Part Two deals with the transfer of data between two devices that are directly connected; that is, the two devices are linked by a single transmission path rather than a network. Even this simple context introduces numerous technical and design issues. First, we need to understand something about the process of transmitting signals across a communications link. Both analog and digital transmission techniques are used. In both cases, the signal can be described as consisting of a spectrum of components across a range of electromagnetic frequencies. The transmission properties of the signal depend on which frequencies are involved. Also, the types of impairments, such as attenuation, that a signal suffers are dependent on frequency. A separate concern is the transmission medium used to transmit signals, which is a factor in determining what performance can be achieved, in terms of data rate and distance. Closely tied to considerations of the signal and the medium is the way in which data are encoded on the signal. Again, the encoding technique is a factor in transmission performance.

Beyond the fundamental concepts of signal, medium, and encoding, Part Two deals with two other important aspects of data communications: reliability and efficiency. In any communications scheme, there will be a certain rate of errors suffered during transmission. A data link control protocol provides mechanisms for detecting and recovering from such errors, so that a potentially unreliable transmission path is turned into a reliable data communications link. Finally, if the capacity of the link is greater than the requirements for a single transmission, then a variety of multiplexing techniques can be used to provide for efficient use of the medium.

ROAD MAP FOR PART TWO

Chapter 3 Data Transmission

The principles of data transmission underlie all of the concepts and techniques presented in this book. To understand the need for encoding, multiplexing, switching, error control, and so on, the reader should understand
the behavior of data signals propagated through a transmission medium. Chapter 3 discusses the distinction between digital and analog data and digital and analog transmission. Concepts of attenuation and noise are also examined.

Chapter 4 Transmission Media
Transmission media can be classified as either guided or wireless. The most commonly used guided transmission media are twisted pair, coaxial cable, and optical fiber. Wireless techniques include terrestrial and satellite microwave, broadcast radio, and infrared. Chapter 4 covers all of these topics.

Chapter 5 Signal Encoding Techniques
Data come in both analog (continuous) and digital (discrete) form. For transmission, input data must be encoded as an electrical signal that is tailored to the characteristics of the transmission medium. Both analog and digital data can be represented by either analog or digital signals; each of the four cases is discussed in Chapter 5.

Chapter 6 Digital Data Communication Techniques
In Chapter 6, the emphasis shifts from data transmission to data communications. For two devices linked by a transmission medium to exchange digital data, a high degree of cooperation is required. Typically, data are transmitted one bit at a time over the medium. The timing (rate, duration, spacing) of these bits must be the same for transmitter and receiver. Two common communication techniques—asynchronous and synchronous—are explored. Following this, the chapter examines the topics of transmission errors and error detection and correction techniques.

Chapter 7 Data Link Control Protocols
True cooperative exchange of digital data between two devices requires some form of data link control. Chapter 7 examines the fundamental techniques common to all data link control protocols, including flow control and error control, and then examines the most commonly used protocol, HDLC.

Chapter 8 Multiplexing
Transmission facilities are, by and large, expensive. It is often the case that two communication stations will not utilize the full capacity of a data link. For efficiency, it should be possible to share that capacity. The generic term for such sharing is multiplexing.
Chapter 8 concentrates on the three most common types of multiplexing techniques. The first, frequency division multiplexing (FDM), is the most widespread and is familiar to anyone who has ever used a radio or television set. The second is a particular case of time division multiplexing (TDM), often known as synchronous TDM. This is commonly used for multiplexing digitized voice streams. The third type is another form of TDM that is more complex but potentially more efficient than synchronous TDM; it is referred to as statistical or asynchronous TDM.

Chapter 9 Spread Spectrum
An increasingly popular form of wireless communications is known as spread spectrum. Two general approaches are used: frequency hopping and direct sequence spread spectrum. Chapter 9 provides an overview of both techniques. The chapter also looks at the concept of code division multiple access (CDMA), which is an application of spread spectrum to provide multiple access.
CHAPTER 3

DATA TRANSMISSION

3.1 Concepts and Terminology
3.2 Analog and Digital Data Transmission
3.3 Transmission Impairments
3.4 Channel Capacity
3.5 Recommended Reading and Web Site
3.6 Key Terms, Review Questions, and Problems
3.6 Appendix 3A Decibels and Signal Strength
CHAPTER 3 / DATA TRANSMISSION

Toto, I've got a feeling we're not in Kansas anymore.

Judy Garland in The Wizard of Oz

KEY POINTS

• All of the forms of information that are discussed in this book (voice, data, image, video) can be represented by electromagnetic signals. Depending on the transmission medium and the communications environment, either analog or digital signals can be used to convey information.

• Any electromagnetic signal, analog or digital, is made up of a number of constituent frequencies. A key parameter that characterizes the signal is bandwidth, which is the width of the range of frequencies that comprises the signal. In general, the greater the bandwidth of the signal, the greater its information-carrying capacity.

• A major problem in designing a communications facility is transmission impairment. The most significant impairments are attenuation, attenuation distortion, delay distortion, and the various types of noise. The various forms of noise include thermal noise, intermodulation noise, crosstalk, and impulse noise. For analog signals, transmission impairments introduce random effects that degrade the quality of the received information and may affect intelligibility. For digital signals, transmission impairments may cause bit errors at the receiver.

• The designer of a communications facility must deal with four factors: the bandwidth of the signal, the data rate that is used for digital information, the amount of noise and other impairments, and the level of error rate that is acceptable. The bandwidth is limited by the transmission medium and the desire to avoid interference with other nearby signals. Because bandwidth is a scarce resource, we would like to maximize the data rate that is achieved in a given bandwidth. The data rate is limited by the bandwidth, the presence of impairments, and the error rate that is acceptable.

The successful transmission of data depends principally on two factors: the quality of the signal being transmitted and the characteristics of the transmission medium. The objective of this chapter and the next is to provide the reader with an intuitive feeling for the nature of these two factors.

The first section presents some concepts and terms from the field of electrical engineering. This should provide sufficient background to deal with the remainder of the chapter. Section 3.2 clarifies the use of the terms analog and digital. Either analog or digital data may be transmitted using either analog or digital signals. Furthermore, it is common for intermediate processing to be performed between source and destination, and this processing has either an analog or digital character.
Section 3.3 looks at the various impairments that may introduce errors into the data during transmission. The chief impairments are attenuation, attenuation distortion, delay distortion, and the various forms of noise. Finally, we look at the important concept of channel capacity.

### 3.1 CONCEPTS AND TERMINOLOGY

In this section we introduce some concepts and terms that will be referred to throughout the rest of the chapter and, indeed, throughout Part Two.

**Transmission Terminology**

Data transmission occurs between transmitter and receiver over some transmission medium. Transmission media may be classified as guided or unguided. In both cases, communication is in the form of electromagnetic waves. With **guided media**, the waves are guided along a physical path; examples of guided media are twisted pair, coaxial cable, and optical fiber. **Unguided media**, also called **wireless**, provide a means for transmitting electromagnetic waves but do not guide them; examples are propagation through air, vacuum, and seawater.

The term **direct link** is used to refer to the transmission path between two devices in which signals propagate directly from transmitter to receiver with no intermediate devices, other than amplifiers or repeaters used to increase signal strength. Note that this term can apply to both guided and unguided media.

A guided transmission medium is **point to point** if it provides a direct link between two devices and those are the only two devices sharing the medium. In a **multipoint** guided configuration, more than two devices share the same medium.

A transmission may be simplex, half duplex, or full duplex. In **simplex** transmission, signals are transmitted in only one direction; one station is transmitter and the other is receiver. In **half-duplex** operation, both stations may transmit, but only one at a time. In **full-duplex** operation, both stations may transmit simultaneously. In the latter case, the medium is carrying signals in both directions at the same time. How this can be is explained in due course. We should note that the definitions just given are the ones in common use in the United States (ANSI definitions). Elsewhere (ITU-T definitions), the term **simplex** is used to correspond to **half duplex** as defined previously, and **duplex** is used to correspond to **full duplex** as just defined.

**Frequency, Spectrum, and Bandwidth**

In this book, we are concerned with electromagnetic signals used as a means to transmit data. At point 3 in Figure 1.3, a signal is generated by the transmitter and transmitted over a medium. The signal is a function of time, but it can also be expressed as a function of frequency; that is, the signal consists of components of different frequencies. It turns out that the **frequency domain** view of a signal is more important to an understanding of data transmission than a **time domain** view. Both views are introduced here.

