CHAPTER 18

Risk and Uncertainty

The future brings surprises. A rainstorm can change the price of wheat. A fire can destroy your house. The invention of the automobile can make you rich if you own rubber plantations or wipe you out if you manufacture buggy whips.

Your wealth tomorrow depends on the **state of the world**. Examples of alternative states of the world are "rain" versus "sunshine," "fire" versus "no fire," and "cars invented" versus "cars not invented."

State of the world

A potential set of conditions.

Markets abound for transferring wealth from one state of the world to another. By placing a bet that it will rain, you increase your wealth in the rainy state of the world while decreasing your wealth in the sunny state. (Of course, you will occupy only one of these states, but at the time you place the bet you don't know which it will be.) Purchasing fire insurance is a mechanism for increasing your wealth in the "fire" state at the expense of decreasing your wealth (by the amount of the insurance premium) in the "no fire" state. Organized markets in stocks and commodities afford numerous opportunities for transferring wealth between states of the world.

In this chapter, we will begin by studying the individual's choice about how much wealth to transfer from one state of the world to another and the determination of the equilibrium price at which he can do so. We will then examine some of the particular markets in which such transactions take place.

18.1 Attitudes Toward Risk

When there are two alternative states of the world, we can use diagrams like those in Exhibit 18.1 to represent your wealth in each of them. The horizontal axis measures your wealth in one state, and the vertical axis measures your wealth in the other. Suppose that your total wealth is $100 but that it will be reduced to $40 if there is a fire. In that case, your position is represented by point *A* in panel A of Exhibit 18.1.

Now suppose that for $20 you purchase an insurance contract that entitles you to collect $60 in the event of a fire. Then if there is no fire, your wealth is reduced to $80, whereas if the fire occurs your wealth is also $80 ($40 plus $60 insurance payment minus $20 to buy the insurance in the first place). Thus, your new position is represented by point *B*.

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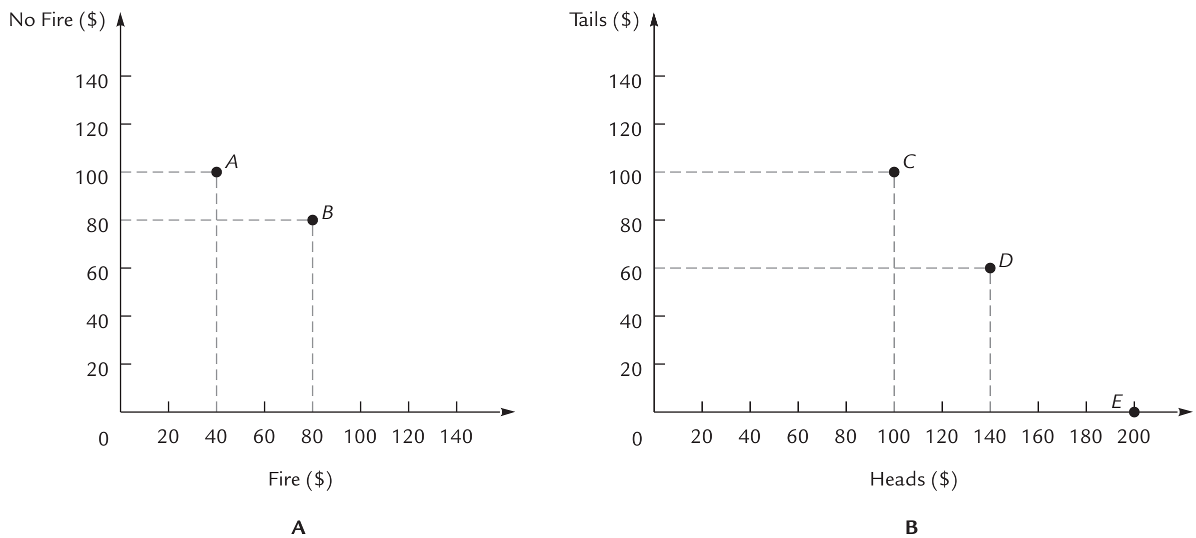
EXHIBIT 18.1 **States of the World**



In either panel the two axes represent your wealth in alternative states of the world. Panel A considers the states in which your house is destroyed by fire and in which it is not. Suppose that your wealth is initially $100 but that it will be reduced to $40 in the event of a fire. Then your position is represented by point *A.* Now suppose that for $20 you purchase an insurance contract that will return $60 in the event of a fire. Then your new position is represented by point *B,* where your wealth is $80 in either state of the world.

Panel B considers the two possible outcomes of a coin toss. If your initial wealth is $100 and if you do not bet on the outcome of the toss, then your position is represented by point *C.* If you wager $40 that the coin will come up heads, you move to point *D.*

For another example, suppose that you are a gambler, that you have total assets of $100, and that you have just bet $40 that a certain tossed coin will come up heads. The possible states of the world are "heads" and "tails." In case of heads, your wealth is $140; in case of tails it is $60. Your position is represented by point *D* in panel B of Exhibit 18.1. If you don't place the bet, your wealth is $100 regardless of whether the coin comes up heads or tails, and your position is represented by point *C.*

Exercise 18.1 What bet would you have to place to move to basket *E* in Exhibit 18.1?

We can think of each of the points in Exhibit 18.1 as a *basket of outcomes,* and we can use indifference curves to represent an individual's preferences among these baskets. However, these baskets of outcomes differ in an important way from the baskets of consumer goods that we studied in Chapter 3. When you own a basket of apples and oranges, you can consume both apples and oranges. But when you own a basket of outcomes, you get only one of the outcomes. Once the state of the world has been determined, we do not need indifference curves to tell us which baskets are preferable to which others. After the coin comes up heads, everyone will agree that point *D* is better than point *C* in panel B of Exhibit 18.1. Or after it comes up tails, everyone will agree that *C* is better than *D.*

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When we talk about preferences between baskets of outcomes, we are referring to the preferences of someone who does not yet know what the state of the world will be. Such preferences are called **ex ante** preferences, as distinguished from the **ex post** preferences of someone who has already learned the state of the world. If we say that Clarence prefers D to C, we mean that he would choose to bet \$40 on heads rather than not bet at all, if he were asked *before* the coin was flipped.

Ex ante

Determined before the state of the world is known.

Ex post

Determined after the state of the world is known.

Characterizing Baskets

Before drawing budget constraints and indifference curves, we need to introduce two concepts that describe important characteristics of any basket of outcomes. One of these is the expected value of a basket; the other is its riskiness.

Expected Values

The **expected value** of a basket is given by the formula

Expected value

The average value over all states of the world, with each state weighted by its probability.

(Probability of state 1) $\times$ (Wealth in state 1) + (Probability of state 2) $\times$ (Wealth in state 2)

For example, suppose that your basket of outcomes is represented by point A in panel A of Exhibit 18.1 and that the probability of a fire is .25 (so that the probability of "no fire" is .75). Then the expected value of your wealth is

$$(.25 \times \$40) + (.75 \times \$100) = \$85$$

In panel B of Exhibit 18.1, if we assume that the coin is unbiased, meaning that it has probability .50 of coming up heads and probability .50 of coming up tails, then the expected value of basket D is

$$(.50 \times \$140) + (.50 \times \$60) = \$100$$

Exercise 18.2 If the coin is unbiased, what is the expected value of basket C? If the coin is weighted so that it comes up heads two-thirds of the time, what are the expected values of baskets C and D? What if the coin is weighted so that it comes up tails two-thirds of the time?

If you repeat the same gamble a large number of times, the average outcome will be approximately equal to the expected value of the gamble. It is possible to formulate this statement more precisely and to prove it mathematically. The careful mathematical formulation is known as the **law of large numbers.**

Law of large numbers

When a gamble is repeated many times, the average outcome is the expected value.

Suppose that state 1 occurs with probability P_1 and state 2 occurs with probability P_2 (so that $P_1 + P_2 = 1$). Then along any line with slope $-P_1/P_2$, all baskets have the same expected value. A family of such "iso-expected value" lines is illustrated in Exhibit 18.2.

Exercise 18.3 In panel B of Exhibit 18.1, what do the iso-expected value lines look like if the coin is unbiased? If the coin comes up heads two-thirds of the time? If it comes up tails two-thirds of the time? In each case, which point lies on the higher line, C or D? Are your answers consistent with your calculations in Exercise 18.2?

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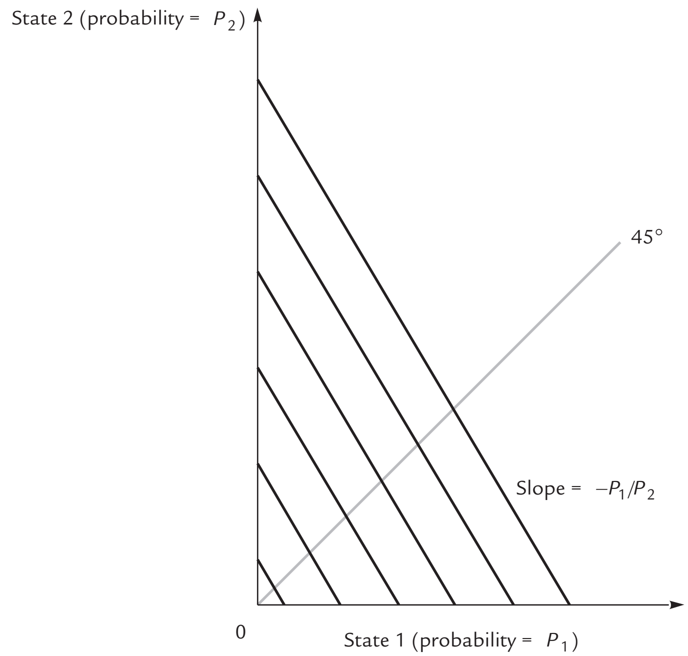
EXHIBIT **18.2** **Baskets with the Same Expected Value**



If the probability of state 1 is P_1 and the probability of state 2 is P_2 (so that $P_1 + P_2 = 1$), then all of the baskets along a line of slope $-P_1/P_2$ have the same expected value. The graph shows a family of such lines.

The baskets along the 45° line are risk-free, because a person holding such a basket will have the same wealth in either state of the world. Moving along an iso-expected value line away from the 45° line in either direction, the baskets become successively riskier.

Riskiness

Riskiness
Variation in potential outcomes.

Risk-free
Having the same value in any state of the world.

Baskets differ not only in expected value but also in **riskiness.** Baskets on the 45° line (shown in Exhibit 18.2) are referred to as **risk-free,** because individuals who hold them know with certainty what their wealth will be regardless of the state of the world. Moving away from the 45° line along an iso-expected value line, the baskets become riskier, carrying more uncertainty about what the future will bring. In panel B of Exhibit 18.1, baskets *C* and *E* have the same expected value, but a person holding basket *C* knows for certain what his wealth will be, whereas a person with basket *E* could come away with either twice as much wealth or with nothing at all.

Opportunities

Suppose that you enter a gambling parlor with \$100 in your pocket. Bets are being taken on a coin flip. If you place no bets, your wealth is \$100 in either state of the world. This is your endowment, and it is represented by point *C* in Exhibit 18.3. Suppose that you are invited to express your opinion about how the coin will turn up and to bet as much as you would like on the outcome. By betting \$50 on tails, you can move yourself to point *X*, where your wealth is \$150 if you win or \$50 if you lose. Other bets can get you to any of the points on the black line shown in Exhibit 18.3. By placing bets, you can trade your endowment for any point along that line. In other words, it is your budget line.

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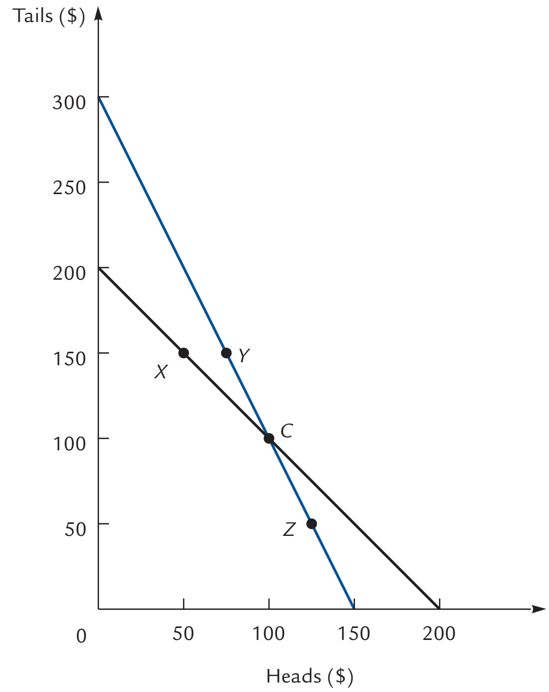
EXHIBIT **18.3** **Opportunities**



If you enter a gambling parlor with $100 in your pocket and choose not to bet on a coin toss, then your wealth will be $100 in either state of the world. Thus, you achieve point *C* without trading—point *C* is your endowment. By betting on either heads or tails at even odds, you can achieve any basket along the black budget line, such as *X.* If the odds are such that tails bettors receive 2 to 1 payoffs, you can achieve any point on the color budget line. The odds give the relative price of wealth in the tails state in terms of wealth in the heads state, and they therefore determine the slope of the budget line.

Exercise 18.4 What would your budget line look like if you were permitted to bet only on heads?

The gambling parlor offers you the opportunity to trade dollars in the heads state of the world for dollars in the tails state at a relative price of 1. This price is reflected in the slope of the budget line, which is 1 in absolute value.

