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Research paper

A discount strategy in word-of-mouth marketing

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ABSTRACT

This paper addresses the issue of discount pricing in word-of-mouth (WOM) marketing. First, we propose a novel discount strategy we refer to as the Influence-Based Discount (IBD) strategy, in which each buyer would enjoy a discount that is linearly proportional to his/her influence in the WOM network. For the purpose of evaluating the performance of an IBD strategy, we model the associated WOM spreading process as a higher-dimensional nonlinear differential dynamical system we refer to as the Dormant-Potential-Adopting (DPA) model. Next, we examine the dynamics of a DPA model through computer experiments. Thereby, we estimate the expected net profit of an IBD strategy and inspect how different factors influence this profit through numerical experiments. On this basis, we suggest some feasible promotional measures.

1. Introduction

In all ages, discount pricing, whereby a price is listed as a discount from an earlier or regular price, is accepted as a common marketing practice. As opposed to a merely low price, a discounted price can make a rational consumer more willing to purchase the product. This is because the information that the product was initially sold at a high price indicates that the product is high quality, and a discounted price signals that the product is an unusual bargain, and there is little point for lower prices [1–3].

Word-of-mouth (WOM), which is loosely defined as the sharing of information about a product between a consumer and his/her friends or colleagues or acquaintances, has long been a prevalent topic among marketing practitioners and researchers [4]. Due to low cost and rapid spreading, WOM outperforms the conventional advertising in terms of marketing profit [5,6]. For instance, a potential customer linked to a prior customer of a product will be 3 to 5 times more likely to purchase the product [7]. With the popularity of online social networks such as Facebook and Twitter, WOM has been turned into the main measure of marketing [8].

A central issue in WOM marketing is influence maximization, in which the objective is to find a set of seeds such that the expected number of the individuals activated from this seed set is maximized [9]. To date, a multitude of seeding algorithms have been proposed [10–14]. In order to evaluate the performance of a seeding algorithm, a number of WOM spreading models have been suggested [15–21]. As it is known, the structure of the WOM network has a remarkable influence on

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the marketing profit [22,23]. As the above-mentioned WOM spreading models are all compartmental, i.e., the individuals in the WOM network are classified simply according to their current states, we cannot use them to accurately estimate the marketing profit. Node-level epidemic modeling, in which each individual is categorized as a single class, is recognized as an effective approach to the study of complex propagation phenomena over arbitrary networks. The reason for this is that the effect of the network structure on the propagation process is fully accounted for Pastor-Satorras et al. [24]. In the past decade, the node-level epidemic modeling technique has been applied to fields as diverse as general epidemics [25–27], malware propagation [28–31], rumor spreading [32,33], and cybersecurity [34–36]. For the purpose of evaluating the performance of a WOM marketing strategy, it is necessary to introduce a proper node-level WOM spreading model.

It seems to be a good idea to introduce discount pricing into WOM marketing, because it may help to enhance the marketing profit. As far as we know, hitherto there has been no systematic theoretical research on this topic. This paper addresses the issue of discount pricing in WOM marketing. First, we propose a novel discount strategy we refer to as the Influence-Based Discount (IBD) strategy, in which each customer would enjoy a discount that is linearly proportional to his/her influence in the WOM network. Second, we propose a node-level WOM spreading model and study its dynamics. Thereby, we estimate the expected net profit of an IBD strategy. Finally, we examine the influence of different factors on the performance of an IBD strategy through numerical experiments. On this basis, we recommend some promotional measures.

The remaining materials are organized in this fashion: Section 2 describes the new discount strategy. Sections 3 and 4 introduce and study a WOM spreading model, respectively. Section 5 examines the performance of the proposed discount strategy. Finally, this work is closed by Section 6.

2. A discount strategy in WOM marketing

This section is dedicated to describing a discount strategy in WOM marketing. First, we present a set of preliminary notions and notations. Second, we formulate the discount strategy.

2.1. Notions and notations

Suppose a supplier intends to conduct a WOM marketing campaign in the time horizon $[0, T]$. Let $V = \{1, 2, \cdots, N\}$ denote the target market of the campaign, i.e., the set of all target individuals in the campaign. Let $G = (V, E)$ denote the associated WOM network, where there is an edge from node i to node j if and only if the target individual i may recommend products to the target individual j. Let $\mathbf{A}(G) = \left(a_{ij}\right)_{N\times N}$ denote the adjacency matrix of the WOM network G, i.e., $a_{ij} = 1$ or 0 according as $(i, j) \in E$ or not. The WOM network consists of a set of social networks over the target individuals, which are already existing prior to the marketing campaign. As most WOM marketing campaigns are short-term, we rationally assume that the WOM network under consideration is fixed and hence is not varying over time.

Define a normalized quantity as follows:

$$d_i = \frac{\sum_{j=1}^{N} a_{ji}}{\max_{1\le k\le N} \sum_{j=1}^{N} a_{jk}}. \tag{1}$$

This quantity measures the influence of the individual i in the WOM network, because it is proportional to the number of the individuals to whom the individual i may recommend products. Therefore, we refer to d_i as the *influence degree* of the individual i. As the WOM network is fixed, d_i is fixed as well.

2.2. The discount strategy

Inspired by the intuition that, as compared to a less influential individual, a more influential individual in the WOM network has a stronger influence on the purchase decisions of other individuals and, hence, is expected to bring a higher profit to the supplier, we describe a discount strategy as follows.

Influence-based discount (IBD) strategy: For a WOM marketing with discount pricing, the discount given to each individual is linearly proportional to his/her influence in the WOM network.

Let θ denote the *basic discount*, which is a constant that is under control of the supplier. According to the IBD strategy, the real discount given to the individual i is θd_i.