**Time Domain Concepts** Viewed as a function of time, an electromagnetic signal can be either analog or digital. An **analog signal** is one in which the signal intensity
varies in a smooth fashion over time. In other words, there are no breaks or discontinuities in the signal.\footnote{A mathematical definition: a signal \( s(t) \) is continuous if \( \lim_{a \to t} s(a) = s(t) \) for all \( a \).} A \textbf{digital signal} is one in which the signal intensity maintains a constant level for some period of time and then abruptly changes to another constant level.\footnote{This is an idealized definition. In fact, the transition from one voltage level to another will not be instantaneous, but there will be a small transition period. Nevertheless, an actual digital signal approximates closely the ideal model of constant voltage levels with instantaneous transitions.} Figure 3.1 shows an example of each kind of signal. The continuous signal might represent speech, and the discrete signal might represent binary 1s and 0s.

The simplest sort of signal is a \textbf{periodic signal}, in which the same signal pattern repeats over time. Figure 3.2 shows an example of a periodic continuous signal (sine wave) and a periodic discrete signal (square wave). Mathematically, a signal \( s(t) \) is defined to be periodic if and only if

\[
\lim_{t \to a} s(t) = s(a)
\]

where the constant \( T \) is the period of the signal (\( T \) is the smallest value that satisfies the equation). Otherwise, a signal is \textbf{aperiodic}.

The sine wave is the fundamental periodic signal. A general sine wave can be represented by three parameters: peak amplitude (\( A \)), frequency (\( f \)), and phase (\( \phi \)). The \textbf{peak amplitude} is the maximum value or strength of the signal over time; typically, this value is measured in volts. The \textbf{frequency} is the rate [in cycles per
second, or Hertz (Hz)] at which the signal repeats. An equivalent parameter is the \textbf{period} \((T)\) of a signal, which is the amount of time it takes for one repetition; therefore, \(T = 1/f\). \textbf{Phase} is a measure of the relative position in time within a single period of a signal, as is illustrated subsequently. More formally, for a periodic signal \(f(t)\), phase is the fractional part \(t/T\) of the period \(T\) through which \(t\) has advanced relative to an arbitrary origin. The origin is usually taken as the last previous passage through zero from the negative to the positive direction.

The general sine wave can be written

\[
s(t) = A \sin(2\pi ft + \phi)
\]

A function with the form of the preceding equation is known as a \textbf{sinusoid}. Figure 3.3 shows the effect of varying each of the three parameters. In part (a) of the figure, the frequency is 1 Hz; thus the period is \(T = 1\) second. Part (b) has the same frequency and phase but a peak amplitude of 0.5. In part (c) we have \(f = 2\), which is equivalent to \(T = 0.5\). Finally, part (d) shows the effect of a phase shift of \(\pi/4\) radians, which is 45 degrees \((2\pi\text{ radians} = 360^\circ = 1\text{ period})\).
In Figure 3.3, the horizontal axis is time; the graphs display the value of a signal at a given point in space as a function of time. These same graphs, with a change of scale, can apply with horizontal axes in space. In this case, the graphs display the value of a signal at a given point in time as a function of distance. For example, for a sinusoidal transmission (e.g., an electromagnetic radio wave some distance from a radio antenna, or sound some distance from a loudspeaker), at a particular instant of time, the intensity of the signal varies in a sinusoidal way as a function of distance from the source.

There is a simple relationship between the two sine waves, one in time and one in space. The wavelength ($\lambda$) of a signal is the distance occupied by a single cycle, or, put another way, the distance between two points of corresponding phase of two consecutive cycles. Assume that the signal is traveling with a velocity $v$. Then the wavelength is related to the period as follows: $\lambda = vT$. Equivalently, $\lambda f = v$. Of particular relevance to this discussion is the case where $v = c$, the speed of light in free space, which is approximately $3 \times 10^8$ m/s.

**Frequency Domain Concepts** In practice, an electromagnetic signal will be made up of many frequencies. For example, the signal

$$s(t) = A \sin(2\pi ft + \phi)$$
is shown in Figure 3.4c. The components of this signal are just sine waves of frequencies $f$ and $3f$; parts (a) and (b) of the figure show these individual components. There are two interesting points that can be made about this figure:

\[
s(t) = \left[ \frac{4}{\pi} \times (\sin(2\pi ft) + \frac{1}{3}\sin(2\pi (3f)t)) \right]
\]

**Figure 3.4** Addition of Frequency Components ($T = \frac{1}{f}$)

---

\[^3\]The scaling factor of $4/\pi$ is used to produce a wave whose peak amplitude is close to 1.
• The second frequency is an integer multiple of the first frequency. When all of
the frequency components of a signal are integer multiples of one frequency,
the latter frequency is referred to as the **fundamental frequency**.

• The period of the total signal is equal to the period of the fundamental
frequency. The period of the component sin(2πft) is \( T = 1/f \), and the period
of \( s(t) \) is also \( T \), as can be seen from Figure 3.4c.

It can be shown, using a discipline known as Fourier analysis, that any signal is
made up of components at various frequencies, in which each component is a sinu-
soid. By adding together enough sinusoidal signals, each with the appropriate ampli-
tude, frequency, and phase, any electromagnetic signal can be constructed. Put
another way, any electromagnetic signal can be shown to consist of a collection of
periodic analog signals (sine waves) at different amplitudes, frequencies, and phases.
The importance of being able to look at a signal from the frequency perspective
(frequency domain) rather than a time perspective (time domain) should become
clear as the discussion proceeds. For the interested reader, the subject of Fourier
analysis is introduced in Appendix A.

So we can say that for each signal, there is a time domain function \( s(t) \) that speci-
fies the amplitude of the signal at each instant in time. Similarly, there is a frequency
domain function \( S(f) \) that specifies the peak amplitude of the constituent frequencies of
the signal. Figure 3.5a shows the frequency domain function for the signal of Figure 3.4c.
Note that, in this case, \( S(f) \) is discrete. Figure 3.5b shows the frequency domain function
for a single square pulse that has the value 1 between \( -X/2 \) and \( X/2 \), and is 0 elsewhere.\(^4\)
Note that in this case \( S(f) \) is continuous and that it has nonzero values indefinitely,
although the magnitude of the frequency components rapidly shrinks for larger \( f \).
These characteristics are common for real signals.

The **spectrum** of a signal is the range of frequencies that it contains. For the
signal of Figure 3.4c, the spectrum extends from \( f \) to \( 3f \). The **absolute bandwidth**
of a signal is the width of the spectrum. In the case of Figure 3.4c, the bandwidth
is \( 2f \). Many signals, such as that of Figure 3.5b, have an infinite bandwidth.
However, most of the energy in the signal is contained in a relatively narrow
band of frequencies. This band is referred to as the **effective bandwidth**, or just
**bandwidth**.

One final term to define is **dc component**. If a signal includes a component of
zero frequency, that component is a direct current (dc) or constant component. For
example, Figure 3.6 shows the result of adding a dc component to the signal of
Figure 3.4c. With no dc component, a signal has an average amplitude of zero, as
seen in the time domain. With a dc component, it has a frequency term at \( f = 0 \) and
a nonzero average amplitude.

**Relationship between Data Rate and Bandwidth** We have said that effective
bandwidth is the band within which most of the signal energy is concentrated. The
meaning of the term *most* in this context is somewhat arbitrary. The important issue

\(^4\)In fact, the function \( S(f) \) for this case is symmetric around \( f = 0 \) and so has values for negative fre-
quencies. The presence of negative frequencies is a mathematical artifact whose explanation is beyond
the scope of this book.
here is that, although a given waveform may contain frequencies over a very broad range, as a practical matter any transmission system (transmitter plus medium plus receiver) will be able to accommodate only a limited band of frequencies. This, in turn, limits the data rate that can be carried on the transmission medium.

To try to explain these relationships, consider the square wave of Figure 3.2b. Suppose that we let a positive pulse represent binary 0 and a negative pulse represent binary 1. Then the waveform represents the binary stream 0101... . The duration of each pulse is 1/(2f); thus the data rate is 2f bits per second (bps). What are the frequency components of this signal? To answer this question, consider again
Figure 3.6  Signal with dc Component

Figure 3.4. By adding together sine waves at frequencies \( f \) and \( 3f \), we get a waveform that begins to resemble the original square wave. Let us continue this process by adding a sine wave of frequency \( 5f \), as shown in Figure 3.7a, and then adding a sine wave of frequency \( 7f \), as shown in Figure 3.7b. As we add additional odd multiples of \( f \), suitably scaled, the resulting waveform approaches that of a square wave more and more closely.

Indeed, it can be shown that the frequency components of the square wave with amplitudes \( A \) and \(-A\) can be expressed as follows:

\[
s(t) = A \times \frac{4}{\pi} \times \sum_{k \text{ odd}, k=1}^{\infty} \frac{\sin(2\pi k ft)}{k}
\]

Thus, this waveform has an infinite number of frequency components and hence an infinite bandwidth. However, the peak amplitude of the \( k \)th frequency component, \( kf \), is only \( 1/k \), so most of the energy in this waveform is in the first few frequency components. What happens if we limit the bandwidth to just the first three frequency components? We have already seen the answer, in Figure 3.7a. As we can
see, the shape of the resulting waveform is reasonably close to that of the original square wave.

