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Should SME exposures be treated as retail or corporate exposures? A comparative analysis of default probabilities and asset correlations in French and German SMEs

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Abstract

We use a one-factor credit risk model to provide new estimates of stationary default probabilities and asset correlations in two large samples of French and German Small and Medium-sized Enterprises. Results show that, on average, SMEs are riskier than large businesses; and the asset correlations in the SME population are very weak (1–3% on average) and decrease with size. On average, the relationship between PDs and asset correlations is not negative, as assumed by Basel II, but positive, especially at the industry level, in the two countries. It is also possible to distinguish different segments inside the SMEs' population: at least between very small and small SMEs and large SMEs.

JEL classification: G21; G28
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1. Introduction

Default correlation is an important determinant of the distribution of losses in a bank loan portfolio. Capturing the correlations between individual exposures is crucial in order to assess the risk at the portfolio level. In most of the credit risk models, the correlations measure the degree of sensitivity of the probability of default (PD) to

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the systematic risk factors that represent the influence of the "state of the economy". Portfolio risk will be greater the more the bank loans tend to vary simultaneously in reaction to the realization of these risk factors. Hence, a crucial element in the estimation of loan loss distribution is a good calibration of parameters – the probabilities of default and their variance – which determine asset correlations.

One objective of the modified risk weights formulas recently proposed by the Basel Committee (BIS, 2002, 2003) was to provide an appropriate treatment of exposures to small and medium-size enterprises (SMEs). Indeed, many comments on the first drafts of the New Basel Accord pointed out a calibration problem for SMEs' credit risk. Most of the criticisms argued that the risk-weight curve was too steep and too high, which induced too-high risk-weights for most of the SMEs, because these firms are generally characterized by relatively high probabilities of default, as compared with large businesses. These criticisms were founded on the fear that too large capital charges for SMEs could lead to credit rationing of small firms and given the importance of these firms in the economy – in France, SMEs account for 60% of total employment – could reduce economic growth. To reply to these criticisms, the Basel Committee (BIS, 2002, 2003) introduced two main changes in the calibration of risk-weight formulas in order to reduce the risk weight on SMEs' exposures. The first one was to propose different risk-weight functions for SMEs and large businesses. Indeed, the committee introduced an adjustment in the risk-weight formula for firms with turnover between €5 and €50 million. More precisely, the correlation formula is adjusted by a term that reduces the value of the correlation proportionately to the size of the firm. In addition, banks are allowed to apply the more favorable retail risk-weight formula to very small businesses (with turnover between €1 and €5 million), provided that the bank's total exposure to any one firm remains below €1 million. The second change was to assume that asset correlations decline with the probabilities of default (PDs). The expected consequence of this amendment is to smooth capital charges for risky small businesses in a recession period, because PDs of the latter tend to increase in such a period, these firms being assumed to be more sensitive to a downturn in macroeconomic conditions than larger firms. However, if this assumed negative relationship between PD and asset correlations is not verified, this amendment could induce too high capital charges for the less risky SMEs, so that the less risky firms would "pay" for the more risky firms.

Despite the importance of asset correlations in credit risk modeling in general and in the calibration of the risk weight formulas of the New Basel Accord in particular, there are few attempts in the literature to compute asset correlations. Lopez (2002) used a KMV type structural model and equity markets data and provided estimates for large corporate or quoted firms. Gordy and Heitfield (2002) and de Servigny and Renault (2002) also used rating agencies' grades and provided estimates for large businesses. The latter address the issue of the consistency of equity correlation as a good proxy for asset correlation. To our knowledge, only Duellmann and Scheule (2003) and RMA (2003) give estimates for other categories of businesses, especially for SMEs. Yet there seems to be a discrepancy between the assumed consequences of macroeconomic risk on the loss distribution in loan portfolios and the lack of estimates of the default correlations for a large part of the population of businesses.

The aim of this paper is to provide new empirical evidence on asset correlations in two very large samples of French and German SMEs by using default data from two main European financial information providers and a single-factor risk model. Our estimation of correlations from default data is more in line with the book-value approach of the management of bank loans than with the previously mentioned approach using structural models and equity markets data.