Other prices are also possible. Suppose that you are offered the opportunity to bet on tails and given *odds* of 2 to 1. This means that for every $1 you bet, you win $2 if tails comes up (but you still lose only $1 if the outcome is heads). Suppose that you are allowed to take either side of this bet: You can bet either on tails at odds of 2 to 1, or on heads, in which case you must grant odds of 2 to 1. You now have an opportunity to trade dollars between the heads state of the world and the tails state of the world. The relative price is 2 "tail-dollars" per "head-dollar." By betting $25 on tails, you can move from point *C* to point *Y* in Exhibit 18.3. In so doing, you are selling 25 head-dollars and receiving 50 tail-dollars in return. Alternatively, you could buy head-dollars and sell tail-dollars, moving to a point like *Z.* Your budget line is the color line in Exhibit 18.3, with an absolute slope of 2, reflecting the relative price of tail-dollars in terms of head-dollars.

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Fair Odds

Fair odds

Odds that reflect the true probabilities of various states of the world.

Odds are said to be **fair odds** if they reflect the actual probabilities of the two states of the world. An unbiased coin is equally likely to come up heads or tails, so the fair odds on the toss of such a coin are 1 to 1. A weighted coin might be twice as likely to come up heads as to come up tails, in which case the fair odds are 2 to 1 for those who bet on tails.

Exercise 18.5 What are the fair odds on a bet that the roll of a die will turn up 1? What are the fair odds on a bet that it will turn up 4 or less? What are the fair odds on a bet that it will turn up an even number?

What is so fair about fair odds? The answer is that at fair odds the expected value of any bet is the same as the expected value of not betting at all. In other words, if two parties bet with each other repeatedly at fair odds, neither will come out very far ahead or very far behind in the long run. If a coin comes up heads twice as often as it comes up tails, and if the payoff for betting on heads is half the payoff for betting on tails, then each party's wins and losses will just cancel out.

When an individual is offered fair odds, any gamble has the same expected value as any other. Therefore,

When an individual is offered fair odds, his budget line coincides with an iso-expected value line.

Preferences and the Consumer's Optimum

The Frequent Gambler

A gambler who bets frequently with the goal of maximizing his winnings is concerned only with the expected values of his wagers. This is because any wager, when it is repeated sufficiently often, returns its expected value on average. In panel B of Exhibit 18.1, if the coin is unbiased, points *C, D,* and *E* all have the same expected value and hence are equally attractive to the frequent, repetitive gambler. If he holds basket *C* every day, he comes away with $100 every day. If he holds basket *E* every day, he comes away with $200 half the time and $0 the other half. Over time, this averages out to the same $100 per day that he can have with basket *C.*

Diversify

To reduce risk.

Risk-neutral

Caring only about expected value.

The frequent gambler is indifferent between two baskets of equal expected value, regardless of the risk associated with each. We say that this is because he can **diversify** his risk by playing repeatedly so that he is guaranteed to win the expected value of any gamble in the long run.[1] When someone's preferences among baskets are determined solely on the basis of their expected values, we describe those preferences as **risk-neutral.** From the definition of risk neutrality, we can see this:

The indifference curves of a risk-neutral individual are identical with the iso-expected value lines.

Risk Neutrality

We have seen that the frequent gambler is risk-neutral. Conceivably, some infrequent gamblers might be risk-neutral as well.

[1] This assumes that he can always borrow enough to keep playing after he is wiped out by a run of bad luck—or by a single turn of bad luck after a large bet.

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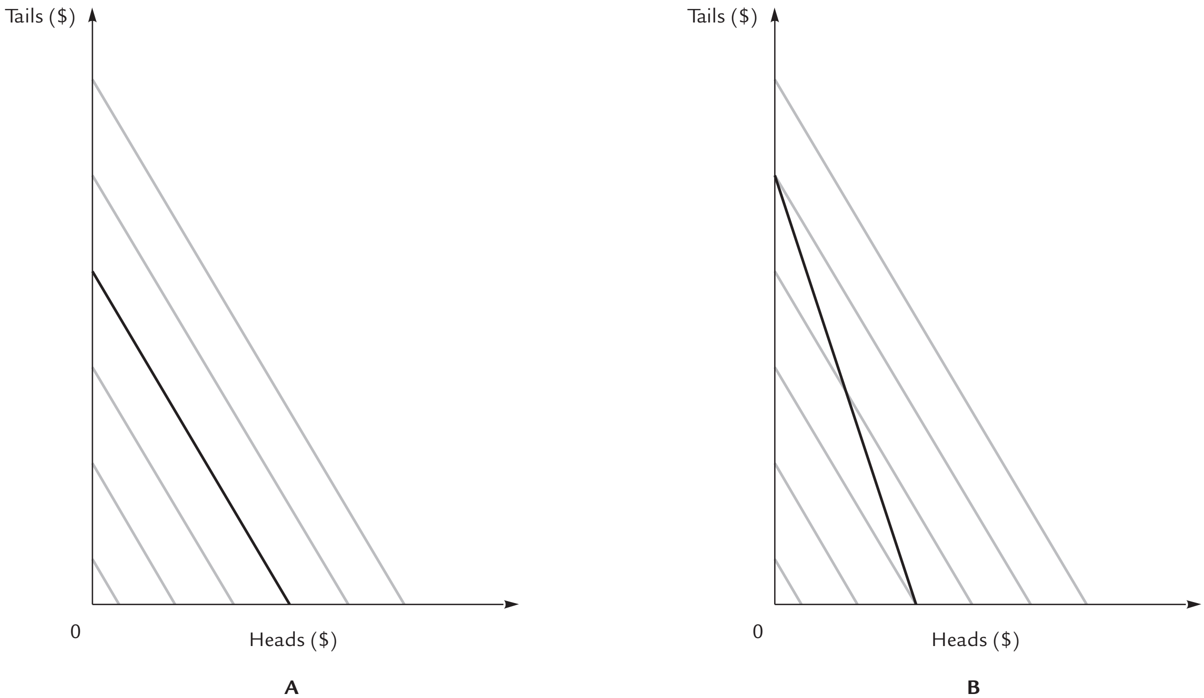
Consider a risk-neutral person who is given the opportunity to play at fair odds. Because he is risk-neutral, his indifference curves are the iso-expected value lines. Because the odds are fair, his budget line is the iso-expected value line through his endowment. The picture is as in panel A of Exhibit 18.4, where the gray iso-expected value lines are the indifference curves and the black budget line coincides with one of them. This individual is indifferent among all of the points on his budget line. Thus,

At fair odds, a risk-neutral individual is indifferent as to how much he bets.

Suppose that the risk-neutral person has an opportunity to play at other than fair odds. This rotates his budget line through his endowment, either clockwise, if the new odds favor betting on tails, or counterclockwise, if the new odds favor betting on heads. The first possibility is illustrated in panel B of Exhibit 18.4. As you can see, he now chooses a point on the vertical axis where his wealth becomes zero in the event that the coin turns up heads.

A risk-neutral individual faced with unfair odds will bet everything he owns on one or the other outcome.

EXHIBIT **18.4** **Risk Neutrality**



A risk-neutral individual has indifference curves that coincide with the iso-expected value lines, shown in gray in both panels. When he is offered fair odds, his budget line coincides with one of the indifference curves, as in panel A. In that case the individual is indifferent among all of the options available to him. When he is offered any odds other than fair odds, his budget line has a different slope than his indifference curves, like the black budget line in panel B. In that case, he will always choose a corner and bet everything he has on one outcome or the other.

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Dangerous Curve

Unlike all of the indifference curves we have encountered previously, the indifference curves of this chapter depend on more than just tastes. They depend also on the probabilities associated with the two states of the world. If a fair coin is replaced by a biased coin, a gambler might change his mind about the desirability of various wagers, even though his underlying tastes have not changed.

Risk Aversion

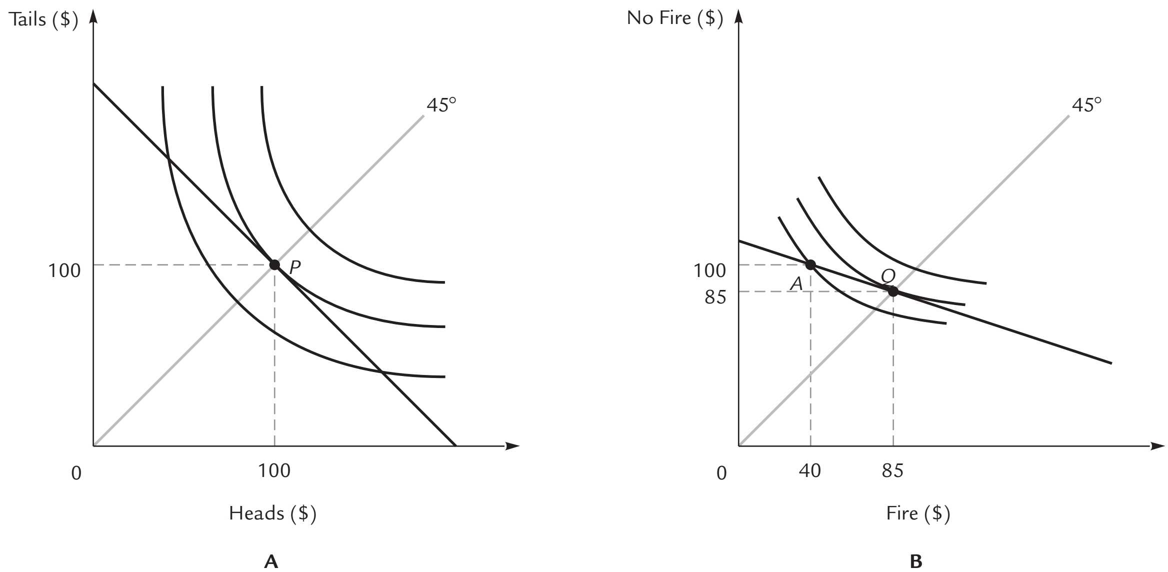
Now let us consider the preferences of someone who is not a frequent gambler. To such a person, the riskiness of his basket can be a significant consideration. He does not expect his gains and losses to cancel out in the long run.

Risk-averse Always preferring the least risky among baskets with the same expected value.

Many people are **risk-averse.** This means that among baskets with the same expected value, they choose the one that is least risky. Consequently, when offered fair odds, they choose the basket that equalizes their incomes in both states of the world. Such baskets are located on the 45° line.

The two panels of Exhibit 18.5 show the indifference curves of typical risk-averse individuals facing fair odds. In panel A the individual has an initial wealth of $100 and

EXHIBIT **18.5** **Risk Aversion**



The two panels illustrate the indifference curves of individuals facing fair odds. In panel A the individual has initial wealth of $100 and is offered the opportunity to bet at even odds on the toss of a fair coin. His endowment is at point *P*, which is already on the 45° line. This is also his optimum, so he places no wager.

In panel B the individual has initial wealth of $100, which will be reduced to $40 in the event of a fire. His endowment is at point *A*. We assume that the probability of "no fire" is 3 times as great as the probability of "fire." Thus, the fair odds for an insurance policy are 3 to 1, and we assume that such a policy is available. This gives the illustrated budget line, which crosses the 45° line at (85, 85). Because he is risk-averse, his optimum is at *Q*. He achieves this point by purchasing $15 worth of insurance.

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is offered the opportunity to bet on a coin toss at fair odds. His optimum point occurs right on the 45° line, at his endowment point *P.* He places no wager.

Panel B shows the situation of a risk-averse person whose wealth is $100, which is reduced to $40 if there is a fire. His endowment is at point *A*. We will assume that "fire" occurs with probability .25, so "no fire" occurs with probability .75.

Suppose that it is possible to buy fire insurance for $1. The insurance pays $4 in the event of fire, and the homeowner can buy as many units of this insurance as he wants to. Buying insurance is exactly like betting that there will be a fire. If there is no fire, he loses his $1. If there is a fire, there is a net gain of $3 (a $4 insurance payment minus the $1 cost of the insurance). Therefore, this particular insurance policy offers 3-to-1 odds when the homeowner bets that a fire will take place. These happen to be the fair odds, because the probability of "no fire" (.75) is 3 times the probability of "fire" (.25).

The homeowner's budget line has an absolute slope of 1/3, reflecting the odds of 3 to 1. Because the homeowner is assumed to be risk-averse, he always eliminates risk when he can bet at fair odds. That is, he chooses the point where his budget line crosses the 45° line, at point *Q* in panel B of Exhibit 18.5. At this point the homeowner is guaranteed that his wealth will be $85 regardless of whether or not the fire occurs. His indifference curves must be like those in the graph, with the optimum at *Q.*

Exercise 18.6 Exactly how much insurance does the homeowner buy?

Risk Preference

Another type of individual is **risk-preferring.** Given a choice between a "sure thing" and a lottery with the same expected value, he chooses the lottery. Such an individual has indifference curves as shown in Exhibit 18.6. They become tangent to the fair-odds budget lines at points along the 45° line, but this is because the individual considers any such point to be the *worst* he can do when trading at fair odds. You can see from Exhibit 18.6 that a risk-preferring person always chooses a lottery in which he risks sacrificing everything he owns in exchange for a chance at great wealth.

Risk-preferring
Always preferring the most risky among baskets with the same expected value.

It is also possible for an individual to be risk-preferring in some situations and risk-averse in others. Consider an individual with the indifference curves and budget line shown in Exhibit 18.7. Starting from an endowment at point *A,* he indulges his risk preference by gambling to get to either point *B* or point *C.* At that point, risk aversion becomes dominant and he gambles no further.

Which Preferences Are Most Likely?