3. A WOM spreading model associated with the IBD strategy

In the previous section, we proposed a discount strategy we refer to as the IBD strategy. The next thing we need to do is to evaluate the performance of an IBD strategy. As this performance is closely related to the spreading process of WOM, this section is devoted to establishing a WOM spreading model. First, we formulate the expected state of the target market. Second, we present the WOM spreading model.

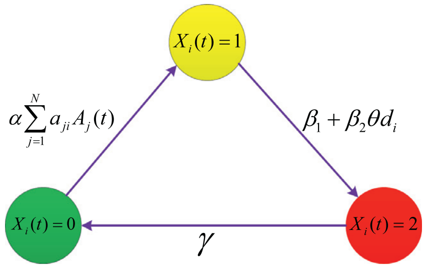

Fig. 1. Diagram of the hypotheses (H_1)-(H_4).

3.1. The expected state of the target market

In what follows, we assume that each and every individual in the target market is in one of three possible states: *dormant, potential*, and *adopting*. Dormant individuals are individuals who are not currently buying any item and have no will of buying items, potential individuals are individuals who are not currently buying any item but have the will of buying items, and adopting individuals (or simply adopters) are individuals who are currently buying items. Let $X_i(t) = 0$, 1, and 2 denote that the individual i is dormant, potential, and adopting at time t, respectively. Then the vector

$$\mathbf{X}(t) = (X_1(t), \cdots, X_N(t)) \tag{2}$$

stands for the state of the target market at time t. At the beginning of the campaign there is no adopter. So, $X_i(0) \neq 2$ for all i.

Let $D_i(t)$, $P_i(t)$, and $A_i(t)$ denote the probabilities of the individual i being dormant, potential, and adopting at time t, respectively.

$$D_i(t) = \Pr\{X_i(t) = 0\}, \quad P_i(t) = \Pr\{X_i(t) = 1\}, \quad A_i(t) = \Pr\{X_i(t) = 2\}. \tag{3}$$

As $D_i(t) = 1 - P_i(t) - A_i(t)$, the vector

$$\mathbf{x}(t) = (P_1(t), \cdots, P_N(t), A_1(t), \cdots, A_N(t)) \tag{4}$$

stands for the expected state of the target market at time t. Let $\mathbf{P}(t) = (P_1(t), \cdots, P_N(t))$, $\mathbf{A}(t) = (A_1(t), \cdots, A_N(t))$. Then $\mathbf{x}(t) = (\mathbf{P}(t), \mathbf{A}(t))$. As $X_i(0) \neq 2$ for all i, we have $\mathbf{A}(0) = \mathbf{0}$. Hereafter, let $\mathbf{0}$ and $\mathbf{1}$ denote an all-zero vector and an all-one vector, respectively.

Let

$$A(t) = \frac{1}{N}\sum_{i=1}^{N} A_i(t). \tag{5}$$

This quantity stands for the expected fraction of adopters at time t. If $\lim_{t\to\infty} A(t)$ exists, we refer to it as the *asymptotic expected fraction of adopters*.

3.2. The WOM spreading model

For the purpose of establishing the WOM spreading model associated with an IBD strategy, we need to introduce a set of hypotheses as follows.

(H_1) (WOM) Due to recommendations by adopters, the dormant individual i has the will of buying items and hence becomes potential at time t at an average rate of $\alpha \sum_{j=1}^{N} a_{ji}A_j(t)$, where $\alpha > 0$ is a constant and is referred to as the *WOM force*.

(H_2) (Rigid demand) Due to rigid demand, each potential individual buys items and hence becomes adopting at any time at an average rate of β_1, where $\beta_1 > 0$ is a constant and is referred to as the *rigid demand*.

(H_3) (Lure) Due to the lure of discount, the potential individual i buys items and hence becomes adopting at an average rate of $\beta_2\theta d_i$, where $\beta_2 > 0$ is a constant and is referred to as the *lure force*.

(H_4) (Viscosity) Each adopter loses the will of buying items and hence becomes dormant at any time at an average rate of γ, where $\gamma > 0$ is a constant and is referred to as the *viscosity*.

The hypothesis (H_1) implies that a more influential individual contributes more to the marketing profit than a less influential individual. The hypothesis (H_3) characterizes the IBD strategy. Fig. 1 shows these hypotheses schematically.

Given a very small $\Delta t > 0$. It follows from the hypotheses (H_1)-(H_4) that for $1 \le i \le N$, $0 \le t \le T - \Delta t$, there hold

$$\Pr\{X_i(t+\Delta t) = 1 \mid X_i(t) = 0\} = \alpha\Delta t \sum_{j=1}^{N} a_{ji}A_j(t) + o(\Delta t), \tag{6}$$

$$\Pr\{X_i(t+\Delta t)=2 \mid X_i(t)=0\}=o(\Delta t), \tag{7}$$

$$\Pr\{X_i(t+\Delta t)=0 \mid X_i(t)=1\}=o(\Delta t), \tag{8}$$

$$\Pr\{X_i(t+\Delta t)=2 \mid X_i(t)=1\}=(\beta_1+\beta_2\theta d_i)\Delta t+o(\Delta t), \tag{9}$$

$$\Pr\{X_i(t+\Delta t)=0 \mid X_i(t)=2\}=\gamma\Delta t+o(\Delta t), \tag{10}$$

and

$$\Pr\{X_i(t+\Delta t)=1 \mid X_i(t)=2\}=o(\Delta t). \tag{11}$$

As a result, there hold

$$\Pr\{X_i(t+\Delta t)=0 \mid X_i(t)=0\}=1-\alpha\Delta t\sum_{j=1}^{N}a_{ji}A_j(t)+o(\Delta t), \tag{12}$$

$$\Pr\{X_i(t+\Delta t)=1 \mid X_i(t)=1\}=1-(\beta_1+\beta_2\theta d_i)\Delta t+o(\Delta t), \tag{13}$$

and

$$\Pr\{X_i(t+\Delta t)=2 \mid X_i(t)=2\}=1-\gamma\Delta t+o(\Delta t). \tag{14}$$

