional default probabilities, calculated as annual default frequencies based on the same data as the conditional transition matrices. These annual default probabilities are used in the Basel risk weight formulas.

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14 According to NBER classification, the US economy last time experienced a recession exceeding 8 quarters in length during the great depression. The recessions in the mid 1970s and in the early 1980s lasted for 16 months, and are therefore situated in-between our ‘long recession’ and ‘prolonged recession’ scenarios in terms of length.
For our numerical analysis, we form portfolios according to the given quality distributions that each have 500 equal sized loans. The loans are ex ante identical in all other respects except the initial obligor rating. We assume that all loans are bullet loans with a maturity of $T$ years ($T$ is also the simulation horizon, a parameter of the framework presented in Section 3). We normalize nominal exposure for each portfolio to 100. As discussed in Section 3, all our simulations are based on the assumption that no new assets are bought, and no existing assets are sold, over the simulation horizon. This assumption is not descriptive of actual bank portfolio dynamics. However, the size of the capital buffers is determined by portfolio dynamics over multi-period recessions, and in such circumstances the assumption of no new business is likely to be much closer to reality.

5. Results

The plan for presenting our numerical results is the following. We first discuss the capital buffers implied by the regulatory capital criterion (8), and verify that these typically dominate the capital buffers implied by the economic capital criterion (10). We then perform a calibration of the parameter $\alpha$ to the data on actual bank capital presented in Section 2, which allows us to generate a ceteris paribus prediction on how capital buffers are likely to change as banks move to the IRB regime within Basel II. Towards the end of this section, we compare our value-at-risk criterion which is based on periodic monitoring of the minimum capital requirement with the value-at-risk criterion applied in Jokivuolle and Peura (2001) where capital adequacy is only monitored at a terminal date. We also contrast the results from our stochastic framework with the results obtained from deterministic scenarios, as applied e.g. by Erwin and Wilde (2001) and Catarineu-Rabell et al. (2002).

5.1. Capital buffers

We present our main comparison on capital ratios under the current Basel regime and under the IRB approach within Basel II subject to the following base case
parameters: portfolio maturity $T$ equal to 2.5 years, a bank income (before credit losses) equal to the unconditional expected credit loss ($\theta = 1$, see formula (13)), an LGD of 45% across all obligors ($\gamma = 0.45$, see formulas (13) and (14)), and a confidence level $\alpha$ of 99%. The average maturity of 2.5 years is the default assumption underlying the Basel II IRB risk weight formulas from October 2002, as is the 45% LGD assumption. The assumption that bank income just covers expected losses is conservative, not so much because actual margin income is always higher, but because of other (fee) income that banks typically accrue through loan sales. The choice of 99% for $\alpha$ is supported by the results in the next subsection where we perform a tentative calibration of this parameter to empirical data. Table 4 contains our main results.

Table 4 points to several conclusions. As for the effects of the new Basel regime, we observe that although the minimum credit risk capital requirement will be reduced for both high quality and average quality portfolios, this capital relief will partially be countered by increases in capital buffers. A high quality portfolio’s minimum capital requirement is reduced by 50% (($8.0 - 4.0)/8.0$) going from the current regime into the IRB regime, while its total capital satisfying the regulatory capital criterion (8) will be reduced only by 26% (($9.7 - 7.2)/9.7$), based on the results of the unconditional scenarios. The corresponding reductions for the average quality portfolio are 18% and 4%, respectively. Therefore the total credit risk capital of a bank with an average quality portfolio, according to our calculations, remains effectively unchanged over the transition into the IRB regime (this is prior to any operational risk charge). We have drawn in Fig. 4 the minimum capital and the capital buffers in the two Basel regimes, both for the high and the average quality portfolio, in order to illustrate this conclusion.

Our results indicate that in the IRB regime, the capital buffer for high quality portfolios is likely to be close to the same order of magnitude as the minimum capital requirement alone. The capital buffer for an average portfolio will be roughly two thirds of the minimum capital charge. In other words, high quality portfolios in the IRB regime will have the highest relative capital buffers, i.e. the buffer as a proportion of the minimum requirement. This is demonstrated by the capital ratios in Table 4, which show that the capital ratio in the IRB regime decreases as portfolio quality deteriorates, while in the current regime the reverse happens.