The paper is organized as follows. Section 2 presents the one-factor model we used to compute the asset correlations. Section 3 presents the data and Section 4 the results of the computation of asset correlations. Section 5 analyzes the relationship between default probabilities and asset correlations. Section 6 concludes.

2. The one-factor model and the computation of asset correlations

We computed the asset correlations by using the one systematic factor probit ordered model (Gordy, 2000; Dietsch and Petey, 2002). In this model, the end of period borrower's i state (default or not) is driven by an unobserved latent random variable U_i, which is defined as a function of a single systematic factor x and a specific idiosyncratic factor ε_i:

$$U_i = wx + \sqrt{1 - w^2}\varepsilon_i \tag{1}$$

where x and ε_i are standard normal and $E[x\varepsilon_i] = 0$. The state of the borrower at the end of the planning horizon depends on the location of the latent variable relative to a "cut-off" value, which defines default. If the latent variable is a standard normal variable, the default cut-off value is simply equal to $\Phi^{-1}(\overline{p})$, where $\Phi^{-1}(\cdot)$ is the inverse standard normal CDF and $\overline{p}$ is the unconditional (long-term) PD for a given rating. Consequently, a borrower is in default when

$$wx + \sqrt{1 - w^2}\varepsilon_i < \Phi^{-1}(\overline{p}). \tag{2}$$

If the realization of the systematic factor is good, the firm will default only if the realization of the specific idiosyncratic factor is worse. The systematic factor represents the state of the economy. It measures the effect of the business cycle on the default rate. Hence, the weighting w of the systematic risk factor measures the sensitivity of borrower i to the evolutions of the general economic conditions. For two borrowers i and j with the same rating grade (or belonging to the same risk class), the (non-conditional) covariance between latent variables is

$$\mathrm{Cov}[U_i; U_j] = w^2. \tag{3}$$

As w increases, all borrowers tend to be more correlated; as w decreases, idiosyncratic risk prevails. Therefore, the degree of correlation between defaults is determined by the sensitivity of the latent variables to the systematic factor.

Therefore, correlations between latent variables are due to the existence of aggregate shocks in the economy. In addition, this correlation determines the shape of the end-of-period value distribution of the portfolio. [1] The one latent factor model shares its structure with the basic framework of Merton (1974) but relies on an indirect approach of estimation, exploiting the dynamics of default rates conditional on ratings classes. In this approach, the current rating is taken to be a sufficient statistic for some measure of credit quality, assuming that firms in the same rating class share the same distance to default. Extending the analogy with Merton's approach, the sensitivity w is called asset values correlation (AVC) or asset correlation. This reflects the economic intuition that, for given levels of debt, defaults will tend to be concentrated in time if the borrowers' asset values are positively correlated.

The default condition (2) allows us to compute $p(x)$, the individual PD conditional on the realization of the systematic factor x. This probability is simply derived from (2), as follows:

$$p(x) = \Pr\left[\varepsilon_i < \frac{\Phi^{-1}(\bar{p}) - wx}{\sqrt{1-w^2}}\right] = \Phi\left[\frac{\Phi^{-1}(\bar{p}) - wx}{\sqrt{1-w^2}}\right] \quad \text{with } \varepsilon_i \sim N(0;1). \tag{4}$$

Thus, any variation in x, the systematic factor, induces a variation in the PD. If borrowers with the same rating grade are more sensitive to the state of the economy, they will more often move together toward the default state. Borrowers would be more affected by x, and therefore, the correlation would be higher in that risk class.

Assuming serial independence for the realizations of the systematic factor and conditional independence between defaults, the probability that two borrowers jointly default is simply $p(x)^2$, with variance

$$\mathrm{Var}[p(x)] = \mathrm{Bivnor}\left(\Phi^{-1}(\bar{p}), \Phi^{-1}(\bar{p}), w^2\right) - \bar{p}^2. \tag{5}$$

The variance of $p(x)$ is estimated using the non-parametric method proposed by Gordy (2000). Finally, the asset correlation w^2 is derived as the solution of Eq. (5).

3. The data

The data comes from the internal ratings systems of two large European financial information providers: Coface in France and Creditreform in Germany. They provide us with records of all changes in rating grades over the years 1995–2001 in France and 1997–2001 in Germany. In addition, the two files contain accounting data for all firms we retained in the panels. Following a quite conventional definition, SMEs are defined as incorporated firms with turnover under €40 million. The final database contains around 440.000 French and 280.000 German firms over the period.