By Total Probability Formula [37], we get

$$\begin{aligned} P_i(t+\Delta t) &= D_i(t)\Pr\{X_i(t+\Delta t)=1 \mid X_i(t)=0\}+P_i(t)\Pr\{X_i(t+\Delta t)=1 \mid X_i(t)=1\} \\ &\quad +A_i(t)\Pr\{X_i(t+\Delta t)=1 \mid X_i(t)=2\} \\ &= \alpha\Delta t[1-P_i(t)-A_i(t)]\sum_{j=1}^{N}a_{ji}A_j(t)+P_i(t)[1-(\beta_1+\beta_2\theta d_i)\Delta t]+o(\Delta t) \end{aligned} \tag{15}$$

and

$$\begin{aligned} A_i(t+\Delta t) &= D_i(t)\Pr\{X_i(t+\Delta t)=2 \mid X_i(t)=0\}+P_i(t)\Pr\{X_i(t+\Delta t)=2 \mid X_i(t)=1\} \\ &\quad +A_i(t)\Pr\{X_i(t+\Delta t)=2 \mid X_i(t)=2\} \\ &= P_i(t)(\beta_1+\beta_2\theta d_i)\Delta t+A_i(t)(1-\gamma\Delta t)+o(\Delta t). \end{aligned} \tag{16}$$

Rearranging the terms, dividing both sides by Δt, and letting $\Delta t\to 0$, we get a system of $2N$ ordinary differential equations as follows.

$$\begin{cases} \dfrac{dP_i(t)}{dt}=\alpha[1-P_i(t)-A_i(t)]\displaystyle\sum_{j=1}^{N}a_{ji}A_j(t)-(\beta_1+\beta_2\theta d_i)P_i(t), & 1\le i\le N, \\ \dfrac{dA_i(t)}{dt}=(\beta_1+\beta_2\theta d_i)P_i(t)-\gamma A_i(t), & 1\le i\le N, \end{cases} \tag{17}$$

where $\mathbf{A}(0)=\mathbf{0}$. We refer to this system as the *Dormant-Potential-Adopting (DPA) model*. Obviously, a DPA model is determined by the following 7-tuple:

$$\mathcal{M}_{DPA}=(G,\alpha,\beta_1,\beta_2,\gamma,\theta,\mathbf{P}(0)). \tag{18}$$

4. The dynamics of a DPA model

This section aims to gain insight into the dynamical properties of a DPA model through computer experiments. For this purpose, below we describe the WOM networks that will be used in our experiments.

First, we know that many real-world networks are small-world, i.e., with a relatively small diameter [38]. We use a well-known network analysis software known as Pajek [39] to generate three small-world networks, G_{SW_1}, G_{SW_2}, and G_{SW_3} (see Fig. 2). These networks have 100 nodes and edge-rewiring probabilities of 0.1, 0.2, and 0.3, respectively. Second, it is known that many real-world networks are scale-free, i.e., with an approximate power-law degree distribution [40]. We use Pajek to generate three scale-free networks, G_{SF_1}, G_{SF_2}, and G_{SF_3} (see Fig. 3). Finally, we choose a real-world email network G_{EM} from [41] (see Fig. 4).

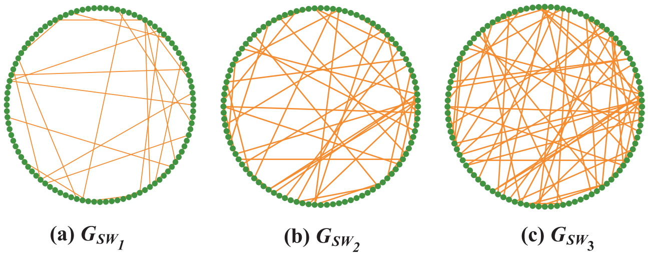

Fig. 2. Three small-world networks with edge-rewiring probabilities of $p = 0.1, 0.2, 0.3,$ respectively.

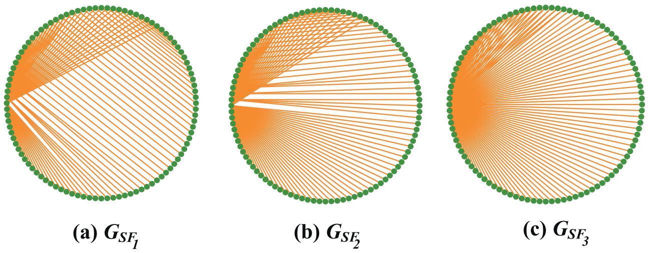

Fig. 3. Three scale-free networks with power-law exponents of r = 2.1, 2.2, and 2.3, respectively.

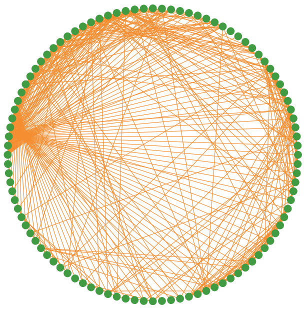

Fig. 4. A real-world email network G_{EM}.