In order to understand this result which appears counterintuitive, we note that in the current regime realized credit losses only generate shocks to bank capital buffers (given a fixed initial portfolio). As the volatility of credit losses increases as portfolio quality decreases, the current regime necessitates higher buffers for lower quality portfolios (both in absolute and in relative terms, since the minimum capital does not depend on portfolio quality). Under the IRB approach, on the other hand, shocks to capital buffers caused by rating migrations are typically more dominant than are shocks caused by actual defaults. The IRB minimum capital requirement

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15 As a point of reference, the median US total capital ratio of 11.9% corresponds to a median capital buffer equal to one half of the minimum requirement under the current Basel regime.
<table>
<thead>
<tr>
<th>Portfolio quality</th>
<th>Business cycle scenario</th>
<th>Current Basel regime</th>
<th></th>
<th></th>
<th>Basel II IRB approach</th>
<th></th>
<th></th>
<th></th>
<th>Minimum</th>
<th>Minimum +</th>
<th>Economic</th>
<th>Capital</th>
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<th>Minimum +</th>
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<th>Capital</th>
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<tr>
<td></td>
<td></td>
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<td>Minimum + buffer</td>
<td>Economic capital</td>
<td>Capital ratio (%)</td>
<td>Minimum capital</td>
<td>Minimum + buffer</td>
<td>Economic capital</td>
<td>Capital ratio (%)</td>
<td>Minimum capital</td>
<td>Minimum + buffer</td>
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<td></td>
<td>Prolonged recession</td>
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</tbody>
</table>

Capital buffer has been calculated from (8) and capital ratio has been calculated from (11). Economic capital has been calculated from (10). Economic capital is in all cases less than Minimum capital + capital buffer. $T = 2.5$ years, $\theta = 1$, $\alpha = 99\%$, $\beta = 99.95\%$. Portfolio nominal exposure is 100.
for low rated credits is of the same order of magnitude as is the actual credit loss in the event of a default, so that the net effect of a default by a low rated asset on a bank’s capital buffer is smaller in the IRB regime than in the current regime. Capital buffer volatility in the IRB regime will therefore be mainly driven by ratings transitions. In high quality portfolios most rating transitions will be downgrades, implying the need for relatively high capital buffers. In low quality portfolios there will be proportionally more upgrades which generate positive shocks to buffer capital. This decreases the relative size of capital buffers, and consequently capital ratios, of low quality portfolios under the IRB regime. In fact, it turns out that for sufficiently low-quality portfolios the size of the capital buffer, both in absolute and in relative terms, is actually less in the IRB regime than under the current Basel regime (Table 4 does not contain these numbers, but numerical examples can be found in Jokivuolle and Peura, 2001).

As to the relation between regulatory and economic capital, we observe that the minimum capital satisfying the dynamic regulatory capital constraint (8) in all the cases in Table 4 is higher than the minimum capital satisfying the economic capital constraint (10). We have calculated the economic capital based on a standard AA confidence level \( \beta = 99.95\% \), but over a non-standard 2.5 year horizon. Typically this confidence level is applied in economic capital calculations over 1 year horizon, making our calculations overly conservative from the perspective of an AA target. However, this conservatism only reinforces our conclusion that the economic capital solving (10) is dominated by the minimum capital solving the dynamic regulatory capital constraint (8).
We note that our previous discussion on capital ratios under Basel II assumes that the confidence level $\alpha$ is equal to 99% and, more importantly, that this value remains unchanged through the changeover into Basel II. Therefore the entire discussion is based on a \textit{ceteris paribus} extrapolation of current bank capital holdings into bank capital holdings under Basel II.

5.2. Calibration of $\alpha$

As is evident from Table 4, the choice of the business cycle scenario has an economically significant impact on the resulting capital buffer. Because we have no data on banks from which to identify the true scenario that banks would use in their capital adequacy analysis, we suggest that the scenario be selected jointly with the value of the confidence level so that these choices together yield capital ratios consistent with empirical evidence. We perform a simple calibration along these lines in this subsection.