Attitudes toward risk typically vary with income. At very low levels of income, people are probably risk-preferring. To see the reason for this, suppose that $5 per year is the minimum income necessary for survival. In that case, an income of $3 per year is no more valuable than an income of zero. Somebody earning $3 per year would be willing to gamble, even at very unfavorable odds, for a chance to earn enough to stay alive.

Even at higher levels of income, we sometimes observe risk preference for similar reasons. If you are determined to purchase a particular sailboat for $20,000 and if your current assets total $19,000, you might be willing to take a very risky bet as long as it offered some chance to win $1,000.

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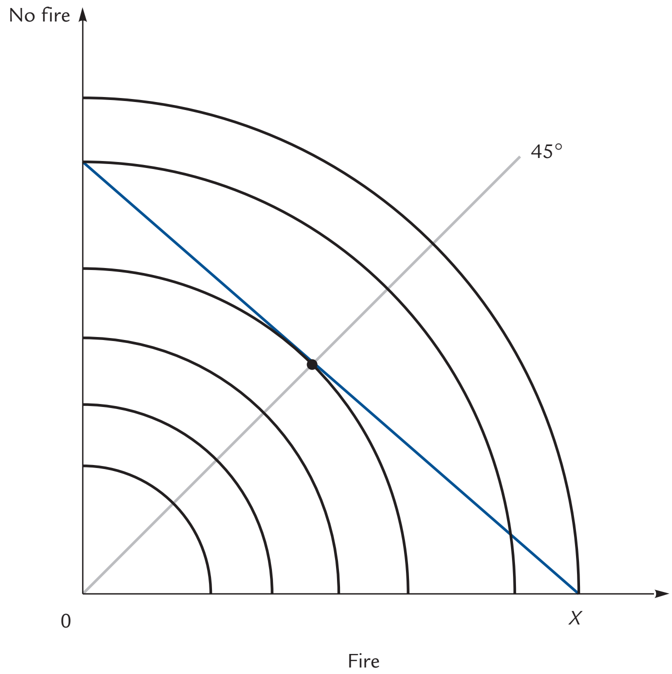
EXHIBIT **18.6** **Risk Preference**



The risk-preferring individual always chooses a corner solution, regardless of the odds he faces. This individual chooses point *X,* where his wealth becomes zero if there is no fire. He can accomplish this by spending all of his income on fire insurance, hoping for a fire that will make him rich.

Nevertheless, most individuals exhibit some degree of risk aversion over most ranges of income. A person earning $20,000 per year is unlikely to be willing to trade a year's income for a 50-50 chance at $40,000, or even a 50-50 chance at $50,000. On the other hand, the same person might very well be willing to trade $20 for a 50-50 chance at $50, or $2 for a 50-50 chance at $5. When small amounts are involved, people tend to exhibit risk-neutral behavior. With large amounts at stake, however, risk aversion is the general rule.

Firms, as opposed to individuals, are more likely to exhibit risk neutrality. This is so for several reasons. First, many firms are frequent gamblers that participate in a large number of risky ventures and can expect their good and bad luck to cancel out over time. Second, unlike individuals, firms face no budget constraints. An individual who risks all his assets and loses is wiped out, whereas a firm that risks all its assets and loses can often borrow enough to continue operating. (Of course, the firm must convince lenders that it is showing good business sense in the long run.)

Those firms that are corporations have an additional reason for risk-neutral behavior. Corporate stockholders are able to diversify their risks by holding small amounts of stock in many different companies. Once diversified, they, like the frequent gambler, earn approximately the expected value of the return on their overall portfolios. For this reason, the stockholders are interested only in maximizing expected return, and they want the corporation to behave in a risk-neutral way.

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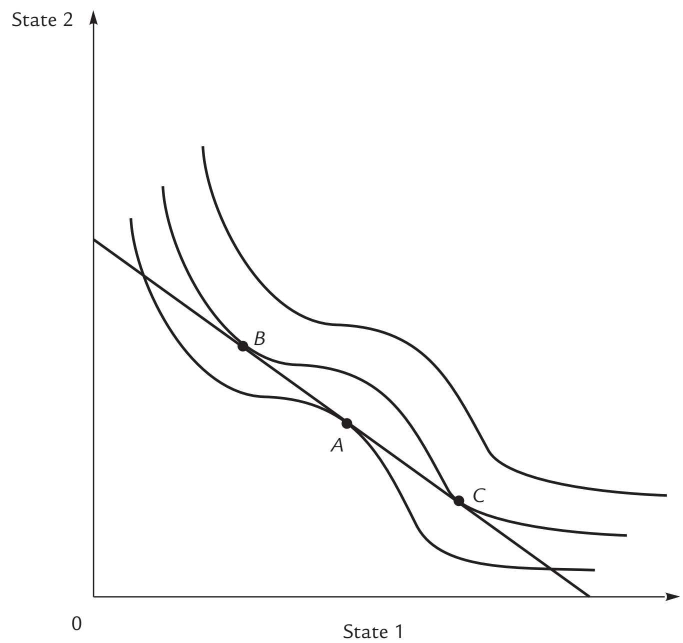
EXHIBIT **18.7** **Risk Preference and Risk Aversion Combined**



The same individual can exhibit both risk preference and risk aversion at different points on his indifference curve map.

Gambling at Favorable Odds

Often we encounter opportunities to gamble at better than fair odds. Suppose that you own a restaurant and have the opportunity to run an advertising campaign that has a 50-50 chance of success. If the campaign succeeds, your profits (net of advertising costs) will increase by $2,000, whereas if it fails, you will lose $1,000. Because success and failure are equally likely, and because the gain from success exceeds the loss from failure, the odds are better than fair. If you run the campaign, you increase the expected value of your wealth. For another example, suppose that you have the opportunity to buy a ticket to a concert that you will enjoy with probability of .75. The ticket costs $1, and you receive $2 worth of pleasure if the concert turns out to be good. Thus, if the concert is bad, you lose $1, and if it is good, you gain $1 ($2 in enjoyment minus $1 for the ticket). Because the concert is more likely to be good than bad, the odds on this gamble are also favorable.

Exercise 18.7 For each of the opportunities described in the preceding paragraph, what odds would be fair? What are the actual odds? What is the expected value of your winnings if you gamble?

We have already seen that a risk-neutral person always accepts any wager in which the odds are better than fair and that he wagers as much as he possibly can at such odds.

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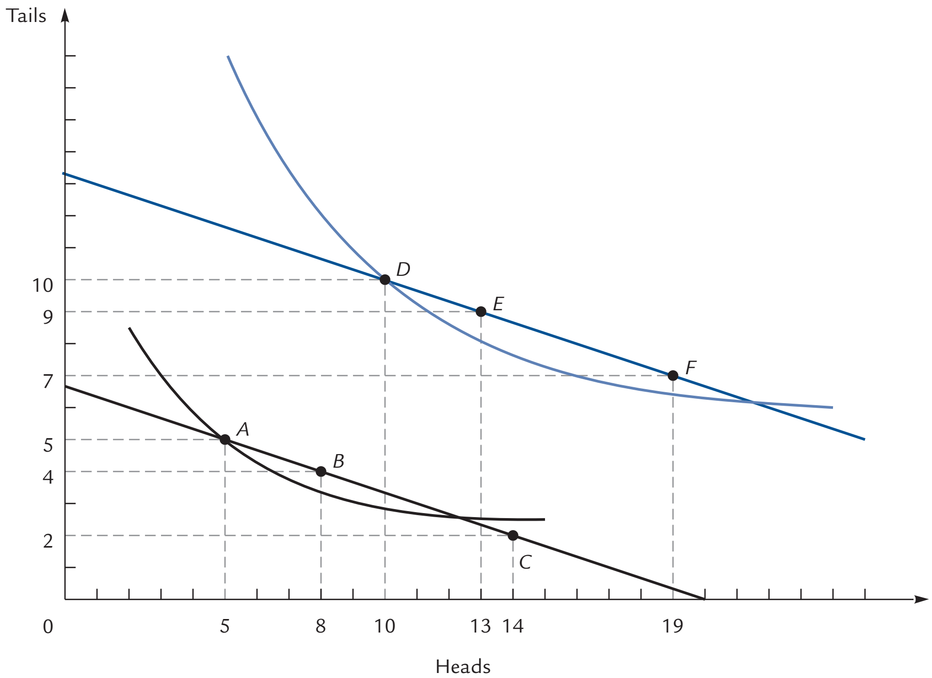
What does a risk-averter do? Does the prospect of a positive expected gain entice him to gamble, or does his risk aversion prevent him from gambling?

Consider an example. Suppose that you are risk-averse, have assets totaling \$5, and have the opportunity to gamble at 3-to-1 odds on the toss of an unbiased coin. If you bet \$1 on heads, then you either lose \$1 (if tails comes up) or win \$3 (if heads comes up).

Your budget line is then the black line in Exhibit 18.8. Your endowment is at point *A*, where you keep your \$5 no matter how the coin turns up. We know that if you were offered the fair odds of 1 to 1, you would not bet at all, so the absolute slope of the indifference curve at *A* must be 1. It follows that the budget line cuts through the indifference curve, as shown in the exhibit.

By betting \$1, you move from point *A* to point *B*, which is an improvement. Thus, if your only options are to bet \$1 or to not bet at all, you choose to bet.

EXHIBIT **18.8** **Gambling at Favorable Odds**



The indifference curves are those of a risk-averter facing the opportunity to bet on the toss of an unbiased coin. His initial wealth is \$5, so point *A* is his endowment. Because he is risk-averse, the absolute slope of the indifference curve at *A* must reflect the fair odds of 1 to 1; in other words, it has an absolute slope of 1.

This individual is invited to bet on heads at the favorable odds of 3 to 1. By betting \$1, he moves to point *B*, which he prefers to point *A*. If he bet \$3, he would move to point *C*, which he likes less than point *A*. Thus, if he is allowed to place the small bet of \$1, he will do so, but if he must place the large bet of \$3, he will decline.

Suppose that this individual has an increase in wealth, to \$10. Then his endowment moves to point *D*. From point *D* a \$1 bet moves him to point *E* and a \$3 bet moves him to point *F*. Either of these is an improvement over point *D*, and if offered either option, he will accept it. With greater initial wealth, he is willing to accept the \$3 bet that he previously considered too large. However, he will continue to reject much larger bets.

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Suppose, alternatively, that the house rules require you to bet either \$3 or nothing at all. A \$3 bet would move you to point *C*, which is less desirable than point *A*. Thus, in this case you would prefer not to bet.

Therefore, the exhibit demonstrates this principle:

A risk-averse person, offered the opportunity to place a sufficiently small bet at favorable odds, always accepts. If only offered the opportunity to place a very large bet at favorable odds, he always declines.

The largest bet that a risk-averter would be willing to make depends on his wealth. Suppose that instead of starting with \$5, you started with \$10. In that case, your endowment would be at point *D* in Exhibit 18.8. A \$1 wager at the favorable odds of 3 to 1 brings you to point *E*, and a \$3 wager brings you to point *F*. Either of these is preferable to point *D*. Thus, even if the house rules require the relatively large \$3 wager, you still choose to bet.

The indifference curves of Exhibit 18.8 are typical. As a risk-averter acquires more wealth, he is willing to enter into larger wagers at favorable odds. However, there is always a limit to what size wager he will accept. Even with the initial wealth of \$10, a person with the indifference curves of Exhibit 18.8 will not bet \$5 on heads.

Risk and Society

Societies, like corporations, must decide when to undertake risky projects. Just as risk-averse stockholders can prefer the corporations they own to behave risk-neutrally, so risk-averse citizens can prefer the societies they inhabit to behave risk-neutrally in some respects. In a society that undertakes a large number of independent investment projects, citizens will be best off in the long run if those projects are evaluated risk-neutrally. However, the individual entrepreneurs who actually decide how to allocate resources often have much personal wealth at stake, so risk aversion enters their decisions.

In some cases, however, entrepreneurial initiatives are intensely personal. In the 1950s Joseph Wilson (later the head of the Xerox Corporation) had a vision of the copying machine as a tool that would transform U.S. business. At the time few shared his vision. Entrepreneurial visions arise every day, and most do not succeed. Should such visions be pursued?

Suppose that Wilson had a 1 in 100 chance of succeeding in his project. Then from a social point of view, the project should be undertaken if the benefits from a success would be more than 100 times the losses from a failure. The frequency with which such projects arise in society justifies a risk-neutral calculation. But visions are the property of individuals, and individuals are risk-averse. From Wilson's point of view, a mere 100-to-1 payoff would not have sufficed. In order to induce him to risk a substantial fraction of his personal wealth for a 1% chance of success, Wilson might have required the prospect of a 500-fold multiplication of his wealth.

From a social point of view, risk-averse individuals underinvest in risky projects. The existence of corporations helps to solve this problem, because, as we have seen, the shareholders, with diversified portfolios, will encourage appropriate risk-taking. However, intensely personal visions cannot always be effectively pursued by large corporations. In such cases, only the prospect of great personal fortune will induce individuals to take great risks. A society that attempted to limit the amassing of great wealth might be a society without copying machines.

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18.2 The Market for Insurance

Many markets have developed to facilitate transfers of risk from one party to another. In this and the next two sections, we will examine a few of these markets. We have already alluded to the insurance market in Section 18.1. Panel A of Exhibit 18.1 depicts the endowment of a homeowner facing the possibility of a fire. In Exhibit 18.5 we can see how the homeowner, when facing a given price, decides how much insurance to buy. But what determines the market price of insurance?