By using a quite conventional SME size classification, we distinguished three size classes of SMEs: small SMEs with turnover between €0.15 and €1 million, medium-

[1] The shape of this distribution is skewed to the right and its degree of asymmetry directly depends on the weight w of the systematic risk factor.

size SMEs with turnover between €1 and €7 million, and large SMEs with turnover between €7 and €40 million. Table 1 presents the distribution by size of the SMEs in the two samples. We also obtain the same information set for large firms (turnover higher than €40 million). Note that all sub-samples are very representative of the incorporated French and German companies' population of each size.

While each original ratings system contains around 15 grades, we built up a simpler nine-position ratings system. Rating grade 1 corresponds to the lowest degree of credit risk and rating grade 9 corresponds to default. In the two countries, legal bankruptcy is used as the definition of default. These ratings systems allow us to build up an annual transition matrix and to compute PD and asset correlation. Table 2 shows the distribution of French and German SMEs in the rating grades classes and in the three size classes at the end of 2001. Because the risk classification comes from two different ratings systems, the two risk scales are not directly comparable. That explains partially why the distribution of the SMEs in the nine risk classes is different from one country to the other. Here, the objective of the study was not to build up a unique ratings system, but only to analyze the building blocks of SMEs' credit risk, and more particularly the asset correlations, in both countries. Note also

Table 1
Size distribution of French and German firms in 2001

Size (turnover in M€)	France		Germany	
	Number of firms	%	Number of firms	%
1 (0,15 to 1)	287.586	64.04	140.660	49.7
2 (1–7)	131.977	29.39	116.175	41.1
3 (7–40)	29.538	6.58	26.112	9.2
Total SMEs	449.101	100	282.947	100
Large firms (>40M€)	6.213		12.081	

Sources: Coface and Creditreform.

Table 2
The risk distribution of French and German SMEs, year 2001

Size	Risk classes (from low risk – class 1 to high risk – class 8)								
	1	2	3	4	5	6	7	8	Default
Panel A: France									
1 <1M€	0.4	39.5	10.9	13.6	13.9	14.6	2.7	2.1	*2.4*
2 1–7M€	5.7	42.8	13.1	13.3	12.7	8	1.2	1.1	*1.6*
3 7–40M€	17.8	32.4	14.5	13.8	12.5	5.7	0.7	0.3	*0.9*
Total	*3.34*	*40.02*	*11.86*	*13.65*	*13.41*	*11.85*	*2.18*	*1.65*	*2.0*
Panel B: Germany									
1 <1M€	0.1	21.7	31.3	9.4	7.1	14.6	5.5	6.2	*4.0*
2 1–7M€	1.0	24.6	32.4	9.0	4.2	11.2	3.7	4.8	*4.1*
3 7–40M€	6.3	47.6	25.8	2.2	4.0	6.1	3.3	2.1	*2.6*
Total	*1.0*	*27.2*	*31.3*	*8.6*	*5.7*	*12.5*	*4.6*	*5.2*	*3.9*

Sources: Coface and Creditreform.

that the higher percentage of firms in default in Germany is due to the fact that the state of the economy was characterized by an increase in the number of bankruptcies in that country in 2001. For both France and Germany, the data contain information on the sector in which firms operate. In the computations of AVCs within sector and risk classes, we will use a two-digit classification of firms. This leads to consider 23 sectors for France and 21 for Germany, using for each country the national classification system. As a consequence, direct comparison of asset correlations is subject to caution because of heterogeneity in the definition of sectors.

4. The results

The ratings grades were used to compute PDs and asset correlations. To compute annual default rates, we considered only the initial rating and the rating at the end of the planning horizon (one year). This implies that transitions within the year other than default (an absorbing state) were neglected. Then, we computed annual PDs and stationary default probabilities – the weighted mean of the annual PDs over the period – for each size and risk class. Before presenting the correlation results, it is interesting to observe how the stationary PDs vary with the firm's size.