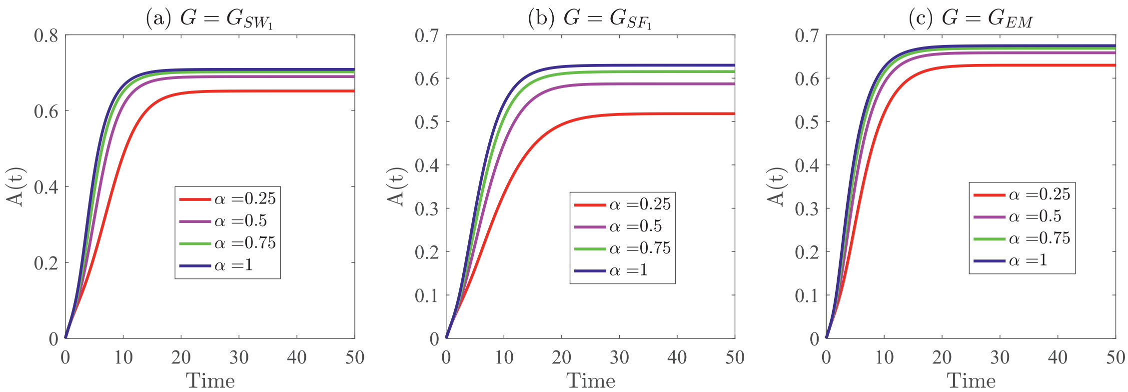

Fig. 5. Time plot of $A(t)$ for each of the DPA models given in Example 1: (a) $G = G_{SW_1}$, (b) $G = G_{SF_1}$, (c) $G = G_{EM}$.

4.1. The influence of the four basic parameters

First, let us examine the influence of the four basic factors (the WOM force, the rigid demand, the lure force, and the viscosity) on the dynamics of a DPA model, respectively. Recall that $A(t)$ denotes the expected fraction of adopters at time t.

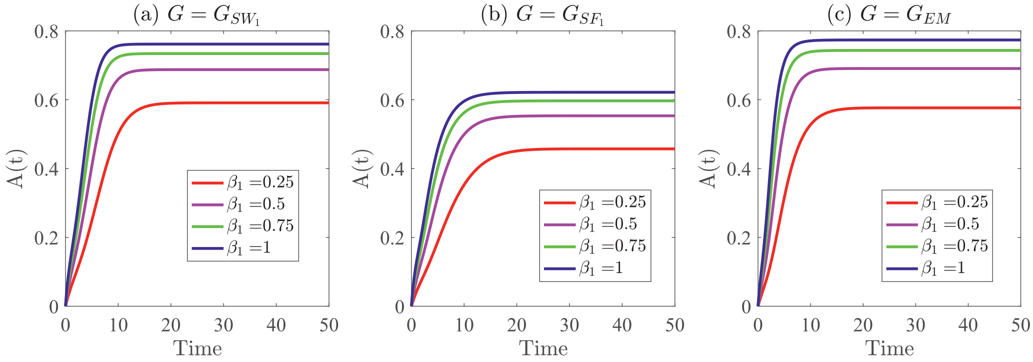

Fig. 6. Time plot of $A(t)$ for each of the DPA models given in Example 2: (a) $G = G_{SW_1}$, (b) $G = G_{SF_1}$, (c) $G = G_{EM}$.

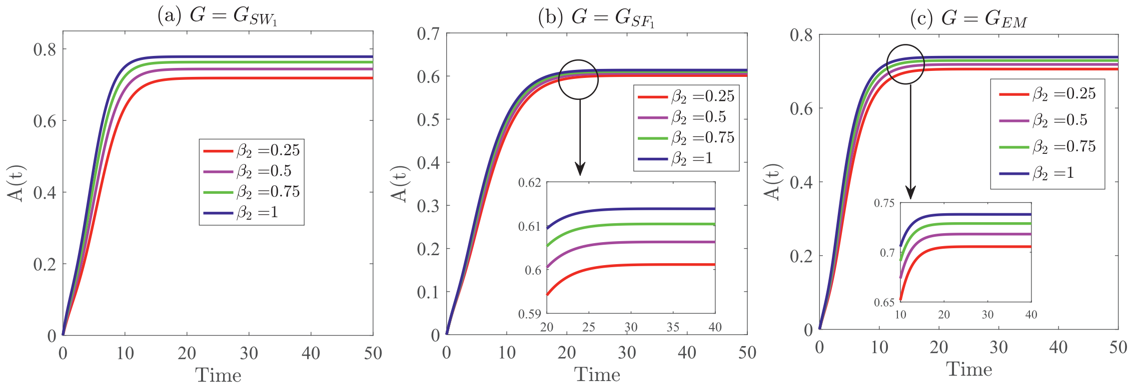

Fig. 7. Time plot of $A(t)$ for each of the DPA models given in Example 3: (a) $G = G_{SW_1}$, (b) $G = G_{SF_1}$, (c) $G = G_{EM}$.

Example 1. Consider a set of DPA models $\mathcal{M}_{DPA} = (G, \alpha, \beta_1, \beta_2, \gamma, \theta, \mathbf{P}(0))$, where $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha \in \{0.25, 0.5, 0.75, 1\}$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\gamma = 0.1$, $\theta = 0.3$, $\mathbf{P}(0) = 0.1 \times \mathbf{1}$. Fig. 5 plots the time plot of $A(t)$ for each of these DPA models. It is seen that $lim_{t\to\infty} A(t)$ exists and is increasing with α.

From this experiment and 100 similar numerical experiments, we conclude that the asymptotic expected fraction of adopters is increasing with the WOM force.

Example 2. Consider a set of DPA models $\mathcal{M}_{DPA} = (G, \alpha, \beta_1, \beta_2, \gamma, \theta, \mathbf{P}(0))$, where $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha = 0.3$, $\beta_1 \in \{0.25, 0.5, 0.75, 1\}$, $\beta_2 = 0.3$, $\gamma = 0.15$, $\theta = 0.4$, $\mathbf{P}(0) = 0.2 \times \mathbf{1}$. Fig. 6 exhibits the time plot of $A(t)$ for each of these DPA models. It is seen that $\lim_{t\to\infty} A(t)$ exists and is increasing with β_1.