We can use Figures 3.4 and 3.7 to illustrate the relationship between data rate and bandwidth. Suppose that we are using a digital transmission system that is capable of transmitting signals with a bandwidth of 4 MHz. Let us attempt to transmit a sequence of alternating 1s and 0s as the square wave of Figure 3.7c. What data rate can be achieved? We look at three cases.

**Case I.** Let us approximate our square wave with the waveform of Figure 3.7a. Although this waveform is a “distorted” square wave, it is sufficiently close to the square wave that a receiver should be able to discriminate between a binary 0
and a binary 1. If we let \( f = 10^6 \) cycles/second = 1 MHz, then the bandwidth of the signal

\[
s(t) = \frac{4}{\pi} \sin((2\pi \times 10^6)t) + \frac{1}{3} \sin((2\pi \times 3 \times 10^6)t) + \frac{1}{5} \sin((2\pi \times 5 \times 10^6)t)
\]

is \((5 \times 10^6) - 10^6 = 4\) MHz. Note that for \( f = 1\) MHz, the period of the fundamental frequency is \( T = 1/10^6 = 10^{-6} = 1\) \(\mu\)s. If we treat this waveform as a bit string of 1s and 0s, one bit occurs every 0.5 \(\mu\)s, for a data rate of \(2 \times 10^6 = 2\) Mbps. Thus, for a bandwidth of 4 MHz, a data rate of 2 Mbps is achieved.

**Case II.** Now suppose that we have a bandwidth of 8 MHz. Let us look again at Figure 3.7a, but now with \( f = 2\) MHz. Using the same line of reasoning as before, the bandwidth of the signal is \((5 \times 2 \times 10^6) - (2 \times 10^6) = 8\) MHz. But in this case \( T = 1/f = 0.5\) \(\mu\)s. As a result, one bit occurs every 0.25 \(\mu\)s for a data rate of 4 Mbps. Thus, other things being equal, by doubling the bandwidth, we double the potential data rate.

**Case III.** Now suppose that the waveform of Figure 3.4c is considered adequate for approximating a square wave. That is, the difference between a positive and negative pulse in Figure 3.4c is sufficiently distinct that the waveform can be successfully used to represent a sequence of 1s and 0s. Assume as in Case II that \( f = 2\) MHz and \( T = 1/f = 0.5\) \(\mu\)s, so that one bit occurs every 0.25 \(\mu\)s for a data rate of 4 Mbps. Using the waveform of Figure 3.4c, the bandwidth of the signal is \((3 \times 2 \times 10^6) - (2 \times 10^6) = 4\) MHz. Thus, a given bandwidth can support various data rates depending on the ability of the receiver to discern the difference between 0 and 1 in the presence of noise and other impairments.

To summarize,

- **Case I:** Bandwidth = 4 MHz; data rate = 2 Mbps
- **Case II:** Bandwidth = 8 MHz; data rate = 4 Mbps
- **Case III:** Bandwidth = 4 MHz; data rate = 4 Mbps

We can draw the following conclusions from the preceding discussion. In general, any digital waveform will have infinite bandwidth. If we attempt to transmit this waveform as a signal over any medium, the transmission system will limit the bandwidth that can be transmitted. Furthermore, for any given medium, the greater the bandwidth transmitted, the greater the cost. Thus, on the one hand, economic and practical reasons dictate that digital information be approximated by a signal of limited bandwidth. On the other hand, limiting the bandwidth creates distortions, which makes the task of interpreting the received signal more difficult. The more limited the bandwidth, the greater the distortion, and the greater the potential for error by the receiver.

One more illustration should serve to reinforce these concepts. Figure 3.8 shows a digital bit stream with a data rate of 2000 bits per second. With a bandwidth of 2500 Hz, or even 1700 Hz, the representation is quite good. Furthermore, we can generalize these results. If the data rate of the digital signal is \( W \) bps, then a very
good representation can be achieved with a bandwidth of $2W$ Hz. However, unless noise is very severe, the bit pattern can be recovered with less bandwidth than this (see the discussion of channel capacity in Section 3.4).

Thus, there is a direct relationship between data rate and bandwidth: The higher the data rate of a signal, the greater is its required effective bandwidth. Looked at the other way, the greater the bandwidth of a transmission system, the higher is the data rate that can be transmitted over that system.

Another observation worth making is this: If we think of the bandwidth of a signal as being centered about some frequency, referred to as the center frequency, then the higher the center frequency, the higher the potential bandwidth and therefore the higher the potential data rate. For example, if a signal is centered at 2 MHz, its maximum potential bandwidth is 4 MHz.
CHAPTER 3 / DATA TRANSMISSION

We return to a discussion of the relationship between bandwidth and data rate in Section 3.4, after a consideration of transmission impairments.

3.2 ANALOG AND DIGITAL DATA TRANSMISSION

The terms analog and digital correspond, roughly, to continuous and discrete, respectively. These two terms are used frequently in data communications in at least three contexts: data, signaling, and transmission.

Briefly, we define data as entities that convey meaning, or information. Signals are electric or electromagnetic representations of data. Signaling is the physical propagation of the signal along a suitable medium. Transmission is the communication of data by the propagation and processing of signals. In what follows, we try to make these abstract concepts clear by discussing the terms analog and digital as applied to data, signals, and transmission.

Analog and Digital Data

The concepts of analog and digital data are simple enough. Analog data take on continuous values in some interval. For example, voice and video are continuously varying patterns of intensity. Most data collected by sensors, such as temperature and pressure, are continuous valued. Digital data take on discrete values; examples are text and integers.

The most familiar example of analog data is audio, which, in the form of acoustic sound waves, can be perceived directly by human beings. Figure 3.9 shows the acoustic spectrum for human speech and for music. Frequency components of typical speech may be found between approximately 100 Hz and 7 kHz. Although much of the energy in speech is concentrated at the lower frequencies, tests have shown that frequencies below 600 or 700 Hz add very little to the intelligibility of speech to the human ear. Typical speech has a dynamic range of about 25 dB; that is, the power produced by the loudest shout may be as much as 300 times greater than the least whisper. Figure 3.9 also shows the acoustic spectrum and dynamic range for music.

Another common example of analog data is video. Here it is easier to characterize the data in terms of the TV screen (destination) rather than the original scene (source) recorded by the TV camera. To produce a picture on the screen, an electron beam scans across the surface of the screen from left to right and top to bottom. For black-and-white television, the amount of illumination produced (on a scale from black to white) at any point is proportional to the intensity of the beam as it passes that point. Thus at any instant in time the beam takes on an analog value of intensity to produce the desired brightness at that point on the screen. Further, as

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5 Note the use of a log scale for the x-axis. Because the y-axis is in units of decibels, it is effectively a log scale also. A basic review of log scales is in the math refresher document at the Computer Science Student Resource Site at WilliamStallings.com/StudentSupport.html.

6 The concept of decibels is explained in Appendix 3A.
the beam scans, the analog value changes. Thus the video image can be thought of as a time-varying analog signal.

Figure 3.10 depicts the scanning process. At the end of each scan line, the beam is swept rapidly back to the left (horizontal retrace). When the beam reaches the bottom, it is swept rapidly back to the top (vertical retrace). The beam is turned off (blanked out) during the retrace intervals.

To achieve adequate resolution, the beam produces a total of 483 horizontal lines at a rate of 30 complete scans of the screen per second. Tests have shown that this rate will produce a sensation of flicker rather than smooth motion. To provide a flicker-free image without increasing the bandwidth requirement, a technique known as **interlacing** is used. As Figure 3.10 shows, the odd numbered scan lines and the even numbered scan lines are scanned separately, with odd and even fields alternating on successive scans. The odd field is the scan from A to B and the even field is the scan from C to D. The beam reaches the middle of the screen’s lowest line after 241.5 lines. At this point, the beam is quickly repositioned at the top of the screen and recommences in the middle of the screen’s topmost visible line to produce an additional 241.5 lines interlaced with the original set. Thus the screen is refreshed 60 times per second rather than 30, and flicker is avoided.

A familiar example of digital data is **text** or character strings. While textual data are most convenient for human beings, they cannot, in character form, be easily stored or transmitted by data processing and communications systems. Such systems are designed for binary data. Thus a number of codes have been devised by which characters are represented by a sequence of bits. Perhaps the earliest common example of this is the Morse code. Today, the most commonly used text code is the
International Reference Alphabet (IRA). Each character in this code is represented by a unique 7-bit pattern; thus 128 different characters can be represented. This is a larger number than is necessary, and some of the patterns represent invisible control characters. IRA-encoded characters are almost always stored and transmitted using 8 bits per character. The eighth bit is a parity bit used for error detection. This bit is set such that the total number of binary 1s in each octet is always odd (odd parity) or always even (even parity). Thus a transmission error that changes a single bit, or any odd number of bits, can be detected.