4.1. The stationary default probabilities vary with firm's size within the SME population

Table 3 presents the stationary PDs in the various sub-portfolios of French and German SMEs and large firms. It shows that, on average, the PDs tend to vary with size. In Germany, the relationship is not monotonic in the SME sample. The PDs are lower in the first size class than in the second one, whatever the ratings class. However, the PDs' differences are not so high. The PDs are also higher in these two first classes, as compared with the last size class. In other words, smaller SMEs seem to be less risky than medium-size SMEs, and these two classes of SMEs seem to be riskier than the largest SMEs. In France, additional results (not shown here) show a similar pattern: PDs are also lower in the very small (personal) businesses (with turnover lower than €0.15 million).

Hence, these results tend to show that we can very likely distinguish three categories of SMEs: the small or very small ones, in which the default risk is lower than in the medium-size SMEs; the medium-size SMEs that are riskier, on average; and the largest SMEs, where credit risk is lower. Table 3 likewise allows comparisons between the SMEs and the large corporate firms. Results show that in France as well as in Germany, the average stationary PDs are, on general, much lower in the large businesses. These results confirm that, in the two countries, default risk tends to decrease, on average, with a firm's size. [2]

[2] Note that this could be due in part to the criterion of default chosen. In fact, legal bankruptcy is a means to resolving financial difficulties and its use appears to be less frequent in the large business sector than in the SME sector.

Table 3
Stationary annual default probabilities in French and German firms (in %)

Risk classes	Size classes			
	SMEs size 1 <1M€	SMEs size 2 1–7M€	SMEs size 3 7–40M€	Total SMEs
Panel A: France				
1 (low)	0.33	0.24	0.15	0.19
2	0.41	0.27	0.19	0.32
3	0.9	0.68	0.48	0.72
4	1.64	1.35	0.84	1.33
5	2.79	2.61	1.53	2.39
6	4.94	4.51	2.44	4.23
7	9.99	9.44	5.49	8.61
8 (high)	14.89	16.24	13.28	13.78
Total	2.63	1.74	0.79	2.21
Panel B: Germany				
1 (low)	0.17	0.43	0.56	0.49
2	0.77	0.98	1.05	0.91
3	1.16	1.68	1.43	1.41
4	1.95	2.56	2.34	2.21
5	2.38	3.04	2.35	2.55
6	2.37	3.55	3.10	2.87
7	5.45	7.63	9.44	6.42
8 (high)	16.22	18.66	18.61	17.28
Total	1.23	0.79	0.14	0.93

The stationary default probabilities are the weighted annual mean default rates over the 1997–2001 periods. Rating transitions other than default within one year were neglected, i.e., only the rating at the beginning of the year is used in the computation of default rates.

4.2. The low level of asset correlations in French and German SMEs

Table 4 presents the asset correlations for the same sub-portfolios of French and German firms. Two main results come to light. First, the values of the asset correlations are very weak. The average value is around 1% in the French and German SMEs. The maximum value per risk class climbs to 10.72% in France and to 6.52% in Germany.

Second, the asset correlations decrease significantly on average with the SME size. They are quite low in the large SMEs, compared with the small and medium-size ones. This result tends to show that the SMEs' credit risk is less sensitive to the systematic risk factor as a firm's size increases. However, results also show that the average correlation is higher in the larger businesses (with turnover higher than €40 million) in the two countries. Average values are 1.45% in Germany and 2.21% in France during the same period.

Two main reasons suggest that estimated asset correlations might be too low. The first reason is the length of the time series. Our period of observation might be too

Table 4
The asset correlations in French and German firms (in %)

Risk classes	SMEs size 1 <1M€	SMEs size 2 1–7M€	SMEs size 3 7–40M€	Large firms >40M€	Total SMEs
Panel A: France					
1 (low)	0.79	2.95	2.79	1.5	2.19
2	0.12	1.95	1.56	0	2.29
3	1.55	0.61	0.71	4.39	2.31
4	1.34	0.95	0.57	2.79	2.67
5	1.53	0.98	0.37	2.77	1.51
6	1.78	1.47	0.82	0	1.99
7	2.67	2.08	2.07	0	2.98
8 (high)	2.71	2.79	10.72	0	3.07
Total	1.54	0.97	0.49	2.21	1.28
Panel B: Germany					
1 (low)	0	0	0	1.21	0.11
2	1.86	1.33	0.57	2.51	1.29
3	1.52	1.29	0.24	0	1.19
4	2.21	1.42	6.52	1.61	2.01
5	3.18	2.02	0.25	0.75	2.59
6	1.21	0.62	0.25	0.49	0.79
7	3.97	1.97	0.57	1.69	2.75
8 (high)	2.71	2.62	2.03	0	2.59
Total	1.23	0.79	0.14	1.45	0.93