From this experiment and 100 similar numerical experiments, we conclude that the asymptotic expected fraction of adopters is increasing with the rigid demand.

Example 3. Consider a set of DPA models $\mathcal{M}_{DPA} = (G, \alpha, \beta_1, \beta_2, \gamma, \theta, \mathbf{P}(0))$, where $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha = 0.3$, $\beta_1 = 0.3$, $\beta_2 \in \{0.25, 0.5, 0.75, 1\}$, $\gamma = 0.1$, $\theta = 0.5$, $\mathbf{P}(0) = 0.3 \times \mathbf{1}$. Fig. 7 depicts the time plot of $A(t)$ for each of the DPA models given in Example 3. It is seen that $\lim_{t\to\infty} A(t)$ exists and is increasing with β_2.

From this experiment and 100 similar numerical experiments, we conclude that the asymptotic expected fraction of adopters is increasing with the lure force.

Example 4. Consider a set of DPA models $\mathcal{M}_{DPA} = (G, \alpha, \beta_1, \beta_2, \gamma, \theta, \mathbf{P}(0))$, where $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha = 0.4$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\gamma \in \{0.25, 0.5, 0.75, 1\}$, $\theta = 0.3$, $\mathbf{P}(0) = 0.4 \times \mathbf{1}$. Fig. 8 portrays the time plot of $A(t)$ for each of the DPA models given in Example 4. It is seen that $\lim_{t\to\infty} A(t)$ exists and is decreasing with γ.

From this experiment and 100 similar numerical experiments, we conclude that the asymptotic expected fraction of adopters is decreasing with the viscosity.

4.2. The influence of the WOM network

Second, let us inspect the influence of the WOM network on the dynamics of the DPA model.

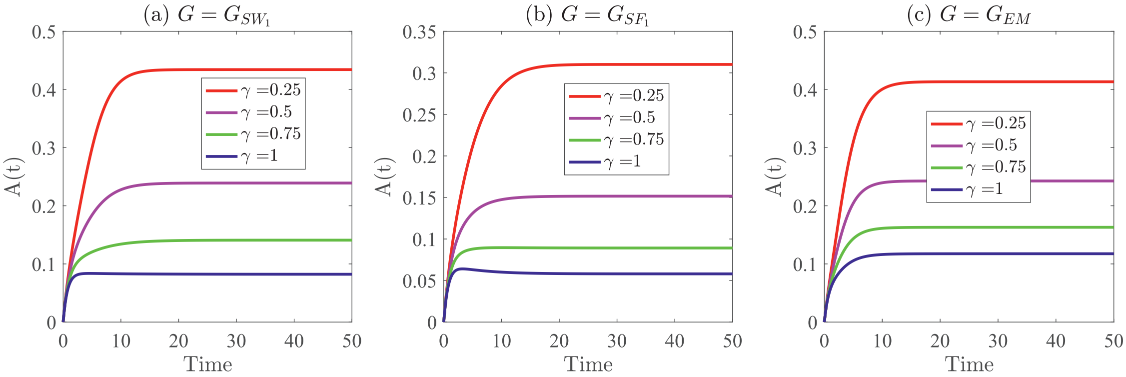

Fig. 8. Time plot of $A(t)$ for each of the DPA models given in Example 4: (a) $G = G_{SW_1}$, (b) $G = G_{SF_1}$, (c) $G = G_{EM}$.

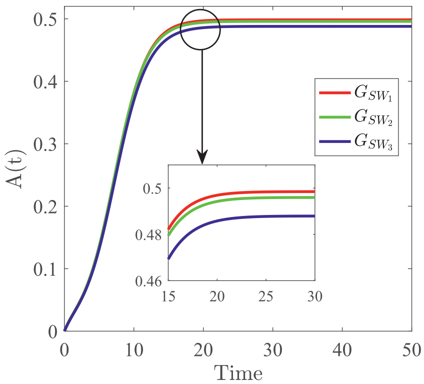

Fig. 9. Time plot of $A(t)$ for each of the DPA models given in Example 5.

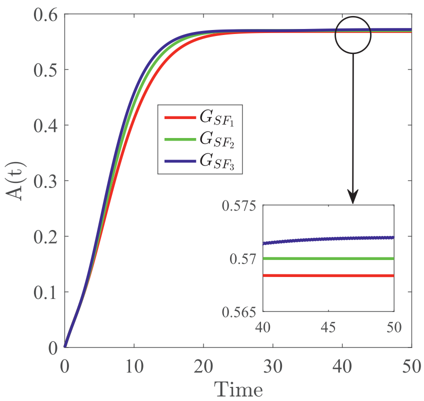

Fig. 10. Time plot of $A(t)$ for each of the DPA models given in Example 6.

Example 5. Consider a set of DPA models $\mathcal{M}_{DPA} = (G, \alpha, \beta_1, \beta_2, \gamma, \theta, \mathbf{A}(0))$, where $G \in \{G_{SW_1}, G_{SW_2}, G_{SW_3}\}$, $\alpha = 0.4$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\gamma = 0.2$, $\theta = 0.3$, $\mathbf{P}(0) = 0.1 \times \mathbf{1}$. Fig. 9 shows the time plot of $A(t)$ for each of these DPA models. It is seen that $\lim_{t\to\infty} A(t)$ exists and is decreasing with the edge-rewiring probability p.

From this experiment and 100 similar numerical experiments, we conclude that the asymptotic expected fraction of adopters is decreasing with the edge-rewiring probability of the small-world WOM network.