Insurance companies are highly diversified. If each individual house catches fire with probability .25, you must experience considerable uncertainty about whether yours will be one of those that burn. By contrast, a company that insures 1,000 houses can be sure that almost exactly 250 of them will burn. If there were no other considerations, an insurance company that offered fair odds would just break even. Any insurance company offering less than fair odds would earn profits, causing entry to the insurance industry and driving the odds down until they were fair. Thus, a $1 insurance policy must buy a $4 payoff in case of fire.[2]

There are, however, other considerations. For one thing, there are costs involved with running an insurance company—costs of maintaining an office, a sales force, an actuarial staff to estimate probabilities, assessors to estimate actual damages when they occur, and so forth. A firm offering fair odds could not cover these costs and would not survive. The odds must be tilted in the company's favor by enough so that these basic operating costs can be met.

However, more interesting and more important reasons exist as to why insurance is not offered at fair odds. In discussing them, we can safely ignore the relatively minor issue of operating costs.

Imperfect Information

First, there are problems of information, such as *moral hazard* and *adverse selection,* which were discussed in Section 9.3. The moral hazard problem arises when people behave more recklessly because they are insured; this means that insurance companies must offer odds that are adjusted accordingly.

The adverse selection problem arises when fair odds are different for different people (as when some are more naturally susceptible to disease than others, which affects the fair odds for health insurance) and the insurance company is unable to tell who is who.

As in Section 9.3, assume that some people are "Healthies," with a 1 in 10 chance of becoming ill, while others are "Sicklies," with a 9 in 10 chance of becoming ill. If the insurance company could distinguish one group from the other, it would offer the appropriate odds to each group. If it can't tell the difference, then it can't simply offer odds that are appropriate for Healthies, because Sicklies will purchase the insurance and bankrupt the company.

The discussion in Section 9.3 suggested a solution: Offer two policies, one at "Healthy" odds and one at "Sickly" odds, but limit the quantity of Healthy insurance that any one person can buy. If the quantity is chosen correctly, each group will voluntarily purchase the right kind of insurance.

[2] With this policy you lose $1 when there is no fire and you gain $3 (the $4 payoff minus the $1 premium) if there is a fire. Therefore, the policy offers the fair odds of 3 to 1.

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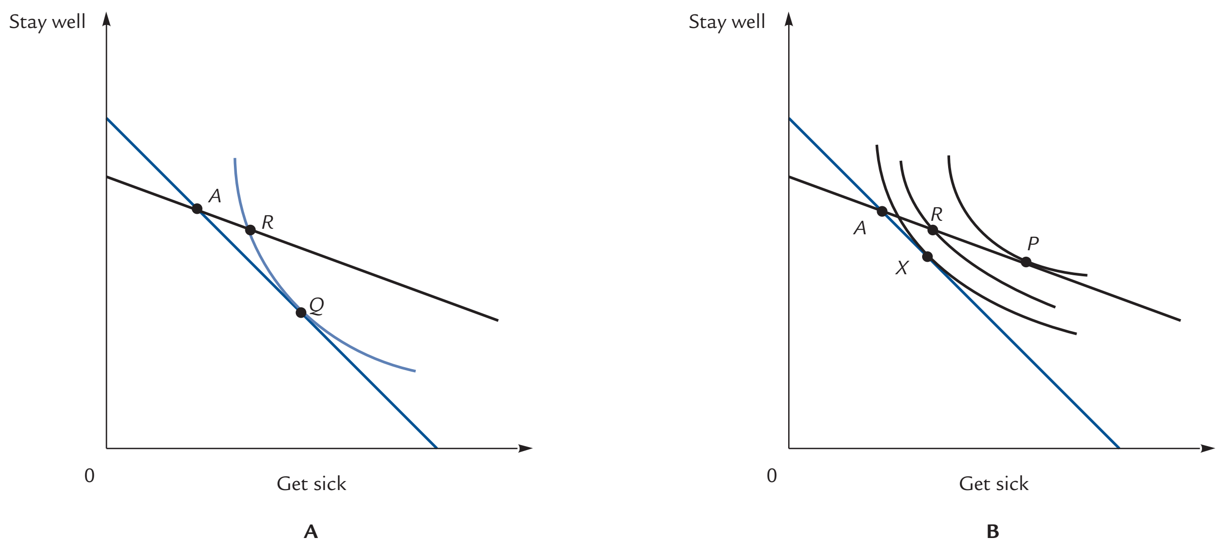
With the machinery of this chapter, we can show exactly how the limit is chosen. In panel A of Exhibit 18.9, Healthies and Sicklies both have endowment point *A*. The black budget line shows fair odds for Healthies and the color line shows fair odds for Sicklies. If Sicklies are offered insurance at fair odds, they choose point *Q* on the blue indifference curve. The point where the blue indifference curve crosses the black budget line is labeled *R*.

People who purchase insurance at Healthy fair odds move down along the black budget line from *A*. If purchases are limited so that nobody is allowed to move past *R*, then no Sickly ever chooses Healthy insurance. By purchasing Sickly insurance, the Sickly can achieve point *Q*, which is preferred to any point between *A* and *R* on the black budget line.

Healthies, on the other hand, who have a different family of indifference curves, might very well choose Healthy insurance. Panel B of Exhibit 18.9 shows that a Healthy would rather buy a limited quantity of Healthy insurance, achieving point *R*, than an unlimited quantity of Sickly insurance, which allows the Healthy to achieve point *X*.

Of course, if Sicklies voluntarily revealed their identities, the insurance company could offer each form of insurance in unlimited quantities to the appropriate group.

EXHIBIT 18.9 Adverse Selection



Healthies and Sicklies both have endowment point *A.* The black budget line represents fair odds for Healthies and the colored budget line represents fair odds for Sicklies. If Sicklies buy Sickly insurance, they choose the quantity to achieve point *Q* on the brown indifference curve in panel A. The point where that curve crosses the black budget line is labeled *R.*

The insurance company offers both types of insurance, but limits the quantity of Healthy insurance so that the purchaser cannot move past point *R* on the black budget line. Sicklies voluntarily choose Sickly insurance, because they prefer point *Q* to any point between *A* and *R* along the black line. Healthies, meanwhile, might have the black indifference curves shown in panel B and choose the limited Healthy insurance, because they prefer point *R* to anything they can achieve on the color budget line.

If Sicklies voluntarily identified themselves, the company could offer unlimited quantities of each type of insurance to the appropriate group. Sicklies would still achieve point *Q,* and Healthies would achieve point *P.*

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Sicklies would still achieve point *Q*, and Healthies would be better off, achieving *P* instead of *X*. But, as discussed in Section 9.3, such voluntary revelation cannot be sustained in equilibrium.[3]

Uninsurable Risks

Uninsurable risk
A risk that cannot be diversified.

Another reason why fair-odds insurance is not always available is that some risks are **uninsurable risks** because they cannot be diversified. This occurs when a large number of people are all adversely affected in the same state of the world.

Suppose that you and your friend must each carry a $10 bill through a bad neighborhood at different times. You can insure against robbery by agreeing that if either one is robbed, the other will ease the burden by paying $5 to the victim. But if you are traveling together, so that if one is robbed the other will also be robbed, then there is no advantage to such a contract and no way you can insure each other.

An insurance company brings together many people, its customers, who effectively insure each other against individual disasters. But a collective disaster cannot be insured against by everybody simultaneously. You cannot buy fair-odds insurance against a nuclear disaster. The insurance company is risk-neutral when it insures 1,000 people against a .25 chance of fire, because it knows that it will have to pay off in only 250 cases. It is no longer risk-neutral, and will not offer fair-odds insurance, when it insures 1,000 people against a .25 chance of a nuclear disaster, because there is a .25 chance that it will have to pay off in 1,000 cases.

18.3 Futures Markets

Futures contract
A contract to deliver a specified good at a specified future date for a specified price.

Futures market
The market for futures contracts.

Spot market
The market for goods for immediate delivery.

Spot price
Price in the spot market.

Suppose that you are a farmer, planting wheat in the spring to be harvested in the fall. You do not know whether the price of wheat will be $3 or $4 a bushel next fall, and you are therefore uncertain both about your future wealth and about the optimal amount of planting to do. If you are risk-averse, you will want to insure against the possibility of a low price.

In practice, this is often accomplished through the medium of a **futures contract.** A futures contract is an agreement to deliver a specified amount of something (in this case wheat) at some future date (in this case next fall) for a price agreed upon today. If the low price of $3 and the high price of $4 are equally likely, then a "fair-odds" delivery price is $3.50. By signing a contract to deliver at this price, you can reduce your risk without sacrificing expected value. At the same time, the buyer of the contract is able to insure against a high price, which is the unfavorable state of the world from the buyer's point of view.

The market for futures contracts is called the **futures market** for short. The market for wheat for immediate delivery is called the **spot market.** The **spot price** of wheat is the price of wheat in the spot market; in other words, it is simply what we would ordinarily call "the" price of wheat.

[3] The discussion of adverse selection is based on M. Rothschild and J. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay in the Economics of Imperfect Information," *Quarterly Journal of Economics* 90 (1976):629–650. Rothschild and Stiglitz show that the solution in Exhibit 18.9 is the only possible equilibrium, although there might be no equilibrium at all.

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Speculation

Nonfarmers can also sell futures contracts. Suppose that in July wheat for September delivery is selling at \$3.50 per bushel. You, however, believe that next September the spot price of wheat is likely to be only \$3.25. In that case, you can sell a futures contract for \$3.50, wait until September, and then buy a bushel of wheat for \$3.25 to deliver in fulfillment of your contract. You will earn a profit of 25¢. On the other hand, if you are in error and the spot price next September turns out to be \$3.75, then you will have to buy at that price and will end up with a net loss of 25¢.

Somebody who tries to outguess the market and earn profits by buying and selling futures contracts is called a **speculator.** Next we will see that when speculators are successful, they have the effect of improving economic efficiency.

Speculator
One who attempts to earn profits in the futures market by predicting future changes in supply or demand.

Suppose that it is now February. A certain amount of grain is stored in grain elevators, and this is the only source of grain for this month and the next. The sellers (that is, the elevator owners) must decide how much to sell in February and how much to save for sale in March. Panel A of Exhibit 18.10 shows the February demand curve for grain. Panel B shows (in dark color) the expected March demand curve as foreseen by the sellers. Sellers choose to supply Q_F bushels in February and save Q_M bushels to supply in March. (If they are risk-averse, they sell futures contracts now, promising delivery of Q_M bushels in March.) These quantities are chosen so that the equilibrium prices in the two months are equal. In Exhibit 18.10 the equilibrium price in each month is P_0.

To see why the equilibrium prices must be equal, let us see what would happen if the expected March spot price exceeded the current price. Sellers, sensing a profit, would save more grain for next month, reducing Q_F and increasing Q_M. This would have the effect of raising the current price and reducing the March price and would continue until the two prices were equal.

Exercise 18.8 Explain what happens if the current price exceeds the expected March spot price.

Dangerous Curve

Actually, sellers equate the current price not to the March price, but to the present value of the March price. We are assuming that the interest rate is small enough so that, for practical purposes, a dollar delivered in March is worth as much as a dollar delivered in February. We are also ignoring storage costs, which, if significant, would make suppliers willing to provide grain at a lower price today than tomorrow. These assumptions simplify the analysis but do not affect the welfare conclusions.

Now suppose that there arrives on the scene a speculator who believes that the market has made a mistake and that the March demand curve will be lower than everyone else expects. He believes that the March demand curve will be the light-colored demand curve shown in panel B of Exhibit 18.10. Anticipating a profit, he sells futures contracts, planning to fulfill them by buying cheap wheat on the spot market in March.

The speculator's advertised willingness to provide March wheat at less than the going price of P_0 drives down the expected price of March wheat and along with it the price of a March futures contract. With the discovery that March wheat is selling for less than P_0, grain suppliers sell more wheat today and save less for March, moving the vertical February supply curve to the right and the vertical March supply curve to the left. This process continues until the speculator no longer perceives any profit to

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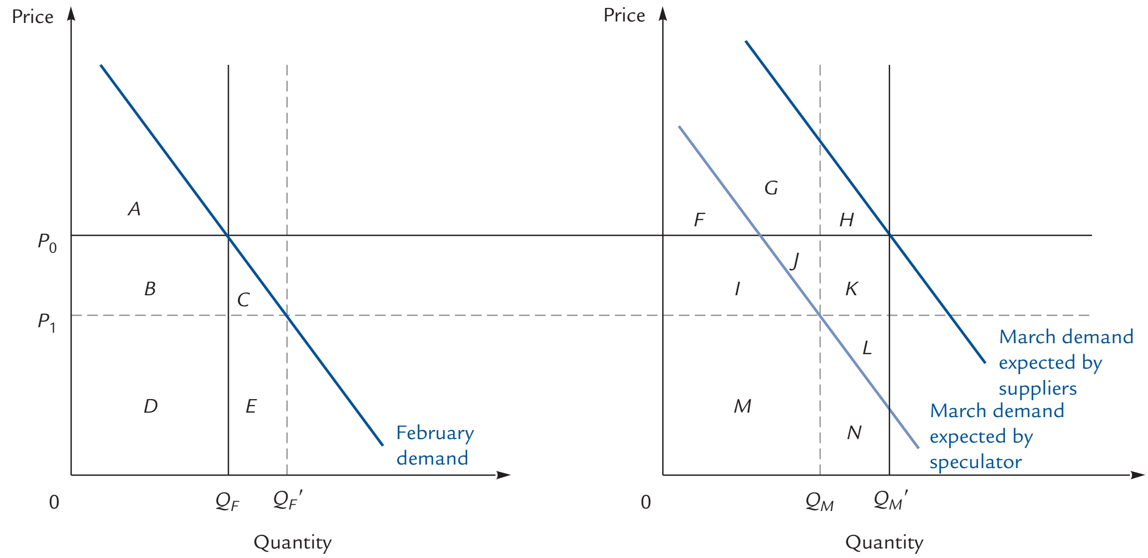
EXHIBIT 18.10 Speculation



A. Supply and Demand for February Grain **B. Supply and Demand for March Grain**

Case 1: Speculator Right

Without Speculator:	With Speculator:
February welfare: $A + B + D$	February welfare: $A + B + C + D + E$
March welfare: $F + I + M + N$	March welfare: $F + I + M$
Total: $A + B + D + F + I + M + N$	Total: $A + B + C + D + E + F + I + M$

Gain due to speculator: $C + E - N$

Case 2: Speculator Wrong

Without Speculator:	With Speculator:
February welfare: $A + B + D$	February welfare: $A + B + C + D + E$
March welfare: $F + G + H + I + J + K + L + M + N$	March: $F + G + I + J + M$
Total: $A + B + D + F + G + H + I + J + K + L$ $+ M + N$	Total: $A + B + C + D + E + F + G$ $+ I + J + M$

Loss due to speculator: $H + K + L + N - C - E$

The February demand curve for grain is shown in panel A. Suppliers expect the March demand curve to be the dark-color curve in panel B. Thus, they supply Q_F bushels in February and Q_M bushels in March, where these quantities are chosen to make the prices equal. The price in either month is P_0.