**Analog and Digital Signals**

In a communications system, data are propagated from one point to another by means of electromagnetic signals. An **analog signal** is a continuously varying electromagnetic wave that may be propagated over a variety of media, depending on

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\[7\text{IRA is defined in ITU-T Recommendation T.50 and was formerly known as International Alphabet Number 5 (IA5). The U.S. national version of IRA is referred to as the American Standard Code for Information Interchange (ASCII). Appendix E provides a description and table of the IRA code.}\]
Voltage at transmitting end

Voltage at receiving end

Figure 3.11  Attenuation of Digital Signals

spectrum; examples are wire media, such as twisted pair and coaxial cable; fiber optic cable; and unguided media, such as atmosphere or space propagation. A digital signal is a sequence of voltage pulses that may be transmitted over a wire medium; for example, a constant positive voltage level may represent binary 0 and a constant negative voltage level may represent binary 1.

The principal advantages of digital signaling are that it is generally cheaper than analog signaling and is less susceptible to noise interference. The principal disadvantage is that digital signals suffer more from attenuation than do analog signals. Figure 3.11 shows a sequence of voltage pulses, generated by a source using two voltage levels, and the received voltage some distance down a conducting medium. Because of the attenuation, or reduction, of signal strength at higher frequencies, the pulses become rounded and smaller. It should be clear that this attenuation can lead rather quickly to the loss of the information contained in the propagated signal.

In what follows, we first look at some specific examples of signal types and then discuss the relationship between data and signals.

Examples  Let us return to our three examples of the preceding subsection. For each example, we will describe the signal and estimate its bandwidth.

The most familiar example of analog information is audio, or acoustic, information, which, in the form of sound waves, can be perceived directly by human beings. One form of acoustic information, of course, is human speech. This form of information is easily converted to an electromagnetic signal for transmission (Figure 3.12). In essence, all of the sound frequencies, whose amplitude is measured in terms of loudness, are converted into electromagnetic frequencies, whose amplitude is measured in volts. The telephone handset contains a simple mechanism for making such a conversion.

In this graph of a typical analog signal, the variations in amplitude and frequency convey the gradations of loudness and pitch in speech or music. Similar signals are used to transmit television pictures, but at much higher frequencies.

Figure 3.12  Conversion of Voice Input to Analog Signal
In the case of acoustic data (voice), the data can be represented directly by an electromagnetic signal occupying the same spectrum. However, there is a need to compromise between the fidelity of the sound as transmitted electrically and the cost of transmission, which increases with increasing bandwidth. As mentioned, the spectrum of speech is approximately 100 Hz to 7 kHz, although a much narrower bandwidth will produce acceptable voice reproduction. The standard spectrum for a voice channel is 300 to 3400 Hz. This is adequate for speech transmission, minimizes required transmission capacity, and allows the use of rather inexpensive telephone sets. The telephone transmitter converts the incoming acoustic voice signal into an electromagnetic signal over the range 300 to 3400 Hz. This signal is then transmitted through the telephone system to a receiver, which reproduces it as acoustic sound.

Now let us look at the video signal. To produce a video signal, a TV camera, which performs similar functions to the TV receiver, is used. One component of the camera is a photosensitive plate, upon which a scene is optically focused. An electron beam sweeps across the plate from left to right and top to bottom, in the same fashion as depicted in Figure 3.10 for the receiver. As the beam sweeps, an analog electric signal is developed proportional to the brightness of the scene at a particular spot. We mentioned that a total of 483 lines are scanned at a rate of 30 complete scans per second. This is an approximate number taking into account the time lost during the vertical retrace interval. The actual U.S. standard is 525 lines, but of these about 42 are lost during vertical retrace. Thus the horizontal scanning frequency is \( \frac{525 \text{ lines}}{30 \text{ scans/s}} = 17.5 \text{ lines per second} \), or \( 63.5 \mu s/\text{line} \). Of the 63.5 \( \mu s \), about 11 \( \mu s \) are allowed for horizontal retrace, leaving a total of 52.5 \( \mu s \) per video line.

Now we are in a position to estimate the bandwidth required for the video signal. To do this we must estimate the upper (maximum) and lower (minimum) frequency of the band. We use the following reasoning to arrive at the maximum frequency: The maximum frequency would occur during the horizontal scan if the scene were alternating between black and white as rapidly as possible. We can estimate this maximum value by considering the resolution of the video image. In the vertical dimension, there are 483 lines, so the maximum vertical resolution would be 483. Experiments have shown that the actual subjective resolution is about 70% of that number, or about 338 lines. In the interest of a balanced picture, the horizontal and vertical resolutions should be about the same. Because the ratio of width to height of a TV screen is 4:3, the horizontal resolution should be about \( \frac{4}{3} \times 338 = 450 \) lines. As a worst case, a scanning line would be made up of 450 elements alternating black and white. The scan would result in a wave, with each cycle of the wave consisting of one higher (black) and one lower (white) voltage level. Thus there would be \( \frac{450}{2} = 225 \) cycles of the wave in 52.5 \( \mu s \), for a maximum frequency of about 4.2 MHz. This rough reasoning, in fact, is fairly accurate. The lower limit is a dc or zero frequency, where the dc component corresponds to the average illumination of the scene (the average value by which the brightness exceeds the reference black level). Thus the bandwidth of the video signal is approximately \( 4 \text{ MHz} - 0 = 4 \text{ MHz} \).

The foregoing discussion did not consider color or audio components of the signal. It turns out that, with these included, the bandwidth remains about 4 MHz. Finally, the third example described is the general case of binary data. Binary data is generated by terminals, computers, and other data processing equipment.
User input at a PC is converted into a stream of binary digits (1s and 0s). In this graph of a typical digital signal, binary one is represented by \(-5\) volts and binary zero is represented by \(+5\) volts. The signal for each bit has a duration of 0.02 ms, giving a data rate of 50,000 bits per second (50 kbps).

Figure 3.13 Conversion of PC Input to Digital Signal

and then converted into digital voltage pulses for transmission, as illustrated in Figure 3.13. A commonly used signal for such data uses two constant (dc) voltage levels, one level for binary 1 and one level for binary 0. (In Chapter 5, we shall see that this is but one alternative, referred to as NRZ.) Again, we are interested in the bandwidth of such a signal. This will depend, in any specific case, on the exact shape of the waveform and the sequence of 1s and 0s. We can obtain some understanding by considering Figure 3.8 (compare Figure 3.7). As can be seen, the greater the bandwidth of the signal, the more faithfully it approximates a digital pulse stream.

Data and Signals In the foregoing discussion, we have looked at analog signals used to represent analog data and digital signals used to represent digital data. Generally, analog data are a function of time and occupy a limited frequency spectrum; such data can be represented by an electromagnetic signal occupying the same spectrum. Digital data can be represented by digital signals, with a different voltage level for each of the two binary digits.

As Figure 3.14 illustrates, these are not the only possibilities. Digital data can also be represented by analog signals by use of a modem (modulator/demodulator). The modem converts a series of binary (two-valued) voltage pulses into an analog signal by encoding the digital data onto a carrier frequency. The resulting signal occupies a certain spectrum of frequency centered about the carrier and may be propagated across a medium suitable for that carrier. The most common modems represent digital data in the voice spectrum and hence allow those data to be propagated over ordinary voice-grade telephone lines. At the other end of the line, another modem demodulates the signal to recover the original data.

In an operation very similar to that performed by a modem, analog data can be represented by digital signals. The device that performs this function for voice data is a codec (coder-decoder). In essence, the codec takes an analog signal that directly represents the voice data and approximates that signal by a bit stream. At the receiving end, the bit stream is used to reconstruct the analog data.