Example 6. Consider a set of DPA models $\mathcal{M}_{DPA} = (G, \alpha, \beta_1, \beta_2, \gamma, \theta, \mathbf{A}(0))$, where $G \in \{G_{SF_1}, G_{SF_2}, G_{SF_3}\}$, $\alpha = 0.4$, $\beta_1 = 0.2$, $\beta_2 = 0.3$, $\gamma = 0.1$, $\theta = 0.4$, $\mathbf{P}(0) = 0.2 \times \mathbf{1}$. Fig. 10 exhibits the time plot of $A(t)$ for each of these DPA models. It is seen that $\lim_{t\to\infty} A(t)$ exists and is increasing with the power-law exponent r.

From this experiment and 100 similar numerical experiments, we conclude that the asymptotic expected fraction of adopters is increasing with the power-law exponent of the scale-free WOM network.

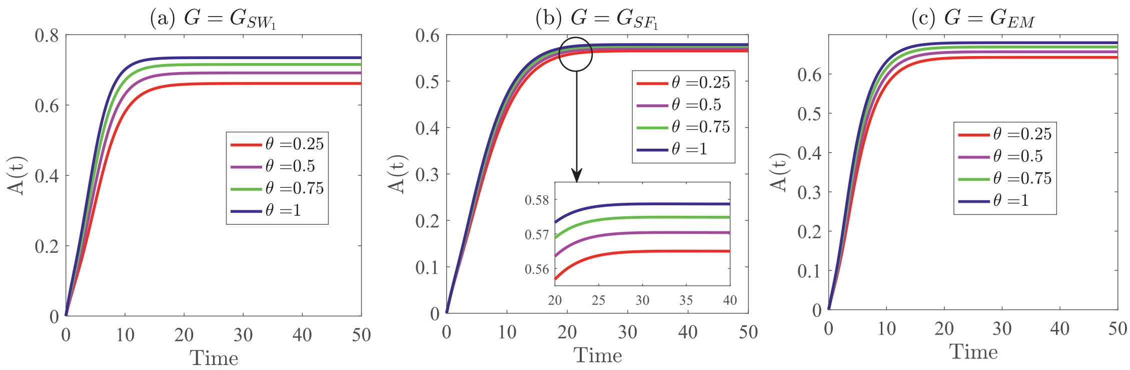

Fig. 11. Time plot of $A(t)$ for each of the DPA models given in Example 7: (a) $G = G_{SW_1}$, (b) $G = G_{SF_1}$, (c) $G = G_{EM}$.

4.3. The influence of the basic discount

Finally, we investigate the influence of the basic discount on the dynamics of the DPA model.

Example 7. Consider a set of DPA models $\mathcal{M}_{DPA} = (G, \alpha, \beta_1, \beta_2, \gamma, \theta, \mathbf{P}(0))$, where $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha = 0.4$, $\beta_1 = 0.2$, $\beta_2 = 0.3$, $\gamma = 0.1$, $\theta \in \{0.25, 0.5, 0.75, 1\}$, $\mathbf{P}(0) = 0.3 \times \mathbf{1}$. Fig. 11 depicts the time plot of $A(t)$ for each of these DPA models. It is seen that $\lim_{t\to\infty} A(t)$ exists and is increasing with θ.

From this experiment and 100 similar numerical experiments, we conclude that the asymptotic expected fraction of adopters is increasing with the basic discount.

5. The performance of an IBD strategy

Based on the discussions made in Sections 2 and 3, an IBD strategy can be described by the ordered pair

$$\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA}). \tag{19}$$

According to Expected Utility Theory [42], we may take the expected net profit of an IBD strategy as the measure of its performance. To estimate the expected net profit, we introduce an added hypothesis as follows.

(H_5) The average gross profit per deal for the supplier is one unit.

It follows from this hypothesis and the DPA model that the expected gross profit of the IBD strategy (19) in the infinitesimal time interval $[t, t+dt)$ is $\sum_{i=1}^{N} (\beta_1 + \beta_2\theta d_i) P_i(t) dt$. So, the expected net profit of the IBD strategy in $[t, t+dt)$ is

$$\sum_{i=1}^{N} (\beta_1 + \beta_2\theta d_i) P_i(t)(1 - \theta d_i) dt. \tag{20}$$

Hence, the expected net profit of the IBD strategy in the time interval $[0, T]$ is

$$EP(\mathcal{A}_{IBD}) = \int_0^T \sum_{i=1}^{N} (\beta_1 + \beta_2\theta d_i) P_i(t)(1 - \theta d_i) dt. \tag{21}$$

This section is devoted to the understanding of the influence of different factors on the performance of an IBD strategy.

5.1. The influence of the four basic parameters

First, let us inspect the influence of the four basic parameters on the performance of an IBD strategy, respectively.

Example 8. Consider a set of IBD strategies $\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA})$, where $T = 50$, $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha \in \{0.1, 0.2, \cdots, 1\}$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\gamma = 0.1$, $\theta = 0.3$, $\mathbf{P}(0) = 0.1 \times \mathbf{1}$. Fig. 12 plots the expected net profit of each of these IBD strategies. It is seen that this quantity is increasing with α.

From this experiment and 100 similar numerical experiments, we conclude that the expected net profit of an IBD strategy is increasing with the WOM force. This finding implies that a supplier may strengthen the WOM force to improve the performance of his/her IBD strategy. In marketing practice, the WOM force can be reinforced by improving the quality or/and the user experience of the promoted products.