Now a speculator arrives on the scene, believing that the March demand curve will be the light-color curve in panel B. Thus, he offers to sell March futures contracts, driving down the price of March grain and leading suppliers to sell more in February and less in March. The quantities adjust to Q_F' and Q_M'.

The table shows the welfare analysis, first when the speculator proves to be right and then when he proves to be wrong. In each case, we must use the appropriate March demand curve—the light-color one if the speculator is right and the dark-color one if he is wrong. If the speculator is right, his arrival increases welfare, and if the speculator is wrong, his arrival decreases welfare.

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be earned by undercutting the price of March wheat, that is, until the quantities have moved to Q_F' and Q_M' and the price has fallen to P_1.

Speculation and Welfare

The table in Exhibit 18.10 calculates the change in welfare due to the arrival of the speculator, first on the assumption that he is right about the March demand curve and then on the assumption that he is wrong. The marginal cost of providing grain that is already in storage has been taken to be zero, so social welfare is simply the area under the demand curve. To calculate March welfare, we must use the actual March demand curve, which is the light-color curve if the speculator is right and the dark-color curve if he is wrong.

To understand the gains and losses better, keep in mind that the distance from Q_F to Q_F' must equal the distance from Q_M' to Q_M. (Either of these distances is the amount of additional grain sold in February instead of March.) From this it is easy to see that N is less than $C + E$, so the speculator really increases social welfare when he is right. Similarly, $C + E$ is less than $H + K + L + N$, so the speculator really decreases social welfare when he is wrong.

Society gains when a speculator correctly alerts it to a coming drop in demand by bidding down the price of futures contracts. This information enables people to increase their current consumption, in recognition of the fact that grain will be less valuable at the margin tomorrow than it is today. Similarly, a speculator who correctly forecasts an increase in tomorrow's demand bids up the price of futures contracts, alerting people today that wheat will be more valuable tomorrow and ought to be conserved.

When the speculator guesses the future correctly, he earns profits and he increases social welfare. When he guesses incorrectly, both he and society lose. By and large, we expect successful speculators to increase the level of their speculative activity over time, and unsuccessful speculators to eventually drop out of the market. Therefore, it is a reasonable expectation that the majority of existing speculators serve a welfare-improving function.

18.4 Markets for Risky Assets

Many assets are valued not for their uses in consumption but for their potential to increase their owners' wealth. Corporate stocks are a prime example; real estate is another. The owner of a stock is often entitled to a stream of dividends of uncertain size. In addition, the value of the stock itself might rise or fall. Both the dividends and the changes in the stock price are referred to as **returns** to the owner of the stock. The expected present value of these returns is called the **expected return** to the stock owner.

Returns

Gains to the holder of a financial asset, including dividends and increases in the asset's value.

Expected return

The expected value of returns.

A risk-neutral stockholder cares only about expected returns. A risk-averse stockholder cares also about the certainty with which those returns will be realized. Such a stockholder is not indifferent between a stock that returns \$5 next year for certain and one that returns either \$0 or \$10 next year with 50-50 probabilities, even though the expected returns are the same in each case.

The risk associated with a given stock can be described by a number called the **standard deviation** in its returns, abbreviated by σ (the Greek letter *sigma*). If you have taken a statistics course, you know a precise definition of the standard deviation. What you need to know here is that σ is a measure of the *spread* in possible outcomes. A stock that returns \$5 with certainty has $\sigma = \$0$. A stock that returns either \$0 or \$10 with

Standard deviation

A precise measure of risk.

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equal probability has σ = \$5. A stock that returns either −\$5 or \$15 with equal probability has σ = \$10.

We shall henceforth measure expected returns and standard deviations as percentages of current asset values. Thus, a stock that currently sells for \$10 and is expected to return \$5 (either by increasing in value or by paying dividends) has an expected return of 50%. If the \$5 return is certain, then σ = 0%. If the return might be either −\$5 or \$15, then σ = 100%, because \$10 is 100% of \$10.

Investors

Buyers of risky assets.

People who buy financial assets in the hope of increasing their wealth are often referred to as **investors.** The language is unfortunate, because the purchase of existing stocks, bonds, and real estate does not constitute investment in the sense of Chapter 17. Economists generally reserve the word *investment* to describe the creation of new factors of production. Nevertheless, we will bow to popular usage and refer to the purchaser of stocks as an "investor."

Portfolios

Portfolios

Combinations of risky assets.

An investor is interested not only in the characteristics of individual stocks, he is also interested in the characteristics of **portfolios,** or combinations of several stocks. In order to compare the characteristics of a portfolio with those of the stocks it comprises, let us consider some examples.

Exhibit 18.11 displays the characteristics of three stocks, each now selling for \$10. The stocks are General Air-Conditioning (GAC), General Surfboards (GSB), and General Snowshoes (GSS). The value of each stock tomorrow depends on the state of the world. Either an ice age begins or it doesn't. Exhibit 18.11 shows what will happen to each stock in each state of the world. It also shows the expected return and the standard deviation for each stock, computed on the probability of an ice age is .50.

For each stock the expected return is the average of the returns in the two states of the world, and the standard deviation (σ) is equal to the absolute value of the difference between the expected return and either of the possible actual returns. For instance, the possible returns to General Surfboards are −40% and 120%. The average of these is 40%, which is the expected return. The possible returns of −40% and 120% differ from

EXHIBIT 18.11 Expected Returns and Standard Deviations

Stock	Current Value	Value If Ice Age Comes	Value If No Ice Age Comes	Expected Return	σ
General Air-Conditioning (GAC)	\$10	\$5 (Return = −50%)	\$25 (Return = 150%)	50%	100%
General Surfboards (GSB)	10	6 (Return = −40%)	22 (Return = 120%)	40	80
General Snowshoes (GSS)	10	25 (Return = 150%)	5 (Return = −50%)	50	100

The table displays the characteristics of three hypothetical stocks. There is a 50% chance of an ice age beginning tomorrow, and each stock's value tomorrow depends on whether the ice age actually arrives. For each stock the expected return is the average of the two possible returns. For each stock the standard deviation is the absolute value of the difference between its return if the ice age arrives and its expected return.

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the expected return of 40% by exactly 80% in absolute value, so for General Surfboards $\sigma = 80\%$.

We can now compute the returns and standard deviations on various portfolios. Consider first a portfolio consisting of one share each of GAC and GSB. Such a portfolio has a current value of \$20 and could either go down to \$11 (the sum of the values of the two stocks if the ice age arrives) or go up to \$47 (the sum of the values if the ice age fails to arrive). These outcomes constitute returns of either −45% or +135%. The expected return is 45% and the standard deviation is 90%.

Exercise 18.9 Verify the numbers in the preceding paragraph.

If you are rash enough to generalize on the basis of this single example, you might be tempted to conclude that the expected return and standard deviation of a portfolio are computed by taking the average expected return and the average standard deviation of the constituent stocks. If you succumbed to such a temptation, you would be right regarding the expected return, but wrong regarding the standard deviation.

Consider a portfolio consisting of one share each of GAC and GSS. The current value of such a portfolio is \$20. In the event of an ice age, its value will be \$5 + \$25 = \$30, and in the event of no ice age, its value will be \$25 + \$5 = \$30. Such a portfolio earns a 50% return with certainty. Its standard deviation is zero.

A portfolio consisting of GAC and GSS is completely diversified. Whenever one of its constituent stocks goes up, the other goes down. As a result, all of the risk is eliminated and σ is equal to zero. In general, the standard deviation of a portfolio is given by the average of the standard deviations of the individual stocks, *minus* a correction term for any diversification that takes place. Because of this correction term, we can say:

The standard deviation of a portfolio is *at most* equal to the average standard deviation of the individual stocks.

By contrast:

The expected return to a portfolio is *exactly* equal to the average expected return of the individual stocks.

We have seen an example of a completely undiversified portfolio (GAC and GSB) and of a completely diversified portfolio (GAC and GSS). It is also possible to construct a portfolio that is partially, but not completely, diversified. Consider the portfolio that combines one share of GSB with one share of GSS. This portfolio, initially worth \$20, will either go up to \$31 or go up to \$27. The possible returns are 55% and 35%. The expected return is 45% (the average of the expected returns on the two stocks). The standard deviation is 10%, much less than the average of the standard deviations on the two stocks, but still not zero because the diversification is not complete.

The Geometry of Portfolios

Any individual stock, and any portfolio, can be represented by a point in a diagram, as in Exhibit 18.12. The points labeled *GSB* and *GSS* represent the stocks General Surfboards and General Snowshoes from Exhibit 18.11.

It is possible for two different stocks to occupy the same position in the diagram. GAC is represented by the same point that represents General Snowshoes.

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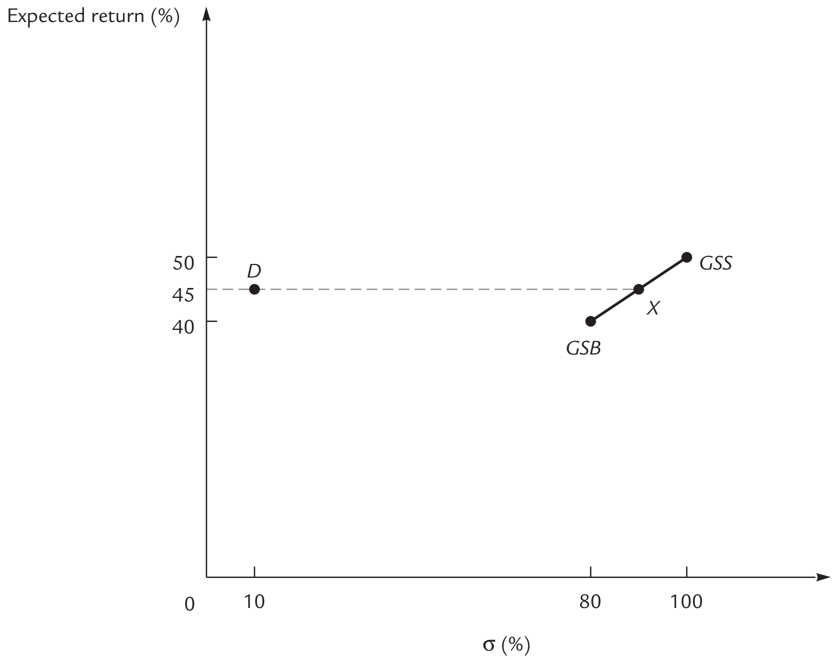
EXHIBIT **18.12** **The Geometry of Portfolios**



Every stock, and every portfolio, is represented by a point in the diagram. The points *GSS* and *GSB* represent General Snowshoes and General Surfboards, which are described in Exhibit 18.11.

The point *X,* which is midway between the stocks *GSS* and *GSB,* represents an asset with the average of their expected returns and the average of their standard deviations. The portfolio containing *GSS* and *GSB* has the average expected return but a smaller standard deviation. Thus, it is represented by a point directly to the left of *X,* namely, *D.*

There is a geometric construction of the portfolio that combines two given stocks. Consider the portfolio consisting of GSB and GSS. We begin by locating the midpoint of the line segment that connects the stocks. That point is labeled *X* in Exhibit 18.12. It represents a portfolio with the average of the two expected returns and the average of the two standard deviations. Because there is some diversification, the portfolio's standard deviation is less than the average of the two stocks' standard deviations. Thus, the portfolio is represented by a point some distance to the left of *X*. The size of the leftward shift depends on the amount of diversification. In this case we find that the combined portfolio is located at point *D.*

Exercise 18.10 What point represents the portfolio consisting of GAC and GSS?

Two portfolios can be combined to make a new portfolio, using the same geometric prescription that is used to combine two stocks.