short to cover at least an entire business cycle, leaving a misleading apparent stability of default rates (low or high). This could induce a selection bias in the measurement of PDs and PDs' volatility. In fact, the period 1997–2001 was mainly characterized by economic growth, with only one "bad" year in Germany and a noticeable decrease in the number of defaults in the SME population at the end of the period in France and in the middle of the period in Germany. The only solution to this shortcoming would be to accumulate new data through time.

The second reason for a possible underestimation of asset correlations stems from the fact that the estimated correlations were computed on a very large sample of businesses (quasi-exhaustive). This implies that the asset correlations gathered in Table 4 are representative, respectively, of the entire French and German economies and represent the limits of diversification within these economies. In general, the size of the banks' SME portfolios will be far smaller than the data we used in the computations. Consequently, we expect that a bank will observe higher asset correlations in its book.

Moreover, the AVCs presented in Table 4 are average correlations within risk-size classes. However, the regulator's problem would be to assess the likelihood of potentially very high correlations in some risk classes. In other terms, the regulator might be more concerned by extreme values of asset correlations for some given portfolios of limited size than with economy-wide average values. To address this concern, we propose a bootstrap-like methodology that permits assessment of the possible variation of the asset correlation inside each size class. To assess the "confidence interval" of asset correlations, we simulated portfolios by drawing random portfolios at the beginning of the period. Then, we computed average PDs and asset correlations in each portfolio, using the specific default rate dynamics of the portfolio. We restricted the analysis to the population of French SMEs, considering the period of German data as too short.

To test the robustness of the evolution of asset correlations with size, we split the entire database into the same size classes as in previous sections. Thus, the simulated portfolios are homogeneous in terms of firm size. The portfolios drawn are intentionally small because the likelihood of finding high correlations is higher in small portfolios. Hence, the simulated portfolios concerning the first SME size classes contain 5000 borrowers (for the smaller firms size class, we drew two different portfolios of 5000 and 10,000 borrowers). The simulated portfolio of large businesses contains only 2000 borrowers, while the total sample contains 4377 firms. [3] The size of each portfolio is maintained constant over the seven-year period by replacing firms in default in a given year with firms that did not default and are still present in the database at the beginning of the next year. Table 5 presents the results of the simulation.

The analysis of Table 5 leads to two main comments. The first main result is that considering portfolios that might be closer to portfolios effectively held by banks, both in terms of size and in terms of the size distribution of borrowers, SMEs exhibit higher average asset correlations than large firms. This confirms the fact that Table 4 describes credit risk features at the macro level of the entire population of firms more than at the micro level of the bank's portfolios. Moreover, within the SME population, the asset correlations remain decreasing with size, confirming the robustness of our results.

The second main result is the difference in volatility of asset correlations across size classes. While the average value decreases with size, the standard deviation increases with size. The volatility is especially high in the large business class (turnover over €40 million). In that size class, the likelihood of observing a correlation higher than a given value is greater than in the other classes, despite the fact that the average value is much lower than in other size classes, for the given size of the simulated portfolios.

The stability of the default correlation in the small SME class (turnover less than €0.75 million) is quite surprising despite the relatively small size of simulated portfolios (5000 and 10,000 exposures to be compared to the over 150,000 French businesses of that size class). An explanation of this result could be that the sensitivity of smaller businesses to economic conditions is relatively uniform across sectors and regions, at least in the French case. The same also seems true for medium-size SMEs. On the contrary, portfolios of larger business loans appear to be potentially more sensitive to the deterioration of economic conditions. In other words, even if the large business PD is much lower than that of SMEs, deterioration of business

[3] This sample is very representative of the population of large businesses in France.