The calibration procedure is the following. We take the observed median capital ratio of US banks, 11.9%, from Table 1 as the ratio against which we match our model ratios. The portfolio that we use is the average portfolio of large US banks, reported in Table 3. Then we iterate on $\alpha$ until we find a value that yields a capital ratio, calculated from (8) and (11), sufficiently close to the observed value. We repeat this for each of the business cycle scenarios, which results in a set of four calibrated (scenario, $\alpha$) pairs. The results are shown in Table 5, where the grey line indicates the value of $\alpha$ which, for any given scenario, yields a model capital ratio closest to the observed value.

Table 5 can be interpreted to indicate the implied confidence level as a function of the assumed scenario. If the average US bank with a capital ratio of 11.9% used a ‘recession’ scenario in its capital adequacy analysis, its capital ratio were consistent with a confidence level of approximately 99.3% associated with regulatory capital

<table>
<thead>
<tr>
<th>$\alpha$ (%)</th>
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</tr>
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<td>99.97</td>
<td>13.8</td>
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</table>

Capital ratios are calculated from (8) and (11). The grey line indicates the value of $\alpha$ which yields a capital ratio closest to the empirically observed value 11.9%. Average quality portfolio, $T = 2.5$ years, $\theta = 1$. 
adequacy. If the average bank used a ‘long recession’ type scenario in its capital adequacy analysis, the implied confidence level were 98.4%, and if the average bank used a ‘prolonged recession’ scenario, the implied confidence level would be 97.2%. Therefore the $\alpha$ value of 99% is roughly consistent with the median bank using something between the ‘recession’ and the ‘long recession’ scenario in its capital simulations. This suggests that the relevant scenario in Table 4 for the median bank is ‘recession’, or perhaps ‘long recession’. If we formulate our ceteris paribus extrapolation of bank capital under Basel II around these scenarios, we observe that the average US bank which now holds 11.9% capital, would have practically the same (absolute) amount of capital under the IRB regime, but an increased capital ratio of roughly 14%, owing to the somewhat lower minimum capital charge. This is prior to consideration of any operational risk charge, which will add to the minimum capital, but is unlikely to influence the absolute size of the buffer.

Table 5 also suggests that not all of the variation observed in capital ratios across banks in Fig. 1 can be attributed to cross-sectional differences in $\alpha$. In particular, the dataset contains some banks with capital ratios in excess of 15%, while Table 5 indicates that exceptionally high confidence levels are required to replicate these capital ratios, given an average bank portfolio. The high observed capital ratios of some banks are likely to be due to higher than average portfolio risk, different business portfolio structure (banks with investment banking arms typically hold more capital than pure commercial banks), or the banks’ growth objectives, which would imply that our ‘no new business’ assumption does not hold. Moreover, some of the observed variation in capital ratios across banks may be due to time-series variation in individual banks’ capital ratios, reflecting the fact that banks are in different phases of their individual credit loss cycles. Time-series variation however is not likely to be the main explanation for the observed heterogeneity in capital ratios.16

Our calibration procedure is subject to many parameter uncertainties, all of which are potentially reflected in the resulting estimate for $\alpha$. If e.g. the underlying asset correlations were increased, the value of $\alpha$ which would produce the empirically observed capital ratios would decline. We acknowledge this, and think of our calibration exercise as an attempt to identify a confidence level which is consistent with the other parameterization of the model. One could in fact suggest that $\alpha$ is the calibrated parameter only because the least of it is known a priori. Also, some of the concerns with the mismeasurement of $\alpha$ may not matter so much from the point of view of our target, which is to use the implied value of $\alpha$ to generate a prediction on bank capital ratios under the forthcoming Basel II. Suppose that we make a systematic error in parameterizing the portfolio model, which results in a biased estimate of implied $\alpha$. As we would be using the same biased parameter estimates as well as the same biased $\alpha$ to calculate capital ratios under Basel II, the effects of the biases would likely be offsetting each other.