The Efficient Set

In panel A of Exhibit 18.13 there are hypothetical dots representing all of the stocks that might be available at a given time. The shaded area represents all of the available portfolios. Any available stock must be in the shaded region, because one can always

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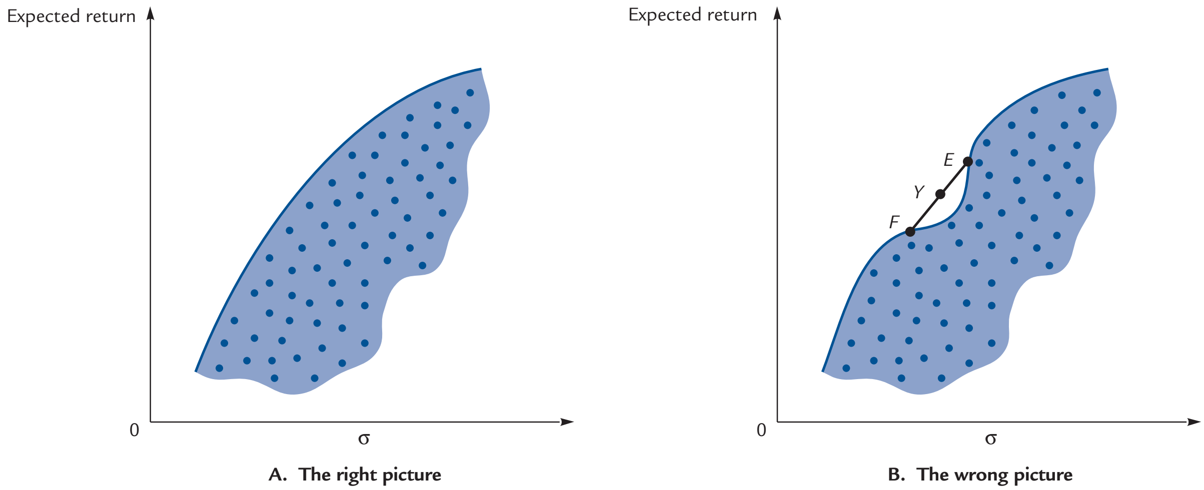
EXHIBIT **18.13** **The Efficient Set**



The dots represent the stocks available in the marketplace, and the shaded region represents all of the portfolios that can be constructed from those stocks. The picture must look like panel A and cannot look like panel B. In panel B the portfolio that combines portfolios *E* and *F* must be located at *Y* or to the left of *Y*, where the picture shows no portfolios. Therefore, the picture is wrong.

In panel A, which is the correct picture, the northwest boundary of the shaded region is the efficient set. No risk-averse investor would choose a portfolio that is not in the efficient set.

hold a portfolio consisting of that stock alone. We have also darkened the northwest boundary of the shaded region.

It is no accident that the northwest boundary is shaped as it is. Panel B suggests another shape, which we shall argue is impossible. If the boundary were shaped as in panel B, then there would be portfolios represented by points *E* and *F*. Combining these portfolios yields a new portfolio, which must be represented either by point Y or by some point to its left. Some such point must therefore be in the shaded region, which is not true. Therefore, the shape depicted in panel B is impossible.

The northwest boundary of the shaded region in Exhibit 18.13 is called the **efficient set,** or the set of **efficient portfolios.** These are the only portfolios that a risk-averse individual would ever hold. The reason is that from any other point in the shaded region the investor can always move either upward (increasing expected returns) or to the left (decreasing risk) or both. Because both upward and leftward movements are desirable to the risk-averse investor, he would never remain at a point that was off the efficient set.

Efficient set

The northwest boundary of the set of all portfolios.

Efficient portfolio

A portfolio in the efficient set.

The Investor's Choice

Because the risk-averse investor views expected return as a "good" and standard deviation as a "bad," his preferences among portfolios are represented by indifference curves such as those shown in Exhibit 18.14. He chooses among the portfolios in the efficient set (also shown) so as to be on the highest possible indifference curve (in this case "highest" means "most northwesterly"). That is, he will pick the portfolio where the efficient set is tangent to an indifference curve, as at point *P* in Exhibit 18.14.

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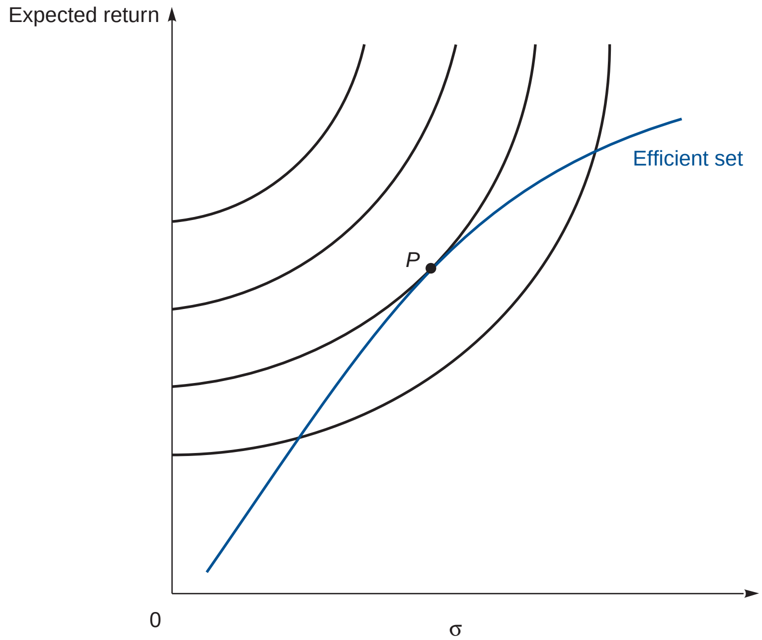
EXHIBIT 18.14 **The Risk-Averse Investor's Choice**



Because the risk-averse investor views expected return as good and standard deviation as bad, his indifference curves are shaped as shown. Of the portfolios in the efficient set, this investor selects the one on the "highest" (most northwesterly) indifference curve, which is at the tangency *P*.

Dangerous Curve

In practice, "choosing" portfolio P is not as easy as we have made it sound. Even though we know that some portfolio has the expected return and standard deviation associated with point *P* in Exhibit 18.14, actually constructing that portfolio—determining the particular combination of stocks from which it is built—can require considerable skill. For this reason, investors often find it in their interest to hire a professional *portfolio manager* to help them construct the portfolio they have chosen.

In asserting that the investor will choose point *P*, we have assumed that expected return and standard deviation are the only characteristics of his portfolio that concern him. Conceivably, he could be concerned with other, more subtle, statistical features as well. Suppose that portfolio *A* could return −6%, −2%, 0%, 2%, or 6%, all with equal probability. Portfolio *B* could return −4% or 4%, each with equal probability. Both portfolios have the same expected return (0%) and the same standard deviation (4%). (If you know the precise definition of standard deviation, you should check this.) Therefore, both portfolios occupy the same position in the graph of Exhibit 18.14. Consequently, the theory embodied in that graph must assume that the investor is indifferent between these two portfolios.

Capital asset pricing model

A model that assumes that investors care only about expected return and risk, where risk is measured by standard deviation.

The assumption that the investor cares only about expected return and standard deviation is the key assumption of the **capital asset pricing model,** which is often used to study markets for risky assets. A body of empirical evidence indicates that this assumption is not far wrong. We will continue to pursue its implications.

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Introduction of a Risk-Free Asset

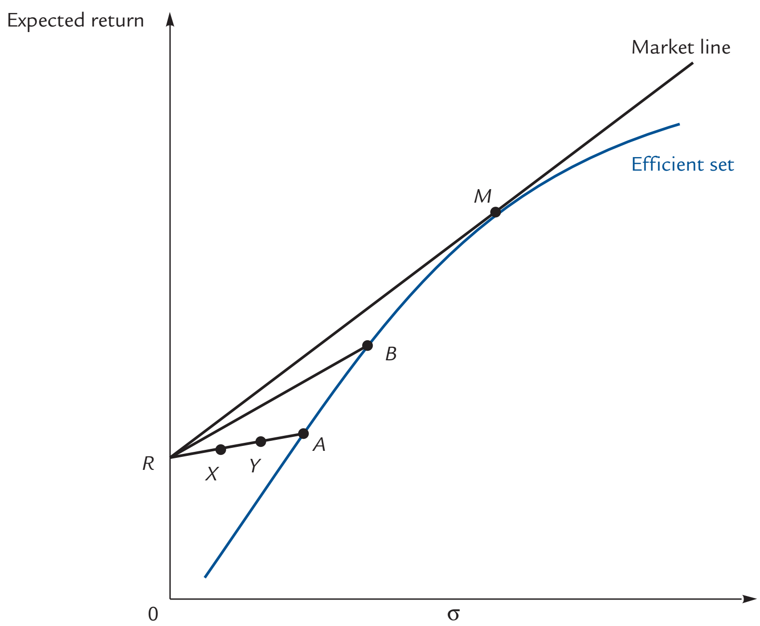
Suppose now that in addition to all of the stocks shown in Exhibit 18.13, a risk-free asset is available. It is often asserted that U.S. Treasury bills constitute such an asset (but see the end of Section 17.1 for some contrary evidence). Whatever this risk-free asset might be, it is represented by a point on the vertical axis, like *R* in Exhibit 18.15.

Let us see what happens when the risk-free asset is combined with a portfolio of stocks. Suppose that an investor holds half of his wealth in the form of stock portfolio *A* and half in the form of the risk-free asset *R*. Then his overall portfolio is represented by the point *X*, midway between *A* and *R*. (A risk-free asset cannot contribute to diversification, so the combined portfolio is represented by *X* rather than some point to the left of *X*.) Similarly, if the investor holds three-fourths of his wealth in portfolio *A* and one-fourth in the risk-free asset *R*, then his overall portfolio is represented by the point *Y*, three-fourths of the way along the line from *R* to *A*.

In general, the investor can achieve any point along the line segment from *R* to *A* by combining portfolio *A* with the risk-free asset. Similarly, he can achieve any point along the line segment from *R* to *B*, or from *R* to any other existing portfolio. The uppermost of these line segments, connecting *R* with *M*, contains the most desirable combinations.

Under some circumstances, the investor can move past *M* along the same line. This is possible when he can hold a *negative* amount of the risk-free asset *R*. For example, if

EXHIBIT 18.15 **A Risk-Free Asset**



Point *R* represents a risk-free asset, possibly a Treasury bill. The investor can achieve any point along the illustrated line segments by combining *R* with portfolios such as *A*, *B*, and *M*. For example, combining *R* and *A* in equal amounts yields point *X*. The line connecting *R* and *M* contains the most desirable possibilities; it is called the market line. If *R* can be held in negative amounts (say by borrowing), then it is possible to move beyond point *M* along the market line. No investor would ever want to be off the market line. Therefore, every investor wants to hold a portfolio consisting partly of *R* and partly of a market portfolio *M*.

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Market line

The line through a risk-free asset and tangent to the efficient set.

Market portfolio

The point of tangency between the market line and the efficient set.

R is a Treasury bill and if the investor is able to borrow at the Treasury bill rate, then such borrowing is equivalent to holding a *negative quantity of Treasury bills.* (*Borrowing* equals *selling bonds* equals buying bonds in negative quantities.) Assuming that this is possible, the investor can achieve any point along the line passing through *R* and *M*. This line is called the **market line.**

The market line is the line through *R* that is just tangent to the efficient set. The **market portfolio** is the portfolio represented by the point where the market line touches the efficient set. In Exhibit 18.15 point *M* represents the market portfolio.

There might be more than one market portfolio, since several portfolios can occupy the same position in the graph.

With the availability of the risk-free asset, an investor is no longer restricted to the old efficient set. He can reach any point on the market line by holding an appropriate combination of risk-free assets and shares of the market portfolio. These options are always preferable to points on the efficient set. For example, an investor holding portfolio *A* in Exhibit 18.15 could move either directly upward to the market line, increasing his expected return, or directly leftward to the market line, decreasing his risk.

Investors choose only points on the market line. Points on the market line are obtained by combining the risk-free asset *R* with the market portfolio *M*. Therefore,

A rational investor always holds a portfolio that combines the risk-free asset with the market portfolio in some proportions.

To see what proportions the investor will choose, we must examine his indifference curves. In Exhibit 18.16 the investor chooses proportions that enable him to reach point *Q*.

Constructing a Market Portfolio

What happens if we create a giant portfolio consisting of all of the risky assets in the economy? Because every asset is held by somebody, this is the same thing as adding up all of the individual investors' portfolios. Because each investor holds a market portfolio at point *M* in Exhibit 18.15, we are adding up many portfolios, all at point *M*. The result must be a portfolio at *M*, or a portfolio to the left of *M* if there is further diversification. But we see from Exhibit 18.15 that there are no portfolios to the left of *M*. It follows that our giant portfolio is itself at point *M*. In other words,

A portfolio that consists of all of the risky assets in the economy, held in proportion to their existing quantities, must be a market portfolio.

The individual investor wants to hold a combination of two assets: the risk-free asset and a market portfolio. But how is he to construct a market portfolio? Actually, we have just described one: the portfolio consisting of all of the assets in the economy. An individual investor can hold a miniaturized copy of this portfolio by holding all of the risky assets in proportion to their existing quantities. By choosing an appropriate mix of this particular market portfolio and the risk-free asset, he can reach point *Q* in Exhibit 18.16, which is his individual optimum.