Thus, Figure 3.14 suggests that data may be encoded into signals in a variety of ways. We will return to this topic in Chapter 5.
Analog and Digital Transmission

Both analog and digital signals may be transmitted on suitable transmission media. The way these signals are treated is a function of the transmission system. Table 3.1 summarizes the methods of data transmission. Analog transmission is a means of transmitting analog signals without regard to their content; the signals may represent analog data (e.g., voice) or digital data (e.g., binary data that pass through a modem). In either case, the analog signal will become weaker (attenuate) after a certain distance. To achieve longer distances, the analog transmission system includes amplifiers that boost the energy in the signal. Unfortunately, the amplifier also boosts the noise components. With amplifiers cascaded to achieve long distances, the signal becomes more and more distorted.
Table 3.1  Analog and Digital Transmission

(a) Data and Signals

<table>
<thead>
<tr>
<th>Analog Data</th>
<th>Digital Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Data</td>
<td>Two alternatives: (1) signal occupies the same spectrum as the analog data; (2) analog data are encoded to occupy a different portion of spectrum.</td>
</tr>
<tr>
<td>Digital Data</td>
<td>Digital data are encoded using a modem to produce analog signal.</td>
</tr>
</tbody>
</table>

(b) Treatment of Signals

<table>
<thead>
<tr>
<th>Analog Transmission</th>
<th>Digital Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Signal</td>
<td>Assumes that the analog signal represents digital data. Signal is propagated through repeaters; at each repeater, digital data are recovered from inbound signal and used to generate a new analog outbound signal.</td>
</tr>
<tr>
<td>Digital Signal</td>
<td>Digital signal represents a stream of 1s and 0s, which may represent digital data or may be an encoding of analog data. Signal is propagated through repeaters; at each repeater, stream of 1s and 0s is recovered from inbound signal and used to generate a new digital outbound signal.</td>
</tr>
</tbody>
</table>

For analog data, such as voice, quite a bit of distortion can be tolerated and the data remain intelligible. However, for digital data, cascaded amplifiers will introduce errors.

Digital transmission, in contrast, assumes a binary content to the signal. A digital signal can be transmitted only a limited distance before attenuation, noise, and other impairments endanger the integrity of the data. To achieve greater distances, repeaters are used. A repeater receives the digital signal, recovers the pattern of 1s and 0s, and retransmits a new signal. Thus the attenuation is overcome.

The same technique may be used with an analog signal if it is assumed that the signal carries digital data. At appropriately spaced points, the transmission system has repeaters rather than amplifiers. The repeater recovers the digital data from the analog signal and generates a new, clean analog signal. Thus noise is not cumulative.

The question naturally arises as to which is the preferred method of transmission. The answer being supplied by the telecommunications industry and its customers is digital. Both long-haul telecommunications facilities and intrabuilding services have moved to digital transmission and, where possible, digital signaling techniques. The most important reasons are as follows:
• **Digital technology:** The advent of large-scale integration (LSI) and very-large-scale integration (VLSI) technology has caused a continuing drop in the cost and size of digital circuitry. Analog equipment has not shown a similar drop.

• **Data integrity:** With the use of repeaters rather than amplifiers, the effects of noise and other signal impairments are not cumulative. Thus it is possible to transmit data longer distances and over lower quality lines by digital means while maintaining the integrity of the data.

• **Capacity utilization:** It has become economical to build transmission links of very high bandwidth, including satellite channels and optical fiber. A high degree of multiplexing is needed to utilize such capacity effectively, and this is more easily and cheaply achieved with digital (time division) rather than analog (frequency division) techniques. This is explored in Chapter 8.

• **Security and privacy:** Encryption techniques can be readily applied to digital data and to analog data that have been digitized.

• **Integration:** By treating both analog and digital data digitally, all signals have the same form and can be treated similarly. Thus economies of scale and convenience can be achieved by integrating voice, video, and digital data.

### 3.3 TRANSMISSION IMPAIRMENTS

With any communications system, the signal that is received may differ from the signal that is transmitted due to various transmission impairments. For analog signals, these impairments can degrade the signal quality. For digital signals, bit errors may be introduced, such that a binary 1 is transformed into a binary 0 or vice versa. In this section, we examine the various impairments and how they may affect the information-carrying capacity of a communication link; Chapter 5 looks at measures that can be taken to compensate for these impairments.

The most significant impairments are

- Attenuation and attenuation distortion
- Delay distortion
- Noise

#### Attenuation

The strength of a signal falls off with distance over any transmission medium. For guided media, this reduction in strength, or attenuation, is generally exponential and thus is typically expressed as a constant number of decibels per unit distance. For unguided media, attenuation is a more complex function of distance and the makeup of the atmosphere. Attenuation introduces three considerations for the transmission engineer. First, a received signal must have sufficient strength so that the electronic circuitry in the receiver can detect the signal. Second, the signal must maintain a level sufficiently higher than noise to be received without error. Third, attenuation varies with frequency.

The first and second problems are dealt with by attention to signal strength and the use of amplifiers or repeaters. For a point-to-point link, the signal strength of the...
transmitter must be strong enough to be received intelligibly, but not so strong as to overload the circuitry of the transmitter or receiver, which would cause distortion. Beyond a certain distance, the attenuation becomes unacceptably great, and repeaters or amplifiers are used to boost the signal at regular intervals. These problems are more complex for multipoint lines where the distance from transmitter to receiver is variable.

The third problem is particularly noticeable for analog signals. Because the attenuation varies as a function of frequency, the received signal is distorted, reducing intelligibility. To overcome this problem, techniques are available for equalizing attenuation across a band of frequencies. This is commonly done for voice-grade telephone lines by using loading coils that change the electrical properties of the line; the result is to smooth out attenuation effects. Another approach is to use amplifiers that amplify high frequencies more than lower frequencies.

An example is provided in Figure 3.15a, which shows attenuation as a function of frequency for a typical leased line. In the figure, attenuation is measured relative to the attenuation at 1000 Hz. Positive values on the y-axis represent attenuation greater than that at 1000 Hz. A 1000-Hz tone of a given power level is applied to the input, and the power, \( P_{1000} \), is measured at the output. For any other frequency \( f \), the procedure is repeated and the relative attenuation in decibels is

\[
N_f = -10 \log_{10} \left( \frac{P_f}{P_{1000}} \right)
\]

The solid line in Figure 3.15a shows attenuation without equalization. As can be seen, frequency components at the upper end of the voice band are attenuated much more than those at lower frequencies. It should be clear that this will result in a distortion of the received speech signal. The dashed line shows the effect of equalization. The flattened response curve improves the quality of voice signals. It also allows higher data rates to be used for digital data that are passed through a modem.

Attenuation distortion can present less of a problem with digital signals. As we have seen, the strength of a digital signal falls off rapidly with frequency (Figure 3.5b); most of the content is concentrated near the fundamental frequency or bit rate of the signal.

**Delay Distortion**

Delay distortion occurs because the velocity of propagation of a signal through a guided medium varies with frequency. For a bandlimited signal, the velocity tends to be highest near the center frequency and fall off toward the two edges of the band. Thus various frequency components of a signal will arrive at the receiver at different times, resulting in phase shifts between the different frequencies.

This effect is referred to as delay distortion because the received signal is distorted due to varying delays experienced at its constituent frequencies. Delay distortion is particularly critical for digital data. Consider that a sequence of bits is being transmitted, using either analog or digital signals. Because of delay distortion, some of the signal components of one bit position will spill over into other bit positions, causing **intersymbol interference**, which is a major limitation to maximum bit rate over a transmission channel.

---

8In the remainder of this book, unless otherwise indicated, we use \( \log(x) \) to mean \( \log_{10}(x) \).
Figure 3.15  Attenuation and Delay Distortion Curves for a Voice Channel
Equalizing techniques can also be used for delay distortion. Again using a leased telephone line as an example, Figure 3.15b shows the effect of equalization on delay as a function of frequency.

**Noise**

For any data transmission event, the received signal will consist of the transmitted signal, modified by the various distortions imposed by the transmission system, plus additional unwanted signals that are inserted somewhere between transmission and reception. The latter, undesired signals are referred to as noise. Noise is the major limiting factor in communications system performance.

Noise may be divided into four categories:

- Thermal noise
- Intermodulation noise
- Crosstalk
- Impulse noise

**Thermal noise** is due to thermal agitation of electrons. It is present in all electronic devices and transmission media and is a function of temperature. Thermal noise is uniformly distributed across the bandwidths typically used in communications systems and hence is often referred to as white noise. Thermal noise cannot be eliminated and therefore places an upper bound on communications system performance. Because of the weakness of the signal received by satellite earth stations, thermal noise is particularly significant for satellite communication.

The amount of thermal noise to be found in a bandwidth of 1 Hz in any device or conductor is

\[ N_0 = kT(W/Hz) \]

where

\[ N_0 = \text{noise power density in watts per 1 Hz of bandwidth} \]

\[ k = \text{Boltzmann’s constant} = 1.38 \times 10^{-23} \text{ J/K} \]

\[ T = \text{temperature, in kelvins (absolute temperature), where the symbol K is used to represent 1 kelvin} \]

**EXAMPLE 3.1** Room temperature is usually specified as \( T = 17^\circ \text{C} \), or 290 K. At this temperature, the thermal noise power density is

\[ N_0 = (1.38 \times 10^{-23}) \times 290 = 4 \times 10^{-21} \text{ W/Hz} = -204 \text{ dBW/Hz} \]

where dBW is the decibel-watt, defined in Appendix 3A.