Table 5
Asset correlations (%) within simulated portfolios over 1994–2001[a]

Size class	Mean	Standard deviation	Confidence interval 90%	Confidence interval 98%	Min	Max
(1a) Very small SME (5000 exposures)	5.16	0.58	4.5–6	4–6.5	3.5	7
(1b) Very small (10,000 exposures)	5.14	0.41	4.5–6	4.5–6	4	6.5
Small SME	3.64	0.90	2.5–5	2–6	1	7
Medium-sized SME	3.99	0.96	2.5–5.5	2–6.5	1.5	7
All SME	3.73	0.99	2.25–55	2–6.5	1	8
Large firms	1.68	1.96	0–5.5	0–7.5	0	12

[a] The asset correlations refer to simulated portfolios of French firms.

conditions might result in a relatively greater increase of the number of defaults in this population, at least for some portfolios. In fact, the credit risk of large businesses comes less from the likelihood of individual defaults than from the likelihood of possible multiple defaults in case of economic downturn.

If the main objective of bank capital regulation is to avoid excessive losses in individual portfolios due to large corporate exposures, this objective could require quite large economic capital requirements. Simulation results show, for instance, that the highest simulated correlation amounts to 12% in this size class.

5. A positive relationship between the PD and the asset correlations

Default probabilities and asset correlations (R in the Basel II risk formulas) are key parameters in the calibration of any credit risk model. They also hold a central position in the new regulatory framework of Basel II (BIS, 2002, 2003). Indeed, the new risk-weight formulas that apply both to corporate exposures and to retail exposures assume a negative relationship between PDs and R. Assuming a decreasing relationship between these two variables means that firms with lower default risk are also more exposed to changes in economic conditions. Conversely, firms with higher default probabilities are less prone to default simultaneously. Their activities may be less dependent on the business cycle. This situation implicitly might reflect an opposition between bigger firms with good credit ratings operating in global markets and sensitive to global macroeconomic conditions, on the one hand, and smaller firms mainly active in local markets and more sensitive to specific factors like personal management skills, on the other hand. Thus, the assumed negative relationship might apply to broadly defined credit portfolios, where the good risks are overwhelmingly large firms and the medium and lower ratings comprise the smaller firms. However, results in Table 4 do not completely verify this pattern. Indeed, if we consider the population of SMEs as a whole and if we compare it to the population of large firms, we can show that the former are less exposed to macroeconomic risk but exhibit higher default risk. In the case of France, SMEs exhibit a PD of 2.21% and an asset correlation of 1.28% to be compared to the PD of 0.28% and the asset correlation of 2.21% for large firms. Similar results could be observed in Germany. However, if we observe the relationship between PDs and AVCs into each size class within the SMEs population, the result no longer holds. The asset correlations, in fact, increase as the default probabilities become larger. The same is true at the industry level.

5.1. A positive relationship between PD and AVC within small size classes

Table 4 did not show a negative relationship between the correlations and the PDs across rating grades. In the French SME population, the correlation increases with the risk of default in the class of smaller firms, and it is U-shaped in the other two classes, which also explains the U-shaped form of the relationship in the total sample. In Germany, the results show a (weak) positive relationship between correlations and PDs on average and in the first two size classes of SMEs. [4] In the largest SMEs and large firms, no clear relationship appears between the two variables. In the case of large firms, the computations within risk classes have to be considered with care as the number of defaults might be too low to obtain meaningful results.

The observed U-shape for asset correlations may be explained by a segmentation argument. In fact, one might identify a first market segment of firms with good credit quality, where defaults are mostly driven by the position in the business cycle. The second segment, with firms of average credit quality, contains the bulk of firms in each size class. In that segment, defaults depend more strongly on the specific factors of management and access to productive and financial resources. The last segment encompasses what could be termed a "subprime" segment of the market: firms with low credit quality that are very sensitive to macroeconomic conditions. For these firms, specific factors lead to higher default rates, but these firms will also be the first firms to default, leading to a higher default concentration in time and higher asset correlations.