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16 The time-series variation in individual banks’ capital ratios that we observe in the US data over 1997–2001 (median of bank level capital ratio standard deviations is 0.65%) is quite insignificant relative to the cross-sectional variation of capital ratios in the data (standard deviation of capital ratios across banks is 2.2%).
5.3. The effects of periodic monitoring and bank income

The framework in this paper extends Jokivuolle and Peura (2001) by assuming that the minimum capital constraint is monitored periodically, and by incorporating bank income into bank capital dynamics. This subsection illustrates the effects of these extensions.

The level of the bank income in our model is measured by the parameter $\theta$ in formula (13). High bank income can act as a partial hedge against credit losses and net increases in capital requirements. It is unlikely, however, that an income flow can completely substitute for the need to hold buffer capital, since income by definition is a flow quantity which in the short term is likely to be dominated by unexpected credit losses and, under the IRB regime, also by changes in the minimum capital requirement. Fig. 5 illustrates the behaviour of the capital ratio as a function of $\theta$, for the average quality portfolio (so that portfolio risk is kept fixed while the income multiple is allowed to change). The other parameters are set equal to their base case values, so the capital ratios here are comparable to those in Table 4.

Fig. 5 shows that capital ratio is a declining function of the level of bank income, and slowly approaches the 8% minimum as the income-to-credit risk multiple increases. The capital ratios associated with the value $\theta = 1$ correspond to the capital ratios of the average quality portfolio under the unconditional scenarios in Table 4, 11.1% and 13.0%, respectively.

Jokivuolle and Peura (2001) solve bank capital buffers in a single-period setting, where the value-at-risk criterion only applies at the terminal simulation date. In our multi-period setting, this criterion corresponds to evaluating capital adequacy only at the terminal date $T$. Using our notation from Section 3, this criterion becomes
Banks report their capital adequacy to their regulators in most countries quarterly. Therefore our quarterly simulation period fits well into this institutional practice. In a multi-quarter analysis (i.e. one where the terminal date extends to many quarters), monitoring of capital adequacy at the terminal date only will result in a downward bias in the estimated probability of capital adequacy violation. Therefore the capital buffer solving (15) is a lower bound on the capital buffer that solves (8), given a common confidence level $\alpha$.

We illustrate here the magnitude of the error which results from applying the terminal value-at-risk criterion (15) instead of the periodic value-at-risk criterion (8). Table 6 shows capital ratios calculated both under periodic monitoring (8) and terminal monitoring (15), as a function of portfolio maturity $T$ and the level of bank income $h$. Also shown is the difference between these capital ratios.

Table 6 indicates that the downward bias in the capital buffer resulting from terminal monitoring is increasing in the level of bank income (given constant portfolio risk). Sufficiently high income flow causes bank capital buffer to drift upwards, so that the terminal distribution of the capital buffer will stochastically dominate the distribution of the capital buffer at some earlier points of time. When the drift of the capital buffer is positive, this effect also increases with maturity.

5.4. Comparison to deterministic scenarios

The current paper, together with Jokivuolle and Peura (2001), is to the best of our knowledge the first study to apply a probabilistic simulation based approach to stress testing capital adequacy within Basel II. Some previous studies have presented deterministic stress tests of capital adequacy under the IRB regime, most notably Catani-neu-Rabell et al. (2002), and Erwin and Wilde (2001). In these papers, the basic idea is to take realized rating transition frequencies, including default frequencies, from a period corresponding to a particularly adverse ratings development, and to apply these transition frequencies to a given initial rating distribution. This yields a

\[
\widehat{B}_0 = \inf \{B_0 : P[B_T \geq 0] \geq \alpha\}.
\]
hypothetical end-of-period ratings distribution for the initial portfolio, and the capital requirement of this stressed portfolio is then compared to the capital requirement of the initial portfolio. The change (increase) in the capital requirement is used as a measure of the required capital buffer.