---

9A Joule (J) is the International System (SI) unit of electrical, mechanical, and thermal energy. A Watt is the SI unit of power, equal to one Joule per second. The kelvin (K) is the SI unit of thermodynamic temperature. For a temperature in kelvins of \( T \), the corresponding temperature in degrees Celsius is equal to \( T - 273.15 \).
The noise is assumed to be independent of frequency. Thus the thermal noise in watts present in a bandwidth of $B$ Hertz can be expressed as

$$N = kTB$$

or, in decibel-watts,

$$N = 10 \log k + 10 \log T + 10 \log B = -228.6 \text{ dBW} + 10 \log T + 10 \log B$$

### EXAMPLE 3.2

Given a receiver with an effective noise temperature of 294 K and a 10-MHz bandwidth, the thermal noise level at the receiver’s output is

$$N = -228.6 \text{ dBW} + 10 \log(294) + 10 \log 10^7$$

$$= -228.6 + 24.7 + 70$$

$$= -133.9 \text{ dBW}$$

When signals at different frequencies share the same transmission medium, the result may be **intermodulation noise**. The effect of intermodulation noise is to produce signals at a frequency that is the sum or difference of the two original frequencies or multiples of those frequencies. For example, the mixing of signals at frequencies $f_1$ and $f_2$ might produce energy at the frequency $f_1 + f_2$. This derived signal could interfere with an intended signal at the frequency $f_1 + f_2$.

Intermodulation noise is produced by nonlinearities in the transmitter, receiver, and/or intervening transmission medium. Ideally, these components behave as linear systems; that is, the output is equal to the input times a constant. However, in any real system, the output is a more complex function of the input. Excessive nonlinearity can be caused by component malfunction or overload from excessive signal strength. It is under these circumstances that the sum and difference frequency terms occur.

**Crosstalk** has been experienced by anyone who, while using the telephone, has been able to hear another conversation; it is an unwanted coupling between signal paths. It can occur by electrical coupling between nearby twisted pairs or, rarely, coax cable lines carrying multiple signals. Crosstalk can also occur when microwave antennas pick up unwanted signals; although highly directional antennas are used, microwave energy does spread during propagation. Typically, crosstalk is of the same order of magnitude as, or less than, thermal noise.

All of the types of noise discussed so far have reasonably predictable and relatively constant magnitudes. Thus it is possible to engineer a transmission system to cope with them. **Impulse noise**, however, is noncontinuous, consisting of irregular pulses or noise spikes of short duration and of relatively high amplitude. It is generated from a variety of causes, including external electromagnetic disturbances, such as lightning, and faults and flaws in the communications system.

Impulse noise is generally only a minor annoyance for analog data. For example, voice transmission may be corrupted by short clicks and crackles with no loss of intelligibility. However, impulse noise is the primary source of error in digital data.
3.4 CHANNEL CAPACITY

We have seen that there are a variety of impairments that distort or corrupt a signal. For digital data, the question that then arises is to what extent these impairments limit the data rate that can be achieved. The maximum rate at which data can be transmitted over a given communication path, or channel, under given conditions, is referred to as the **channel capacity**.

There are four concepts here that we are trying to relate to one another.

- **Data rate**: The rate, in bits per second (bps), at which data can be communicated
• **Bandwidth**: The bandwidth of the transmitted signal as constrained by the transmitter and the nature of the transmission medium, expressed in cycles per second, or Hertz

• **Noise**: The average level of noise over the communications path

• **Error rate**: The rate at which errors occur, where an error is the reception of a 1 when a 0 was transmitted or the reception of a 0 when a 1 was transmitted

The problem we are addressing is this: Communications facilities are expensive and, in general, the greater the bandwidth of a facility, the greater the cost. Furthermore, all transmission channels of any practical interest are of limited bandwidth. The limitations arise from the physical properties of the transmission medium or from deliberate limitations at the transmitter on the bandwidth to prevent interference from other sources. Accordingly, we would like to make as efficient use as possible of a given bandwidth. For digital data, this means that we would like to get as high a data rate as possible at a particular limit of error rate for a given bandwidth. The main constraint on achieving this efficiency is noise.

**Nyquist Bandwidth**

To begin, let us consider the case of a channel that is noise free. In this environment, the limitation on data rate is simply the bandwidth of the signal. A formulation of this limitation, due to Nyquist, states that if the rate of signal transmission is \(2B\), then a signal with frequencies no greater than \(B\) is sufficient to carry the signal rate. The converse is also true: Given a bandwidth of \(B\), the highest signal rate that can be carried is \(2B\). This limitation is due to the effect of intersymbol interference, such as is produced by delay distortion. The result is useful in the development of digital-to-analog encoding schemes and is, in essence, based on the same derivation as that of the sampling theorem, described in Appendix F.

Note that in the preceding paragraph, we referred to signal rate. If the signals to be transmitted are binary (two voltage levels), then the data rate that can be supported by \(B\) Hz is \(2B\) bps. However, as we shall see in Chapter 5, signals with more than two levels can be used; that is, each signal element can represent more than one bit. For example, if four possible voltage levels are used as signals, then each signal element can represent two bits. With multilevel signaling, the Nyquist formulation becomes

\[
C = 2B \log_2 M
\]

where \(M\) is the number of discrete signal or voltage levels.

So, for a given bandwidth, the data rate can be increased by increasing the number of different signal elements. However, this places an increased burden on the receiver: Instead of distinguishing one of two possible signal elements during each signal time, it must distinguish one of \(M\) possible signal elements. Noise and other impairments on the transmission line will limit the practical value of \(M\).
Some of the literature uses SNR; others use S/N. Also, in some cases the dimensionless quantity is referred to as SNR or S/N and the quantity in decibels is referred to as or Others use just SNR or S/N to mean the dB quantity. This text uses SNR and SNRdb.

Shannon Capacity Formula

Nyquist’s formula indicates that, all other things being equal, doubling the bandwidth doubles the data rate. Now consider the relationship among data rate, noise, and error rate. The presence of noise can corrupt one or more bits. If the data rate is increased, then the bits become “shorter” so that more bits are affected by a given pattern of noise.

Figure 3.16 illustrates this relationship. If the data rate is increased, then more bits will occur during the interval of a noise spike, and hence more errors will occur.

All of these concepts can be tied together neatly in a formula developed by the mathematician Claude Shannon. As we have just illustrated, the higher the data rate, the more damage that unwanted noise can do. For a given level of noise, we would expect that a greater signal strength would improve the ability to receive data correctly in the presence of noise. The key parameter involved in this reasoning is the signal-to-noise ratio (SNR, or S/N), which is the ratio of the power in a signal to the power contained in the noise that is present at a particular point in the transmission. Typically, this ratio is measured at a receiver, because it is at this point that an attempt is made to process the signal and recover the data. For convenience, this ratio is often reported in decibels:

\[
\text{SNR}_{\text{dB}} = 10 \log_{10} \frac{\text{signal power}}{\text{noise power}}
\]

This expresses the amount, in decibels, that the intended signal exceeds the noise level. A high SNR will mean a high-quality signal and a low number of required intermediate repeaters.

The signal-to-noise ratio is important in the transmission of digital data because it sets the upper bound on the achievable data rate. Shannon’s result is that the maximum channel capacity, in bits per second, obeys the equation

\[
C = B \log_2 (1 + \text{SNR})
\]  

(3.1)

where \(C\) is the capacity of the channel in bits per second and \(B\) is the bandwidth of the channel in Hertz. The Shannon formula represents the theoretical maximum that can be achieved. In practice, however, only much lower rates are achieved. One reason for this is that the formula assumes white noise (thermal noise). Impulse noise is not accounted for, nor are attenuation distortion or delay distortion. Even in

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10Some of the literature uses SNR; others use S/N. Also, in some cases the dimensionless quantity is referred to as SNR or S/N and the quantity in decibels is referred to as SNR_{dB} or (S/N)_{dB}. Others use just SNR or S/N to mean the dB quantity. This text uses SNR and SNR_{dB}.
an ideal white noise environment, present technology still cannot achieve Shannon capacity due to encoding issues, such as coding length and complexity.

The capacity indicated in the preceding equation is referred to as the error-free capacity. Shannon proved that if the actual information rate on a channel is less than the error-free capacity, then it is theoretically possible to use a suitable signal code to achieve error-free transmission through the channel. Shannon’s theorem unfortunately does not suggest a means for finding such codes, but it does provide a yardstick by which the performance of practical communication schemes may be measured.

Several other observations concerning the preceding equation may be instructive. For a given level of noise, it would appear that the data rate could be increased by increasing either signal strength or bandwidth. However, as the signal strength increases, so do the effects of nonlinearities in the system, leading to an increase in intermodulation noise. Note also that, because noise is assumed to be white, the wider the bandwidth, the more noise is admitted to the system. Thus, as $B$ increases, SNR decreases.