This non-monotonic relationship between average default probabilities and asset correlations is quite surprising. There are good reasons to expect a positive relationship between these two variables: the most financially fragile borrowers would be the most sensitive to variations in economic activity, *ceteris paribus*. Consequently, the concentration of defaults would be higher in higher risk classes, leading to higher default correlations. One possible reason for not observing this relation could come from the diversification of the entire population of SMEs across economic sectors. Indeed, average default rates as well as default correlations may be industry-specific, i.e., related to the specificity of each industry in terms of competition, sensitivity to macroeconomic conditions, access to financial resources, capabilities of managers, differences in bankruptcy legislation, and so on. In other words, there would be a greater chance to find the expected systematic relationship between PD and R if the portfolios contain businesses belonging to the same industry, if such a relationship

[4] Duellmann and Scheule (2003) obtain similar results by using a different panel of German SMEs.

exists. So, to explore this hypothesis, we built up more homogeneous portfolios in terms of activity, that is, 23 industry portfolios and computed average default rates and correlations within the eight risk classes in each industry portfolio. [5]

5.2. A positive relationship between PD and AVC at the industry level

Results at the industry level verify the existence of the assumed positive relationship between PDs and correlations (Table 6). Indeed, if we compute the correlation between the values of the PDs and the correlations over the 23 industries, we find an average and significant correlation of 0.25 in both countries (with a *p*-value less than 0.01). [6] Moreover, Table 6 shows that default correlations are generally higher in higher risk classes, especially in the French sample. In other words, the relationship is positive in most of the industries, even if it is strictly monotonic in only 2 of the 23 sectors.

Therefore, when we consider homogeneous groups of firms within a particular industry, the results show more clearly the existence of a positive relationship between PDs and correlations whatever the country. This result points to the importance of industry diversification. Hence, concentration of exposure, in terms of size and activity, seems to play an important role in the measurement of credit risk.

Changing the segmentation criterion of the SME population leads to different results in terms of the relationship between PD and AVC. The activity-based segmentation leads to the positive relationship observed between PD and AVC. In that case, each sector contains firms that are homogeneous in terms of activities but heterogeneous in economic terms. Specifically, within each sector, lower credit quality (as measured by the credit rating) is associated with higher sensitivity to macroeconomic risk. On the contrary, a segmentation of borrowers based on size leads to the observed U-shaped or slightly positive relationship. This result might be explained by the fact that portfolios of firms of similar size benefit from the diversification across sectors. Moreover, these diversification gains are most important for borrowers with low credit ratings. One consequence of this result is that a regulatory model, that assumes any relationship between PD and AVC other than a positive one, makes the implicit assumption that the portfolios are large and cross-sector diversified. Moreover, the assumed negative relationship will in fact overestimate the capital charge for good risks relative to bad risks, at least for SMEs.

[5] Unfortunately, despite the large size of our sample, it was difficult to control for size, i.e., to build portfolios crossing size and activity, since many of these portfolios were almost empty, therefore making estimations meaningless.

[6] The results might be sensitive to the risk classification. To test the robustness of the results, we performed the same computations within only seven risk classes, merging classes seven and eigth into one class. The overall correlation is then 0.32. This suggests that these two risk classes are not significantly different in terms of sensitivity to systematic risk.

Table 6
Asset correlations across risk classes within industries [a]

Risk class	Activity																			
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20
Panel A: France																				
Lowest	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Very low	0	4	6.5	0	5	0	4	0	0.1	3	0	0	0	0.5	7	6.5	8	7	9	5.5
Low	0.3	0	2.5	0	3.5	0	0	0.4	3.5	2	0	6	0	13	7.5	8.5	9	6	7	11
Med-low	4	9	8.5	3	12.5	0	8	10.5	4.5	6.5	4	9.5	6	2	9.5	12	11	8.5	4.5	11
Med-high	5.5	0.5	12	1	12.5	0	1	2	7.5	10.5	1.5	13	9	3.5	9.5	14.5	12	12	5.5	10.5
High	10	11	15.5	16.5	7.5	0	0	0	9	9	16.5	6.5	5	3.5	12	13.5	15	12	8	16
Very high	12	9.5	28.5	1	17.5	10	0	0	13	10	0	0.3	4.5	3.5	14.5	12	15.5	12.5	16.5	20
Highest	12	40	5	5.5	18.5	0	15	0	9.5	16	20.5	7	23	5	8	19.5	17.5	9	2	13.5
Panel B: Germany																				
	S1–3	S4–5	S7	S8–10	S11–13	S14–15	S16–17	S18	S19	S20–21	S22	S23	S24–25	S29–30	S31	S32	S33	S34	S36–37	
Lowest	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.5	0	0	0	
Very low	2.5	0	0.2	1.5	1.5	2.5	0	0.1	2	0	0	2.5	4	1	4.5	0	0	0	3.5	
Low	3	0.3	0.3	0	5	1.5	0	1.5	1.5	0	0	1.5	2.5	5.5	2	0.5	1	0	1	
Med-low	2	0	3	0	0	1.5	5	2.5	2	1	1	5.5	2.5	2.5	0	1.5	3	0	2	
Med-high	2	0	1.5	0	10	6	0	0.4	2.5	0	0	2	1.5	5.5	4	5	8	0	2.5	
High	1	1.5	0.4	3.5	2	0.3	0	0	1.5	1	1	0	0	1	1	0	6	0	2	
Very high	3	0	1.5	9	0	0	0	5	3.5	1	1	1	3	0	10.5	2	10.5	0	6.5	
Highest	3	0	0.3	2	6.5	4	11	3.5	3.5	0.2	0.2	3.5	3	4	7	2	0	5.5	3	