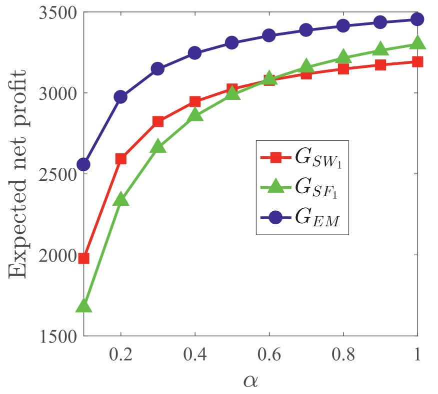

Fig. 12. The expected net profit of an IBD strategy versus the WOM force.

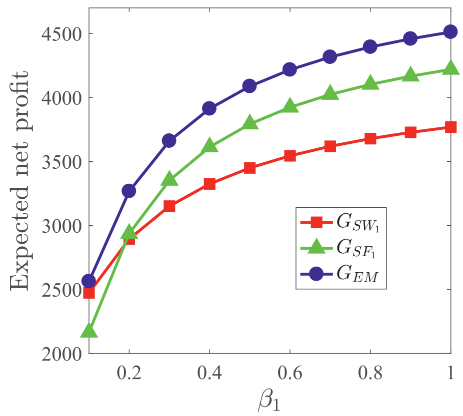

Fig. 13. The expected net profit of an IBD strategy versus the rigid demand.

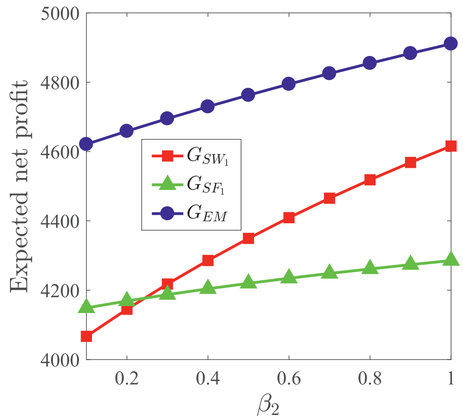

Fig. 14. The expected net profit of an IBD strategy versus the lure force.

Example 9. Consider a set of IBD strategies $\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA})$, where $T = 50$, $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha = 0.4$, $\beta_1 \in \{0.1, 0.2, \cdots, 1\}$, $\beta_2 = 0.4$, $\gamma = 0.1$, $\theta = 0.4$, $\mathbf{P}(0) = 0.2 \times \mathbf{1}$. Fig. 13 shows the expected net profit of each of these IBD strategies. It is seen that this quantity is increasing with β_1.

From this experiment and 100 similar numerical experiments, we conclude that the expected net profit of an IBD strategy is increasing with the rigid demand. In marketing practice, the rigid demand is not under control of the supplier.

Example 10. Consider a set of IBD strategies $\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA})$, where $T = 50$, $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha = 0.5$, $\beta_1 = 0.3$, $\beta_2 \in \{0.1, 0.2, \cdots, 1\}$, $\gamma = 0.15$, $\theta = 0.3$, $\mathbf{P}(0) = 0.3 \times \mathbf{1}$. Fig. 14 shows the expected net profit of each of these IBD strategies. It is seen that this quantity is increasing with β_2.

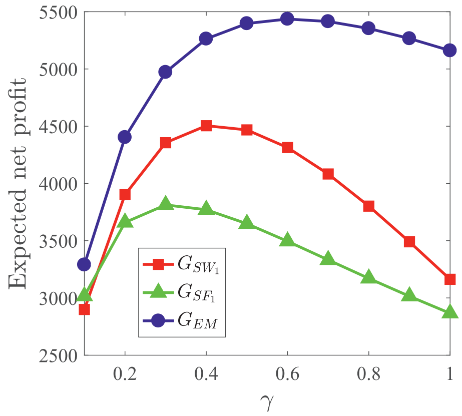

Fig. 15. The expected net profit of an IBD strategy versus the viscosity.

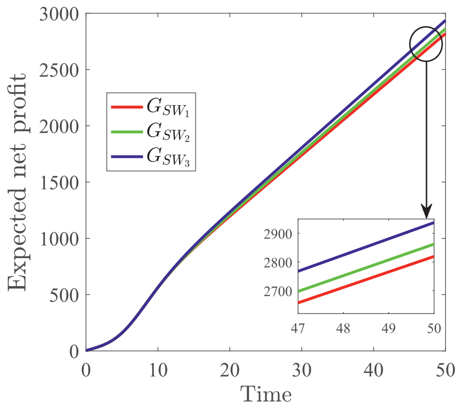

Fig. 16. The expected net profit of an IBD strategy versus the edge-rewiring probability.

From this experiment and 100 similar numerical experiments, we conclude that the expected net profit of an IBD strategy is increasing with the lure force. This finding implies that the supplier may reinforce the lure force to improve the performance of his/her IBD strategy. In marketing practice, the lure force can be intensified by taking promotional measures such as providing small gifts or offering coupons.

Example 11. Consider a set of IBD strategies $\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA})$, where $T = 50$, $G \in \{G_{SW_1}, G_{SF_1}, G_{EM}\}$, $\alpha = 0.4$, $\beta_1 = 0.2$, $\beta_2 = 0.3$, $\gamma \in \{0.1, 0.2, \cdots, 1\}$, $\theta = 0.4$, $\mathbf{P}(0) = 0.4 \times \mathbf{1}$. Fig. 15 shows the expected net profit of each of these IBD strategies. It is seen that this quantity is first increasing then decreasing with γ.

From this experiment and 100 similar numerical experiments, we conclude that the expected net profit of an IBD strategy is first increasing then decreasing with the viscosity. This finding implies that before reaching the turning point, the supplier may increase the viscosity to improve the performance of the IBD strategy. In marketing practice, the viscosity can be enhanced by taking measures such as continually pushing new discount information.

5.2. The influence of the WOM network

Second, we examine the influence of the WOM network on the performance of an IBD strategy.