**EXAMPLE 3.4** Let us consider an example that relates the Nyquist and Shannon formulations. Suppose that the spectrum of a channel is between 3 MHz and 4 MHz and $\text{SNR}_{\text{dB}} = 24$ dB. Then

$$B = 4 \text{ MHz} - 3 \text{ MHz} = 1 \text{ MHz}$$

$$\text{SNR}_{\text{dB}} = 24 \text{ dB} = 10 \log_{10}(\text{SNR})$$

$$\text{SNR} = 251$$

Using Shannon’s formula,

$$C = 10^6 \times \log_2(1 + 251) \approx 10^6 \times 8 = 8 \text{ Mbps}$$

This is a theoretical limit and, as we have said, is unlikely to be reached. But assume we can achieve the limit. Based on Nyquist’s formula, how many signaling levels are required? We have

$$C = 2B \log_2 M$$

$$8 \times 10^6 = 2 \times (10^6) \times \log_2 M$$

$$4 = \log_2 M$$

$$M = 16$$

**The Expression $E_b/N_0$**

Finally, we mention a parameter related to SNR that is more convenient for determining digital data rates and error rates and that is the standard quality measure for digital communication system performance. The parameter is the ratio of signal energy per bit to noise power density per Hertz, $E_b/N_0$. Consider a signal, digital or analog, that contains binary digital data transmitted at a certain bit rate $R$. Recalling that 1 Watt $= 1$ J/s, the energy per bit in a signal is given by $E_b = ST_b$, where $S$ is the signal power and $T_b$ is the time required to send one bit. The data rate $R$ is just $R = 1/T_b$. Thus
or, in decibel notation,

\[
\left( \frac{E_b}{N_0} \right)_{\text{dB}} = S_{\text{dBW}} - 10 \log R - 10 \log k - 10 \log T
\]

\[
= S_{\text{dBW}} - 10 \log R + 228.6 \text{ dBW} - 10 \log T
\]

The ratio \( E_b/N_0 \) is important because the bit error rate for digital data is a (decreasing) function of this ratio. Given a value of \( E_b/N_0 \) needed to achieve a desired error rate, the parameters in the preceding formula may be selected. Note that as the bit rate \( R \) increases, the transmitted signal power, relative to noise, must increase to maintain the required \( E_b/N_0 \).

Let us try to grasp this result intuitively by considering again Figure 3.16. The signal here is digital, but the reasoning would be the same for an analog signal. In several instances, the noise is sufficient to alter the value of a bit. If the data rate were doubled, the bits would be more tightly packed together, and the same passage of noise might destroy two bits. Thus, for constant signal to noise ratio, an increase in data rate increases the error rate.

The advantage of \( E_b/N_0 \) over SNR is that the latter quantity depends on the bandwidth.

**EXAMPLE 3.5** For binary phase-shift keying (defined in Chapter 5), \( E_b/N_0 = 8.4 \text{ dB} \) is required for a bit error rate of \( 10^{-4} \) (one bit error out of every 10,000). If the effective noise temperature is \( 290^{\circ}\text{K} \) (room temperature) and the data rate is \( 2400 \text{ bps} \), what received signal level is required?

We have

\[
8.4 = S(\text{dBW}) - 10 \log 2400 + 228.6 \text{ dBW} - 10 \log 290
\]

\[
= S(\text{dBW}) - (10)(3.38) + 228.6 - (10)(2.46)
\]

\[
S = -161.8 \text{ dBW}
\]

We can relate \( E_b/N_0 \) to SNR as follows. We have

\[
\frac{E_b}{N_0} = \frac{S}{N_0 R}
\]

The parameter \( N_0 \) is the noise power density in Watts/Hertz. Hence, the noise in a signal with bandwidth \( B \) is \( N = N_0 B \). Substituting, we have

\[
\frac{E_b}{N_0} = \frac{S B_T}{N R}
\]

Another formulation of interest relates \( E_b/N_0 \) to spectral efficiency. Shannon’s result (Equation 3.1) can be rewritten as:
Using Equation (3.2), and equating $R$ with $C$, we have

$$\frac{S}{N} = 2^{C/B} - 1$$

This is a useful formula that relates the achievable spectral efficiency $C/B$ to $E_b/N_0$.

**EXAMPLE 3.6** Suppose we want to find the minimum $E_b/N_0$ required to achieve a spectral efficiency of 6 bps/Hz. Then

$$E_b/N_0 = \frac{1}{6}(2^6 - 1) = 10.5 = 10.21 \text{ dB}.$$
Review Questions

3.1. Differentiate between guided media and unguided media.
3.2. Differentiate between an analog and a digital electromagnetic signal.
3.3. What are three important characteristics of a periodic signal?
3.4. How many radians are there in a complete circle of 360 degrees?
3.5. What is the relationship between the wavelength and frequency of a sine wave?
3.6. Define *fundamental frequency*.
3.7. What is the relationship between a signal’s spectrum and its bandwidth?
3.8. What is attenuation?
3.9. Define *channel capacity*.
3.10. What key factors affect channel capacity?

Problems

3.1 a. For multipoint configuration, only one device at a time can transmit. Why?
b. There are two methods of enforcing the rule that only one device can transmit. In the centralized method, one station is in control and can either transmit or allow a specified other station to transmit. In the decentralized method, the stations jointly cooperate in taking turns. What do you see as the advantages and disadvantages of the two methods?

3.2 A signal has a fundamental frequency of 1000 Hz. What is its period?

3.3 Express the following in the simplest form you can:
   a. \( \sin(2\pi ft - \pi) + \sin(2\pi ft + \pi) \)
   b. \( 2\pi ft + \sin(2\pi ft - \pi) \)

3.4 Sound may be modeled as sinusoidal functions. Compare the relative frequency and wavelength of musical notes. Use 330 m/s as the speed of sound and the following frequencies for the musical scale.

<table>
<thead>
<tr>
<th>Note</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>264</td>
<td>297</td>
<td>330</td>
<td>352</td>
<td>396</td>
<td>440</td>
<td>495</td>
<td>528</td>
</tr>
</tbody>
</table>

3.5 If the solid curve in Figure 3.17 represents \( \sin(2\pi t) \), what does the dotted curve represent? That is, the dotted curve can be written in the form \( A \sin(2\pi ft + \phi) \); what are \( A \), \( f \), and \( \phi \)?

3.6 Decompose the signal \( (1 + 0.1 \cos 5t)\cos 100t \) into a linear combination of sinusoidal functions, and find the amplitude, frequency, and phase of each component. *Hint:* Use the identity for \( \cos a \cos b \).
3.7 Find the period of the function \( f(t) = (10 \cos t)^2 \).

3.8 Consider two periodic functions \( f_1(t) \) and \( f_2(t) \), with periods \( T_1 \) and \( T_2 \), respectively. Is it always the case that the function \( f(t) = f_1(t) + f_2(t) \) is periodic? If so, demonstrate this fact. If not, under what conditions is \( f(t) \) periodic?

3.9 Figure 3.4 shows the effect of eliminating higher-harmonic components of a square wave and retaining only a few lower harmonic components. What would the signal look like in the opposite case; that is, retaining all higher harmonics and eliminating a few lower harmonics?

3.10 Figure 3.5b shows the frequency domain function for a single square pulse. The single pulse could represent a digital 1 in a communication system. Note that an infinite number of higher frequencies of decreasing magnitudes is needed to represent the single pulse. What implication does that have for a real digital transmission system?

3.11 IRA is a 7-bit code that allows 128 characters to be defined. In the 1970s, many newspapers received stories from the wire services in a 6-bit code called TTS. This code carried upper- and lower case characters as well as many special characters and formatting commands. The typical TTS character set allowed over 100 characters to be defined. How do you think this could be accomplished?

3.12 For a video signal, what increase in horizontal resolution is possible if a bandwidth of 5 MHz is used? What increase in vertical resolution is possible? Treat the two questions separately; that is, the increased bandwidth is to be used to increase either horizontal or vertical resolution, but not both.

3.13 a. Suppose that a digitized TV picture is to be transmitted from a source that uses a matrix of \( 480 \times 500 \) picture elements (pixels), where each pixel can take on one of 32 intensity values. Assume that 30 pictures are sent per second. (This digital source is roughly equivalent to broadcast TV standards that have been adopted.) Find the source rate \( R \) (bps).

b. Assume that the TV picture is to be transmitted over a channel with 4.5-MHz bandwidth and a 35-dB signal-to-noise ratio. Find the capacity of the channel (bps).

c. Discuss how the parameters given in part (a) could be modified to allow transmission of color TV signals without increasing the required value for \( R \).