Unfortunately, practical considerations prevent the investor from really holding all of the risky assets in proportion to their existing quantities. A shopping center in Dubuque, Iowa, might represent a .0001% share of the nation's economy. It is unrealistic to suggest that .0001% of an investor's portfolio should consist of shares in this shopping center. Typically, practical considerations make it necessary for an investor to approximate the market portfolio with a very small number of assets. To some extent

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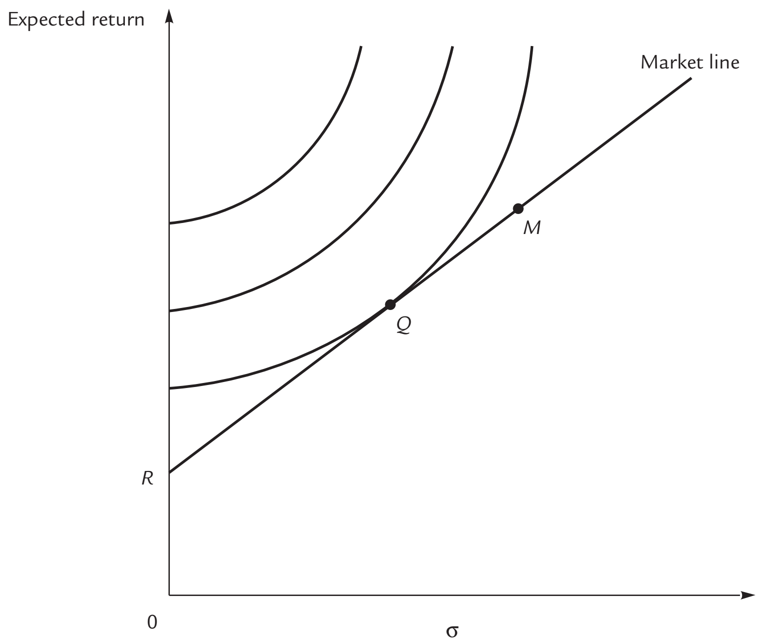
EXHIBIT **18.16** **The Investor's Choice Revisited**



When there is a risk-free asset, the investor is no longer restricted to the old efficient set. He can now reach any point on the market line by combining the risk-free asset with the market portfolio in appropriate proportions. This investor chooses proportions that enable him to reach point *Q*.

The investor can never do better than to be on the market line. Thus, his portfolio of risky assets will always be the market portfolio. There is never any reason to hold any other portfolio of risky assets.

he can alleviate this problem by holding shares in mutual funds that in turn hold shares in a large and highly diversified collection of assets. Also, the services of a portfolio manager can be helpful.

18.5 Rational Expectations

In this section, we will examine how prices are set in a market where suppliers have to make decisions in the face of uncertain demand. We will discover that equilibrium prices depend very much on the way in which suppliers form their expectations. We will also discover an important reason why economists studying such markets are liable to make predictions that are drastically wrong.

A Market with Uncertain Demand

Suppose that lettuce is sold in a central marketplace. Each day lettuce farmers must decide how much lettuce to load onto their trucks and bring to the market. If they knew what the price was going to be, this decision would be easy. They would simply bring lettuce until the marginal cost of supplying another head was equal to the price. Unfortunately,

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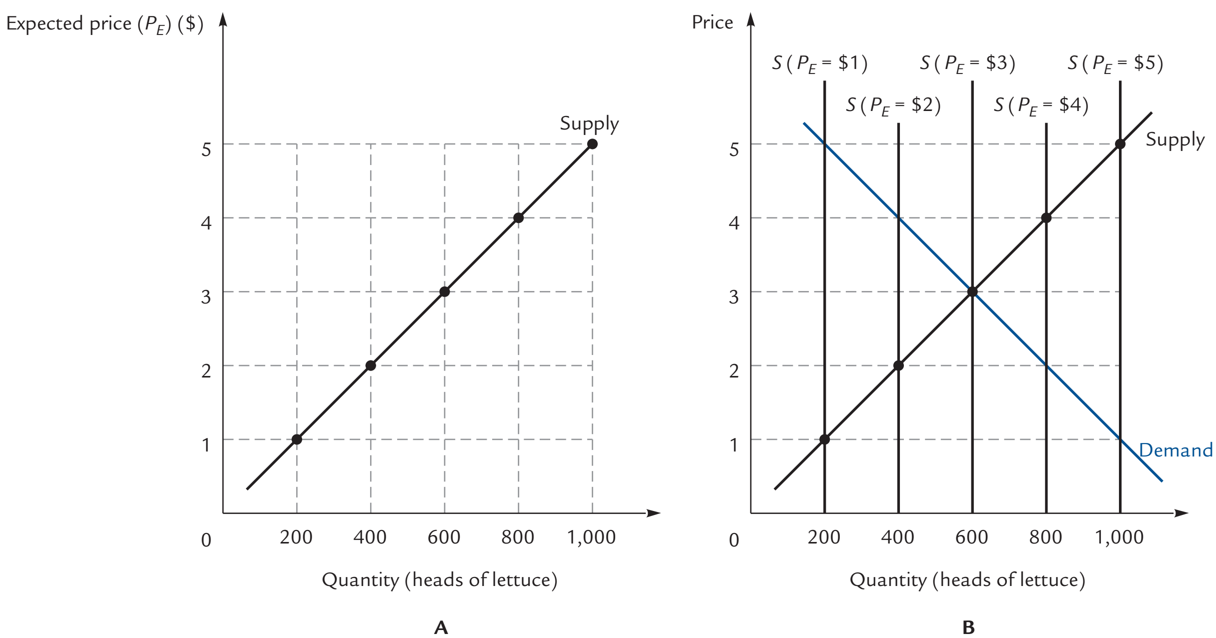
demand, and therefore price, fluctuates from day to day. The best that farmers can do is to form an *expectation* of the price. The amount they bring to market on a given day is given by an upward-sloping "supply curve," as shown in panel A of Exhibit 18.17. The difference between this curve and a true supply curve is that in this case the vertical axis measures not price, but expected price, which we denote by the symbol P_E.

When the farmers actually arrive at the marketplace, the supply curve for lettuce is vertical. The quantity of lettuce is equal to what the farmers have irrevocably decided to bring with them, and all of the lettuce must be sold or it will rot. The location of the vertical supply curve depends on the farmers' expectation of price at the time they start out in the morning. According to Exhibit 18.17, if the farmers expect a price of $1, the supply is 200 heads of lettuce; if they expect a price of $2, the supply is 400 heads, and so forth. Panel B of the exhibit shows the curve from panel A together with the various possible vertical supply curves, each labeled with the corresponding expected price.

Panel B of Exhibit 18.17 also shows the demand curve for lettuce on a particular day. The market price depends both on the location of this demand curve and on what expectation the farmers have when they start out. If the farmers expect a price of $2, they bring 400 heads of lettuce to market and the actual price is $4. If they expect a price of $4, they bring 800 heads and the actual price is $2. If they expect a price of $3, they bring 600 heads and the actual price is $3. Only in this last case does the farmers' expectation prove to be correct.

EXHIBIT 18.17 **Expectations and Supply**



The curve in panel A shows how much lettuce the farmers bring to market at each expected price. It is like a supply curve, except that it depends on expected price rather than actual price. When the farmers arrive at the market, the supply curve is vertical. The position of the vertical supply curve depends on the farmers' expectation of the price. Panel B shows the supply curve from panel A superimposed on several possible vertical supply curves. The actual price depends on the expected price (which determines the vertical supply curve) and the actual demand.

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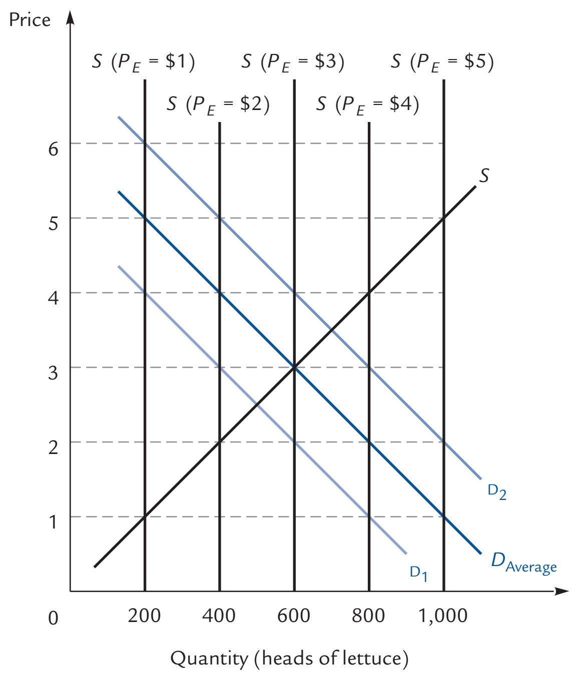
Exercise 18.11 What is the actual price of lettuce if the farmers expect a price of \$1? If they expect a price of \$5?

Each day demand is different. Suppose, for example, that the curves D_1 and D_2 in Exhibit 18.18 represent the lower and upper limits of demand. Some days demand is as low as D_1, some days it is as high as D_2, and on the average day it is given by the demand curve $D_{Average}$ between them. If farmers consistently expect a price of \$1, they will find that the actual price is sometimes as low as \$4, sometimes as high as \$6, and about \$5 on the average day. In other words, they will consistently find that their predictions are drastically wrong.

Exercise 18.12 Explain how farmers' expectations are confounded if they consistently expect a price of \$5.

Now, farmers are not omniscient; nobody expects them to make correct predictions all the time. But farmers are not foolish either, and when their predictions are consistently off in a systematic way, we expect them to revise those predictions. Farmers who expect a price of \$1 will consistently find that they have underestimated, and therefore they will not persist in their belief.

EXHIBIT **18.18** **Rational Expectations**



Demand fluctuates between D_1 and D_2; it is $D_{Average}$ on the average day. If farmers expect lettuce to sell at \$1, they bring 200 heads of lettuce to market and the price on the average day is \$5 (where $D_{Average}$, crosses the quantity 200). The farmers' expectation is systematically wrong.

If, on the other hand, the farmers expect lettuce to sell at \$3, they bring 600 heads of lettuce to market and the price on the average day is \$3. Thus, the expectation of a \$3 price is correct on average; we say that it is a rational expectation.

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A similar argument can be made about any expected price except for an expected price of \$3. If farmers expect a price of \$3, then the price will be as low as \$2 some days, as high as \$4 other days, and \$3 on average. Farmers will have no reason to revise their expectations either upward or downward. In this case we say that the farmers have **rational expectations.**

Rational expectations
Expectations that, when held by market participants, lead to behavior that fulfills those expectations on average.

An expectation is rational when it does not lead to systematic, correctable errors in prediction. Nevertheless, a rational expectation is not always, nor even usually, a correct expectation. In our example the price might be \$2 half of the time and \$4 half of the time, in which case the rational expectation of \$3 will *never* be correct.

Geometrically, the rational expectation occurs where the average day's demand curve crosses the farmers' upward-sloping supply curve.

Why Economists Make Wrong Predictions

Now let us embellish our model by making an assumption about why demand fluctuates. Suppose that the demand for lettuce is strictly determined by the income of the local lumberjacks. Exhibit 18.19 shows some possible demand curves. When the lumberjacks earn \$100, the demand curve is D_{100}; when they earn \$150, it is D_{150}, and so on. Let us also assume that on the average day lumberjacks earn \$150.

If the farmers have rational expectations, they always expect a price of \$3, which is correct on the average day. Thus, they always bring 600 heads of lettuce to market. The actual price on any given day is perfectly predictable on the basis of the lumberjacks' income. When the lumberjacks earn \$100, the price of lettuce is \$2; when the lumberjacks earn \$150, the price is \$3; and so on.

Exercise 18.13 What is the price of lettuce when the lumberjacks' income is \$200? When it is \$250? When it is \$300?

Exercise 18.14 Suppose that the lumberjacks' income averaged \$250. What would be the rational expectation of the price of lettuce? How many heads of lettuce would farmers bring to the market? What would be the actual price when the lumberjacks earned \$100? When they earned \$200? When they earned \$300?

Suppose now that an econometrician comes to study this market. He is pleased to discover that he can predict the price of lettuce on the basis of the lumberjacks' income, as detailed in the preceding paragraph. He might even be so bold as to summarize his knowledge in an equation:

$$\text{Price of lettuce} = \frac{1}{50} \times \text{Lumberjack's income}$$

For example, because the lumberjacks earn \$150 on average days, the price of lettuce is $1/50 \times \$150 = \$3$ on average days, which is correct.

One day a paper mill is built in the area. The owners of the mill announce that they will be purchasing a lot of lumber. As a result, the lumberjacks' income will now average \$250 per day. What does the econometrician predict about the price of lettuce?

Drawing on past experience, the econometrician knows that lumberjack income of \$250 implies a lettuce price of \$5. Thus, he predicts that the price of lettuce will now be \$5 on the average day.

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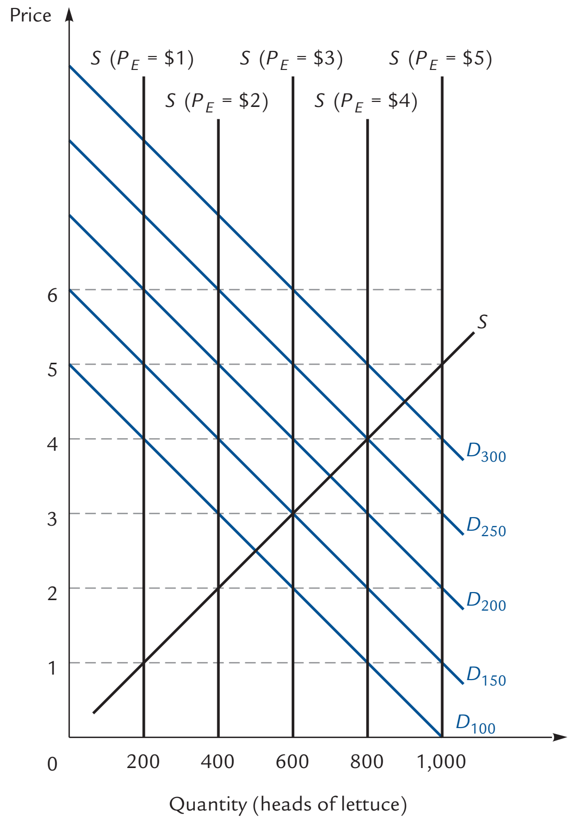
EXHIBIT **18.19** **Lumberjacks' Income and the Price of Lettuce**



The demand curve for lettuce depends on the lumberjacks' income. When their income is \$100, the demand curve is D_{100}; when their income is \$150, the demand curve is D_{150}; and so on. Initially, the lumberjacks earn \$150 on average. Thus, the rational expectation of the price is \$3 (where D_{150} crosses the upward-sloping supply curve), so farmers supply 600 heads of lettuce. When the lumberjacks really do earn \$150, the rational expectation is fulfilled. On days when the lumberjacks earn \$250, the price of lettuce is \$5.