Source: Coface and A–K Creditreform and our computations.

[a] Appendix A gives the definitions of activities.

6. Conclusion

In this paper, we show that the best way to adjust risk-weight formulas for SMEs would be to propose a different treatment of these firms. We use an internal ratings-based approach of credit risk, and we run a one-factor model to provide estimates of stationary default probabilities and asset correlations in two very large populations of around 440,000 SMEs in France and 280,000 in Germany. The study retains legal bankruptcy as the definition of default in the two countries.

The results show that, in the two samples of French and German SMEs, the sensitivity to one systematic risk factor ("the state of the economy") is quite low and that the estimated default correlations are on the order of 2–4%, on average. A bootstrap-like test confirms that the effective correlations in the small and medium-size businesses are very low. The results also show that we can distinguish three types of borrowers in the corporate sector: very small SMEs, where the default correlations are very stable; an intermediate group of medium-size SMEs, where the likelihood of high default correlations remains relatively low; and large businesses, which exhibit small correlations on average, but where the risk of having a loan portfolio with high correlation is great.

Moreover, our results do not show a negative relationship between asset correlations and PDs as assumed in the Basel formulas. This relationship is U-shaped in France and positive in Germany. Consequently, the choice of high correlation values in the risk-weight formulas could induce too high a capital charge for SMEs if we consider the very low values of the correlation in this population. An alternative rule could be to impose stringent concentration rules for large corporate portfolios. Credit risk differs within the SME population at least between small SMEs and larger SMEs. Large SMEs should probably receive more favorable treatment than large firms because they seem to be less sensitive to systematic risk than the latter. On the other hand, even if smaller SMEs are on an individual basis riskier than the large SMEs, their very weak sensitivity to systematic risk and the positive effects of large portfolio diversification invite the exposures on these firms to be treated as retail exposures.

Our results also suggest two paths for future research. First, the results could be affected by our definition of default. Hence, it would be interesting to see if the results remain valid when choosing another criterion of default, such as default on a bank loan or other source of debt. Second, because the SME sector is a central part of the productive system in most of the European countries, it would be interesting to see if the relationships between correlations and PDs and between correlations and firm size are the same in other European countries as in the two countries considered in this study.

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Appendix A. Definition of activities (Table 6)

France

S1: Food industry
S2: Textile
S3: Clothing
S4: Wood industry
S5: Paper, publishing
S6: Chemicals, pharmacy
S7: Plastics
S8: Glass industry
S9: Metallurgy
S10: Mechanics
S11: Electronics
S12: Optics, medical equipment
S13: Car, plane, ship manufacturing
S14: Accessories
S15: Building
S16: Car trade
S17: Wholesalers
S18: Retailers
S19: Hotels, restaurants
S20: Transport
S21: Real estate
S22: Mining, quarrying
S23: Waste industry

Germany

S1–3: Agriculture
S4–5: Food industry
S6–7: Accessories
S8–10: Chemical and petrol industry
S11–13: Plastics, glass and pharmacy
S14–15: Metal industry
S16–17: Papers
S18: Building materials
S19: Building
S20–21: Textile

S22: Clothing
S23: Mechanics
S24–27: Industrial equipment and electronics
S29: Automobiles
S31: Transport
S32: Retailing
S33: Printing, media
S34: Public utilities
S36: Services

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