Example 12. Consider a set of IBD strategies $\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA})$, where $T = 50$, $G \in \{G_{SW_1}, G_{SW_2}, G_{SW_3}\}$, $\alpha = 0.4$, $\beta_1 = 0.2$, $\beta_2 = 0.3$, $\gamma = 0.1$, $\theta = 0.3$, $\mathbf{P}(0) = 0.1 \times \mathbf{1}$. Fig. 16 shows the expected net profit of each of these IBD strategies. It is seen that this quantity is increasing with the edge-rewiring probability p.

From this experiment and 100 similar numerical experiments, we conclude that the expected net profit of an IBD strategy is increasing with the randomness of the associated small-world WOM network.

Example 13. Consider a set of IBD strategies $\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA})$, where $T = 50$, $G \in \{G_{SF_1}, G_{SF_2}, G_{SF_3}\}$, $\alpha = 0.5$, $\beta_1 = 0.3$, $\beta_2 = 0.4$, $\gamma = 0.1$, $\theta = 0.4$, $\mathbf{P}(0) = 0.2 \times \mathbf{1}$. Fig. 17 depicts the expected net profit of each of these IBD strategies. It is seen that this quantity is increasing with the power-law exponent r.

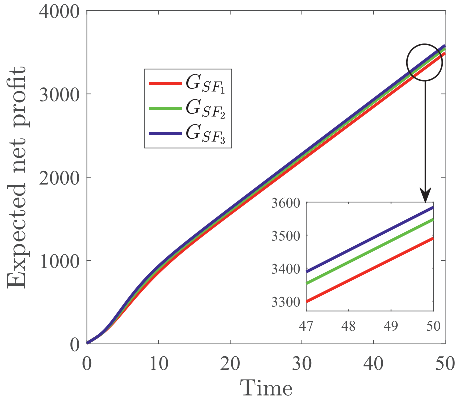

Fig. 17. The expected net profit of an IBD strategy versus the power-law exponent.

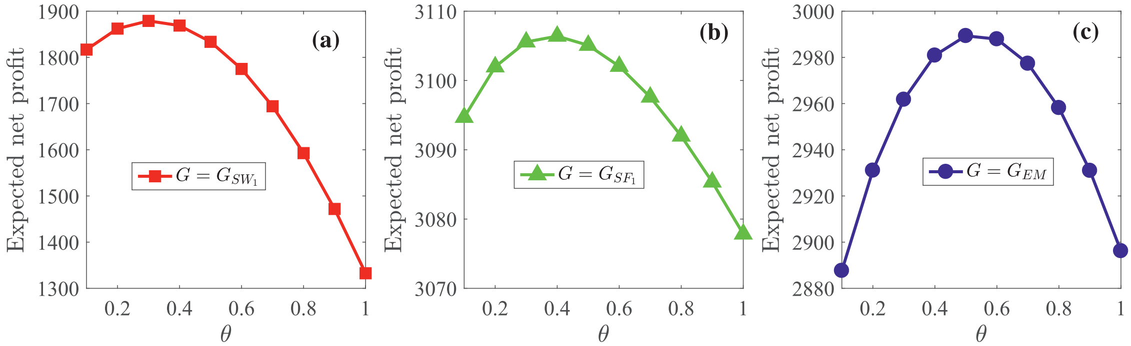

Fig. 18. The expected net profit of an IBD strategy versus the basic discount.

From this experiment and 100 similar numerical experiments, we conclude that the expected net profit of an IBD strategy is increasing with the heterogeneity of the associated scale-free WOM network.

5.3. The influence of the basic discount

Finally, we study the influence of the basic discount on the performance of the IBD strategy.

Example 14. Consider a set of IBD strategies $\mathcal{A}_{IBD} = (T, \mathcal{M}_{DPA})$, where $T = 50$, $\mathbf{P}(0) = 0.2 \times \mathbf{1}$, $\theta \in \{0, 0.1, \cdots, 1\}$.

(a) $G = G_{SW_1}$, $\alpha = 0.2$, $\beta_1 = 0.1$, $\beta_2 = 0.2$, $\gamma = 0.4$.
(b) $G = G_{SF_1}$, $\alpha = 0.5$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\gamma = 0.1$.
(c) $G = G_{EM}$, $\alpha = 0.3$, $\beta_1 = 0.1$, $\beta_2 = 0.2$, $\gamma = 0.3$.

Fig. 18 depicts the expected net profit of each of these IBD strategies. It is seen that this quantity is first increasing then decreasing with θ.

From this experiment and 100 similar numerical experiments, we conclude that the expected net profit of an IBD strategy is first increasing then decreasing with the basic discount. This phenomenon can be explained as follows. When the basic discount is very lower, the marketing campaign is less attractive, leading to a lower marketing profit. Whereas when the basic discount is very high, the high gross profit is severely diminished by the high discounts, leading to a lower marketing profit as well. In marketing practice, an in-between basic discount would help the supplier to achieve the maximum possible expected net profit.

6. Conclusions and remarks

In the context of WOM marketing, a novel discount strategy has been proposed. By measuring the expected marketing profit of a discount strategy, the performance of the discount strategy has been evaluated. On this basis, some promotional measures have been recommended.

Towards this direction, there are several problems that are yet to be resolved. First, the influential degree defined in the proposed discount strategy may be replaced with other influence indexes [43–46], resulting in different discount strategies.

As a result, a comparative study of different discount strategies is rewarding. Second, the static discount strategy proposed in this work may be made dynamic to further enhance the marketing profit [47–51]. Finally, a WOM marketing campaign with discount pricing is essentially a non-cooperative game between the supplier and the target market. Hence, it is appropriate to address the discount pricing problem in the framework of game theory [52–55].

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