Now a paper mill arrives, raising the lumberjacks' income to \$250 on average. The new rational expectation for the price is \$4. Farmers bring 800 heads of lettuce to market. On an average day the lumberjacks earn \$250 and the price of lettuce is \$4.

An econometrician extrapolating from past experience would predict that on days when the lumberjacks earn \$250, the price of lettuce is \$5. Thus, he would predict that when the paper mill arrives, the price of lettuce will go up to \$5 on average. But he is wrong, because past experience is no longer relevant. When farmers have rational expectations, the additional lettuce that they bring to market invalidates the old relationship between the lumberjacks' income and the price of lettuce.

But what actually happens? By examining Exhibit 18.19, you can see that the new rational-expectations price of lettuce is \$4 (where supply crosses the new average demand curve D_{250}). Farmers now bring 800 heads of lettuce to market each day. When the lumberjacks earn \$250, on the average day the price of lettuce is \$4, not \$5 as the econometrician predicted.

Where did the econometrician go wrong? All past experience supports his equation. It has always been true in the past that on days when the lumberjacks earn \$250,

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the price of lettuce is \$5. What happened is that the arrival of the paper mill caused farmers to change their expectations and bring a different amount of lettuce to market. This, in turn, invalidated the econometrician's equation. The correct new equation is:

$$\text{Price of lettuce} = \left(\frac{1}{50} \times \text{Lumberjack's income}\right) - \$1$$

Exercise 18.15 Suppose that a tree disease reduces the lumberjacks' average income to \$100. What is the new equation for the price of lettuce?

Example-Tweedledum and Tweedledee

Tweedledum and Tweedledee have identical skills and have therefore always had identical incomes. In years when their skills are in demand, their incomes are both high, and at other times their incomes are both low. An econometrician, having carefully collected data, can confidently assert the truth of the equation

$$\text{Tweedledee's income} = \text{Tweedledum's income}$$

If he can observe Tweedledum's income, the econometrician can use his equation to predict Tweedledee's income, and he will always be right.

One day Tweedledee hired just such an econometrician to advise him on how to increase his income. The econometrician, having discovered the preceding equation, advised Tweedledee, "It's simple. Your income is always the same as Tweedledum's income, so if you want your income to rise, just give all of your money to Tweedledum." Tweedledee tried it, but it didn't work.

This simple example illustrates that even when equations predict very well, they can be entirely useless as guides to policy. The reason is that changes in policy can invalidate the equations.[4] The equality between Tweedledee's and Tweedledum's incomes existed for a *reason;* because their incomes were derived from selling identical skills in the marketplace. The econometrician's suggestion leads to behavior that eliminates this reason for equality, and as a result the equality itself disappears.

Similarly, we can imagine the econometrician of the preceding subsection advising farmers to try to attract a paper mill to the area, promising that the price of lettuce will rise to \$5. When it rises to only \$4, the farmers are disappointed, just like Tweedledee. Again the reason is that the policy change eliminates the reason underlying the validity of the very equation that was used to justify the policy change.

The example of Tweedledee and Tweedledum illustrates that this problem with policy evaluation can occur even in exceptionally simple examples. The lumberjacks/lettuce example illustrates that the problem is particularly likely to arise in the presence of rational expectations, because changes in policy lead to changes in those expectations and hence to changes in behavior.

Rational expectations play a central and exciting role in modern macroeconomics, although they are fundamentally a microeconomic concept.[5] Two important areas of research are the attempt to understand the ways in which econometricians can be led

[4] This point was made forcefully in R. E. Lucas, Jr., "Econometric Policy Evaluation: A Critique," in *The Phillips Curve and Labor Markets,* vol. 1 of *Carnegie-Rochester Conference Series on Public Policy,* Karl Brunner and Allan H. Meltzer, eds. (Amsterdam: North Holland, 1976), pp. 19–46.

[5] Rational expectations were introduced by the economist John Muth to study problems in agricultural economics.

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astray in their predictions and the development of new econometric techniques that are appropriate for studying markets in which expectations are rational.

Summary

In many cases an individual's wealth depends on the state of the world. It is possible to transfer income from one state of the world to another in a variety of ways. The gambler who bets that a tossed coin will turn up heads transfers income from the state of the world in which tails comes up to the state of the world in which heads comes up. The homeowner who buys fire insurance transfers income from the state of the world in which his house is undamaged to the state of the world in which his house burns down. The investor who buys a share of stock in a company that makes digital tapes transfers income from the state of the world in which digital tape technology is unsuccessful in the marketplace to the state of the world in which digital tapes completely replace compact discs.

There are thus many ways that an individual can distribute his income across states of the world. We can draw indifference curves to illustrate his preferences among these distributions. The indifference curves depend both on the consumer's tastes and on the probabilities of the various states of the world.

A risk-neutral individual is one who always chooses the lottery with the highest expected value, without regard to risk. We expect a frequent gambler to be risk-neutral, since his good and bad luck wash out over time. For someone who is risk-neutral, the indifference curves are straight lines whose absolute slope is the ratio of the probabilities of the states of the world. When he is offered the opportunity to gamble at fair odds, the risk-neutral person is indifferent among all of the opportunities on his budget line. When offered the opportunity to gamble at favorable odds, he will always bet everything he has.

In many situations we expect people to be risk-averse. Among baskets with the same expected value, a risk-averter always chooses the one with no risk, that is, the one on the 45° line. Thus, at points along the 45° line the risk-averter's indifference curves have an absolute slope that reflects the fair betting odds. When offered the opportunity to bet at favorable odds, the risk-averter always accepts a small wager, but never a large one. Usually, an increase in income will increase the size of the largest wager that the risk-averter will accept at given odds.

Many markets exist to facilitate the transfer of risk across individuals. One is the market for insurance. In a world of perfect information, much insurance would be offered at fair odds (except for a slight tilting in favor of the insurance company to allow it to cover its costs). However, there are important reasons why we do not observe this practice. Among these are moral hazard and adverse selection, which were introduced in Chapter 9. Another problem is that some risks are undiversifiable, hence uninsurable.

The futures market is another market for transferring risk. It enables farmers to reduce their risks by contracting now for the prices of future deliveries. It also creates the opportunity for speculation, which is welfare-improving when speculators are right and detrimental to welfare when speculators are wrong.

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The stock market is yet another market for trading risky assets. In addition to individual stocks, investors can hold portfolios, created by combining various stocks in different proportions. The portfolio consisting of two stocks in equal proportions has the average expected return of the two but may have less than their average standard deviation (riskiness), because of diversification.

By combining the market portfolio (which consists of all of the risky assets in the economy held in proportion to their actual quantities) with a risk-free asset, the investor can create a portfolio that is superior to any other given portfolio in terms of risk and expected return. Thus, the only portfolio of risky assets that an investor would ever want to hold is the market portfolio. In practice, however, it is necessary to approximate this portfolio, which can require considerable expertise.

When there is uncertainty about the future, people may form rational expectations, which are expectations that are correct on the average day. If there is a change in circumstances, such as the arrival of a new industry or a change in some government policy, then the rational expectations may change, and consequently so may people's behavior. As a result, equations that have always predicted accurately in the past may prove drastically wrong following a policy change.

Author Commentary www.cengage.com/economics/landsburg

AC1. What is the best way to deter crime: with *harsher* punishments or with *more certain* punishments? The theory of risk aversion supplies the answer.

AC2. See this article for a challenge to the theory that attitudes toward risk vary with income.

AC3. This article also challenges risk theory.

AC4. For investors, the optimal portfolio is highly diversified. For charitable givers, exactly the opposite is true. For the reasons, see this article.

AC5. For another aspect of portfolio diversification, read this article.

Review Questions

R1. Describe the indifference curves of (a) a person who is risk-neutral, (b) a person who is risk-averse, and (c) a person who is risk-preferring.

R2. Under what circumstances might a person be expected to be risk-neutral? Why is a firm more likely to be risk-neutral than an individual?

R3. Explain why the stockholders and the executives of a corporation might have different preferences with regard to the corporation's behavior toward risk. Describe some possible remedies and their pros and cons.

R4. What is moral hazard? Give some examples.

R5. What is adverse selection? Give some examples.

R6. Describe a possible equilibrium in an insurance market with adverse selection. In what sense is it suboptimal?

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R7. What is an uninsurable risk? Give some examples.

R8. Explain what a futures contract is. How can a farmer or the owner of a grain elevator use futures contracts to eliminate risk?

R9. Explain what happens to the current and future supply of wheat when a speculator expects the price to fall. In what circumstances is this socially beneficial?

R10. What is the efficient set of portfolios? Explain why it is shaped as it is.

R11. Explain why the market portfolio is the only portfolio of risky assets that any investor would want to hold.

R12. What determines the daily equilibrium price in a market where demand fluctuates and suppliers have rational expectations?

R13. Explain how the arrival of a paper mill can cause a change in the relationship between lumberjacks' income and the price of lettuce.

Problem Set

1. According to Dr. Johnson, "He is no wise man who will quit a certainty for an uncertainty." Comment.

2. **True or False:** If nothing is worth dying for, then going to war is irrational.

3. Whenever John is offered the opportunity to take either side of a bet in which the odds are even slightly unfair, he invariably does bet something. **True or False:** John is certainly not risk-averse.

4. Jill likes to bet on heads when the odds are fair, but will bet on tails only if offered very favorable odds. Draw her indifference curves.

5. **True or False:** A risk-preferring person will always bet, no matter how much the odds are against him.

6. Bookmakers organize betting on football games in the following way: First, they determine a "point spread" that one team is expected to beat with 50-50 probability. Then bettors are allowed to bet on whether the team will beat the spread. They may take either side of the bet and are offered slightly unfavorable odds either way. Show the budget line faced by the bettors. What will a risk-averse bettor do in these circumstances? What will a risk-preferring bettor do? Can you think of any reason why a risk averter might still bet?

7. **True or False:** If "sickly" people could insure against illness at the same rates available to healthy people, they would end up preferring illness to good health.

8. **True or False:** Speculators are less harmful to society than they at first appear, because they sometimes err in forecasting the future and their losses due to these errors compensate the rest of us for their gains when they are right.

9. Suppose that it is known for certain that the demand for wheat this year is identical to the demand for wheat next year. This year's wheat crop of 100 tons has just been harvested. Everybody believes that next year's wheat crop, which has already been planted, will also be 100 tons. Now a speculator arrives on the scene, convinced that next year's crop will be only 80 tons.

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a. If wheat can be stored costlessly, what will the speculator do? What happens to this year's wheat supply and to next year's? (If it helps you, assume an interest rate of 0%.)

b. How long does the speculator continue this activity? What is this year's wheat supply when he is finished? What is next year's wheat supply when he is finished if he turns out to be right? What is it if he turns out to be wrong?

c. Use a graph to show the social gains with and without a speculator, on the assumption that the speculator is right. If he is right, does he improve social welfare?

d. Use a graph to show the social gains with and without a speculator, on the assumption that the speculator is wrong. If he is wrong, does he improve social welfare?

10. **True or False:** Nobody would ever hold a stock that was below the efficient set, since there is always an alternative with less risk or greater expected return.

11. Suppose that exactly half of all terrorists who take hostages kill their hostages. The government is considering a new policy under which all terrorist kidnappings will be met with massive military force intended to kill the kidnapper immediately. Unfortunately, it is estimated that in 90% of cases, the victim will die in the assault. **True or False:** Obviously, one drawback of this plan is that more hostages will die.

12. In New York State, the drinking age is 18. Studies show that 18-year-old drivers have a much higher crash rate than do 16- and 17-year-olds. The same studies indicate that if the drinking age were raised to 19, 30% of all crashes by 18-year-olds could be prevented, saving 25 to 35 lives per year. The *New York Times* has editorialized that the drinking age should be raised to 19, as 25 to 35 lives would be well worth saving. Assuming that all of the numbers in the studies are correct, comment on the *Times's* assertion that raising the drinking age would save 25 to 35 lives per year.

13. Suppose that in reality the number of cars demanded, Q, depends on the real interest rate, r, according to an equation of the form

$$Q = Ar + B$$

where A and B are constants. An econometrician believes that the number of cars demanded depends on the *nominal* interest rate, i, and uses data to estimate the coefficients C and D in the equation

$$Q = Ci + B$$

a. Express the estimated coefficients C and D in terms of the "true" coefficients A and B and the inflation rate, π.

b. Explain why this model will make good predictions as long as the inflation rate is constant.

c. Suppose that it is considered desirable to raise the demand for cars and that the government can affect i by adopting policies that lead to a change in π. What will the econometrician advise?

d. When the new policy is adopted, what happens to C and D? Explain why the recommended policy won't work.

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