3.14 Given an amplifier with an effective noise temperature of 10,000 K and a 10-MHz bandwidth, what thermal noise level, in dBW, may we expect at its output?

3.15 What is the channel capacity for a teleprinter channel with a 300-Hz bandwidth and a signal-to-noise ratio of 3 dB, where the noise is white thermal noise?

3.16 A digital signaling system is required to operate at 9600 bps.

a. If a signal element encodes a 4-bit word, what is the minimum required bandwidth of the channel?

b. Repeat part (a) for the case of 8-bit words.
3.17 What is the thermal noise level of a channel with a bandwidth of 10 kHz carrying 1000 watts of power operating at 50°C?

3.18 Given the narrow (usable) audio bandwidth of a telephone transmission facility, a nominal SNR of 56 dB (400,000), and a certain level of distortion,
   a. What is the theoretical maximum channel capacity (kbps) of traditional telephone lines?
   b. What can we say about the actual maximum channel capacity?

3.19 Study the works of Shannon and Nyquist on channel capacity. Each places an upper limit on the bit rate of a channel based on two different approaches. How are the two related?

3.19 Consider a channel with a 1-MHz capacity and an SNR of 63.
   a. What is the upper limit to the data rate that the channel can carry?
   b. The result of part (a) is the upper limit. However, as a practical matter, better error performance will be achieved at a lower data rate. Assume we choose a data rate of 2/3 the maximum theoretical limit. How many signal levels are needed to achieve this data rate?

3.20 Given the narrow (usable) audio bandwidth of a telephone transmission facility, a nominal $SNR_{db}$ of 56 dB (400,000), and a distortion level of <0.2%,
   a. What is the theoretical maximum channel capacity (kbps) of traditional telephone lines?
   b. What is the actual maximum channel capacity?

3.21 Given a channel with an intended capacity of 20 Mbps, the bandwidth of the channel is 3 MHz. Assuming white thermal noise, what signal-to-noise ratio is required to achieve this capacity?

3.22 The square wave of Figure 3.7c, with $T = 1$ ms, is passed through a lowpass filter that passes frequencies up to 8 kHz with no attenuation.
   a. Find the power in the output waveform.
   b. Assuming that at the filter input there is a thermal noise voltage with $N_0 = 0.1 \mu$ Watt/Hz, find the output signal to noise ratio in dB.

3.23 If the received signal level for a particular digital system is $-151$ dBW and the receiver system effective noise temperature is 1500 K, what is $E_b/N_0$ for a link transmitting 2400 bps?

3.24 Fill in the missing elements in the following table of approximate power ratios for various dB levels.

<table>
<thead>
<tr>
<th>Decibels</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Losses</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Gains</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

3.25 If an amplifier has a 30-dB voltage gain, what voltage ratio does the gain represent?

3.26 An amplifier has an output of 20 W. What is its output in dBW?

APPENDIX 3A DECIBELS AND SIGNAL STRENGTH

An important parameter in any transmission system is the signal strength. As a signal propagates along a transmission medium, there will be a loss, or attenuation, of signal strength. To compensate, amplifiers may be inserted at various points to impart a gain in signal strength.

It is customary to express gains, losses, and relative levels in decibels because

- Signal strength often falls off exponentially, so loss is easily expressed in terms of the decibel, which is a logarithmic unit.
- The net gain or loss in a cascaded transmission path can be calculated with simple addition and subtraction.
The decibel is a measure of the ratio between two signal levels. The decibel gain is given by

\[ G_{\text{dB}} = 10 \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}} \]

where

- \( G_{\text{dB}} \) = gain, in decibels
- \( P_{\text{in}} \) = input power level
- \( P_{\text{out}} \) = output power level
- \( \log_{10} \) = logarithm to the base 10

Table 3.2 shows the relationship between decibel values and powers of 10.

<table>
<thead>
<tr>
<th>Power Ratio</th>
<th>dB</th>
<th>Power Ratio</th>
<th>dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^1 )</td>
<td>10</td>
<td>( 10^{-1} )</td>
<td>−10</td>
</tr>
<tr>
<td>( 10^2 )</td>
<td>20</td>
<td>( 10^{-2} )</td>
<td>−20</td>
</tr>
<tr>
<td>( 10^3 )</td>
<td>30</td>
<td>( 10^{-3} )</td>
<td>−30</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>40</td>
<td>( 10^{-4} )</td>
<td>−40</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>50</td>
<td>( 10^{-5} )</td>
<td>−50</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>60</td>
<td>( 10^{-6} )</td>
<td>−60</td>
</tr>
</tbody>
</table>

The decibel is a measure of the ratio between two signal levels. The decibel gain is given by

\[ G_{\text{dB}} = 10 \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}} \]

where

- \( G_{\text{dB}} \) = gain, in decibels
- \( P_{\text{in}} \) = input power level
- \( P_{\text{out}} \) = output power level
- \( \log_{10} \) = logarithm to the base 10

Table 3.2 shows the relationship between decibel values and powers of 10.

There is some inconsistency in the literature over the use of the terms \textit{gain} and \textit{loss}. If the value of \( G_{\text{dB}} \) is positive, this represents an actual gain in power. For example, a gain of 3 dB means that the power has doubled. If the value of \( G_{\text{dB}} \) is negative, this represents an actual loss in power. For example, a gain of −3 dB means that the power has halved, and this is a loss of power. Normally, this is expressed by saying there is a loss of 3 dB. However, some of the literature would say that this is a loss of −3 dB. It makes more sense to say that a negative gain corresponds to a positive loss. Therefore, we define a decibel loss as

\[ L_{\text{dB}} = -10 \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}} = 10 \log_{10} \frac{P_{\text{in}}}{P_{\text{out}}} \]  \hspace{1cm} (3.3)  

**EXAMPLE 3.7** If a signal with a power level of 10 mW is inserted onto a transmission line and the measured power some distance away is 5 mW, the loss can be expressed as

\[ L_{\text{dB}} = 10 \log(10/5) = 10(0.3) = 3 \text{ dB}. \]

Note that the decibel is a measure of relative, not absolute, difference. A loss from 1000 mW to 500 mW is also a loss of 3 dB.

The decibel is also used to measure the difference in voltage, taking into account that power is proportional to the square of the voltage:

\[ P = \frac{V^2}{R} \]
where

\[ P = \text{power dissipated across resistance } R \]
\[ V = \text{voltage across resistance } R \]

Thus

\[ L_{dB} = 10 \log \frac{P_{in}}{P_{out}} = 10 \log \frac{V_{in}^2/R}{V_{out}^2/R} = 20 \log \frac{V_{in}}{V_{out}} \]

**EXAMPLE 3.8** Decibels are useful in determining the gain or loss over a series of transmission elements. Consider a series in which the input is at a power level of 4 mW, the first element is a transmission line with a 12-dB loss (−12-dB gain), the second element is an amplifier with a 35-dB gain, and the third element is a transmission line with a 10-dB loss. The net gain is (−12 + 35 − 10) = 13 dB. To calculate the output power \( P_{out} \):

\[ G_{dB} = 13 = 10 \log \left( \frac{P_{out}}{4 \text{ mW}} \right) \]
\[ P_{out} = 4 \times 10^{1.3} \text{ mW} = 79.8 \text{ mW} \]

Decibel values refer to relative magnitudes or changes in magnitude, not to an absolute level. It is convenient to be able to refer to an absolute level of power or voltage in decibels so that gains and losses with reference to an initial signal level may be calculated easily. The **dBW** (decibel-Watt) is used extensively in microwave applications. The value of 1 W is selected as a reference and defined to be 0 dBW. The absolute decibel level of power in dBW is defined as

\[ \text{Power}_{dBW} = 10 \log \frac{\text{Power}_{W}}{1 \text{ W}} \]

**EXAMPLE 3.9** A power of 1000 W is 30 dBW, and a power of 1 mW is −30 dBW.

Another common unit is the **dBm** (decibel-milliWatt), which uses 1 mW as the reference. Thus 0 dBm = 1 mW. The formula is

\[ \text{Power}_{dBm} = 10 \log \frac{\text{Power}_{mW}}{1 \text{ mW}} \]

Note the following relationships:

\[ +30 \text{ dBm} = 0 \text{ dBW} \]
\[ 0 \text{ dBm} = -30 \text{ dBW} \]

A unit in common use in cable television and broadband LAN applications is the **dBmV** (decibel-millivolt). This is an absolute unit with 0 dBmV equivalent to 1 mV. Thus

\[ \text{Voltage}_{dBmV} = 20 \log \frac{\text{Voltage}_{mV}}{1 \text{ mV}} \]

In this case, the voltage levels are assumed to be across a 75-ohm resistance.