The deterministic approach to stress testing bank capital is reminiscent of historical simulation approaches to risk measurement in market risk contexts. The appeal of this approach is in the use of adverse rating transitions from a period that actually took place in history. The obvious shortcoming of such an analysis is the absence of any probabilities attached to the chosen rating transition scenario. In other words, this approach to stress testing of capital adequacy does not yield a complete description of bank capital dynamics, unlike our stochastic analysis which yields a well-defined stochastic process for bank capital. Moreover, the deterministic approach may yield misleading results if the granularity of the bank’s portfolio is significantly different from the granularity of the portfolio which has generated the transition matrix.

6. Conclusions

We have presented a framework for capital adequacy analysis which is based on simulating the difference between a bank’s actual (book) capital and its minimum (book) capital requirement, i.e. the bank’s capital buffer. The framework is an extension of a typical value-at-risk analysis applied to bank credit portfolios. Our framework is entirely probabilistic and parameterized with obligor level data on bank portfolios. As such we believe the framework is well suited for measuring and stress testing bank capital adequacy, a task which the Basel Committee (2002a) has proposed as an additional requirement to banks under the Pillar 2 of the Basel II regime.

The aim of stress testing bank capital adequacy is to determine the sufficiency, over a business cycle, of bank’s free capital buffer. Given serious illiquidity of bank lending portfolios and the capital market imperfections which are particularly severe in economic downturns, hedging through holding buffer capital is widely seen as instrumental in curbing the procyclical effects of risk sensitive minimum capital requirements. Our framework is a quantitative tool for generating an estimate of the initial buffer capital required for the sufficiency of bank capital over a downturn.

Our analysis has been mainly illustrative in nature. For example, there is empirical evidence that realized LGD varies countercyclically (Allen and Saunders, 2002; Altman et al., 2002), while our simulations have been based on a constant LGD assumption. Incorporating the systematic variation in the LGD into the model would lead to an increase in the resulting capital buffers. Also we have assumed a quite simplistic form for the bank’s income flow. A practitioner interested in applying our framework could enrich these aspects of the model within the context of the Monte Carlo simulation. The simulation framework e.g. allows one to vary the specification of the stochastic profit flow subject to the constraint that it be driven by the factors of the portfolio model, as well as by ratings changes and defaults of individual obligors. Also the planned growth of the bank’s portfolio could be accounted for. As a part of this exercise, one would like to model the natural procyclicality in bank
lending growth rates, perhaps as a function of the evolution of the obligor ratings distribution. In general, we believe this type of bank capital simulation is a natural way of complementing currently commonplace economic capital calculations, which are based on conventional credit value-at-risk models, but fail to account for regulatory capital constraints and fall short of explaining observed levels of bank capitalization.

An important point illustrated by Catarineu-Rabell et al. (2002) is that a bank’s chosen internal rating methodology has an impact on its regulatory capital volatility, and therefore on its needed capital buffers. Catarineu-Rabell et al. compare a stock market based rating system (such as KMV-Moody’s EDF) to traditional ‘over-the-cycle’ agency ratings in terms of their implied capital buffers in a deterministic setting. In this paper, we have used transition matrices of agency ratings, implying that our analysis applies to a bank whose portfolio is mainly agency rated, or correspondingly internally rated using a system analogous to agency ratings in terms of the volatility of rating changes. However, analysis in our framework can also be based on a ‘point-in-time’ stock market based rating system.

Our numerical results indicate that the introduction of rating sensitive capital requirements will necessitate higher bank capital buffers than are currently observed, at least for high and average quality bank portfolios, because of the increased volatility of the minimum capital requirement. The increase in capital buffers will consume up to 50% of the savings in minimum capital requirements for high quality portfolios, and basically all the savings in minimum capital requirements for average quality portfolios. This happens prior to considering the additional operational risk charge which will directly feed to the minimum requirement, but is not likely to significantly influence the size of the buffer.

Acknowledgements

We have received valuable comments from two anonymous referees, Heikki Koskenkylä, Tuomas Takalo and seminar participants at the Bank of Finland, Helsinki School of Economics, Swedish School of Economics, University of Antwerp, and the Capital Allocation 2002 Europe seminar. We thank Janne Villanen for excellent research assistance. Esa Jokivuolle thanks Helsingin Kauppakorkeakoulun Tukisäätiö for financial support.

References


