PART II

Corporate Financing and Agency Costs

3
Outside Financing Capacity

3.1 Introduction

> A would-be borrower is said to be *rationed* if he cannot obtain the loan that he wants even though he is willing to pay the interest that the lenders are asking, perhaps even a higher interest. In practice such credit rationing seems to be commonplace: Some borrowers are constrained by fixed lines of credit which they must not exceed under any circumstances; others are refused loans altogether. As far as one can tell, these rationing phenomena are more than the temporary consequences of short-term disequilibrium adjustment problems. Indeed they seem to inhere in the very nature of the loan market.

This quotation from Bester and Hellwig (1987) is a good description of the puzzle of credit rationing. Why are lenders not willing to raise interest rates if the demand for loans exceeds their supply at the prevailing rates? One possible explanation is that interest rate ceiling regulations prevent such adjustment toward market equilibrium; however, such regulations have mostly been phased out and credit rationing is still a key feature of loan markets.

In the last thirty years, economists, following the impetus of Jaffee and Russell (1976), Keeton (1979), and Stiglitz and Weiss (1981), have come to the view that credit rationing is actually an equilibrium phenomenon driven by the asymmetry of information between borrowers and lenders. They have used both moral hazard and adverse selection arguments to explain why a lender would not want to raise interest rates even if the borrower were willing to pay higher rates, and why loans markets are personalized (there is usually no organized market for a standard commodity named "2-year loan at 10% interest rate") and clear through quantities (credit limits) as well as through prices (interest rates).

Both explanations start from the observation that a higher interest rate reduces the borrower's stake in the project: an interest rate increase has no effect on the borrower in the event of bankruptcy as long as the borrower is protected by limited liability. But it lowers the borrower's income in the absence of bankruptcy. The moral-hazard explanation is that this reduced stake may demotivate the borrower, induce her to pursue projects with high private benefits, or to neglect the project in favor of alternative activities, or even (in extreme cases) engage in outright fraud. That is, an increase in the interest rate may lower the probability of reimbursement indirectly through reduced performance.[1] The adverse selection explanation is that, in a situation where lenders cannot directly tell good and bad borrowers apart, higher interest rates tend to attract low-quality borrowers; for, low-quality borrowers are more likely to default on their loan and therefore are less affected by a rise in the interest rate than high-quality borrowers. Lenders may then want to keep interest rates low in order to face a better sample of borrowers.

This chapter analyzes credit rationing and the role of net worth. It emphasizes the moral-hazard

1. This moral-hazard explanation emphasizes the reduction in profit (technically speaking, in the sense of first-order stochastic dominance). Stiglitz and Weiss (1981) consider a different form of moral hazard. They observe that if the contract between the borrower and the lenders is a standard debt contract and if the lenders cannot observe the riskiness of the project chosen by the borrower, the borrower may have an incentive to choose an excessively risky project at the cost of sacrificing expected profit. Hart (1985) criticized this approach and observed that the conflict of interest between borrower and lenders relative to the choice of project riskiness could be solved by replacing the debt contract by profit sharing. To reintroduce divergent preferences between the two parties, one can either assume that the profit is costly to verify or completely unverifiable (see the descriptions of the costly state verification and of the nonverifiable income models in the supplementary section) or else introduce the form of moral hazard considered in this chapter. See Section 7.2.3 for models with both forms of moral hazard.

explanation, leaving the adverse selection explanation for Chapter 6. Section 3.2 develops the simplest model of credit rationing and uses it to illustrate the role of net worth. In this model, an "entrepreneur" or a "borrower" does not have enough money to finance a fixed-size project and must therefore resort to outside funding. The project may "succeed" and generate some income or "fail" and produce nothing. A key feature of this model is that lenders face an agency problem as the borrower may mismanage the project. She may take a private benefit and thereby reduce the probability that the project succeeds. The private benefit is inefficient in that its value to the borrower is smaller than the foregone profit; yet the borrower, who receives the entire private benefit and only part of the profit, may choose to enjoy the private benefit. The borrower must then keep a sufficient stake in the outcome of the project in order to have an incentive not to waste the money. Consequently, the project's income cannot be fully pledged to outside investors, which in turn implies that the project may not receive financing even if the expected income when the manager behaves exceeds the investment cost, that is, even if the project has positive net present value (NPV). That is, there may be credit rationing.

This first analysis points up some determinants of credit rationing. Borrowers with little cash on hand, with large private benefits from misbehaving, and whose performance conveys little information about managerial choices (in the technical sense of a low likelihood ratio) are more likely to see their positive-NPV projects turned down by the capital market.

It is also shown that investors optimally write covenants preventing the borrower from issuing in the future and without their approval claims on the firm's income, even if these new claims are junior to, and therefore do not directly dilute, theirs in bankruptcy. Because new claims alter managerial incentives, they may indirectly dilute the initial investors' stake anyway. Finally, Section 3.2 takes a first look at the sensitivity of investment to cash flow. While we argue that this question is best addressed in a dynamic setup (see Chapter 5), the basic model gives us some first insights as to whether investment can be predicted to increase with cash flow and whether this effect is likely to be more pronounced for firms with weak balance sheets.

Section 3.3 uses this basic model to illustrate the phenomenon of debt overhang, according to which the borrower may not be able to raise new funds for a profitable project if she has already committed future income linked with existing assets and if he cannot renegotiate some "debt forgiveness" or more generally some "claim forgiveness" or "claim dilution" with initial investors.

Banks, financial markets, and rating agencies generally feel that firms should not lever beyond some maximum level, called the debt capacity. Section 3.4 derives a rationale for debt capacity (or, more generally, borrowing capacity) and studies its determinants. In contrast with Section 3.2, which analyzes a fixed-size investment model, Section 3.4 views investment as a continuous variable, and shows that the firm's productive investments are optimally set equal to a given multiple of its equity. Equivalently, because investments are equal to equity plus leverage, this finding can be interpreted as the existence of a maximum leverage or gearing ratio.

The continuous investment extension serves another purpose besides the derivation of the outside financing capacity. It is also a convenient modeling device which will allow us to tackle in a simple way more complex issues related to choices of firm size such as diversification (Chapter 4), growth prospects (Chapter 5), asset repurchases (Chapter 14), the investment cycle (Chapter 14), as well macroeconomic models requiring an aggregation across borrowers (Chapters 13 and 14).

Because in the basic model, the project either succeeds or fails, and delivers nothing in the latter alternative, any claim is but a share to income in the case of success under limited liability. Put differently, it does not generate a diversity of claims such as debt and equity. "Debt capacity" is an abuse of terminology (it really is an "outside financing capacity") in that the outsiders' claim can but need not be interpreted as debt: if the profit in the case of success is 10, a claim of 4 can be interpreted either as a 40% equity stake, or as a risky debt claim with nominal value 4 which is defaulted upon in the case of failure. We later capture one feature of debt, namely, its priority over equity by introducing a leftover value of the assets in the case of failure. We show that it is optimal for investors to have priority in the case of default and for the entrepreneur to be the residual claimant (Chapter 5 will investigate another feature of debt, namely, the borrower's promise to pay fixed amounts to investors as a going concern, i.e., before liquidation; and Chapter 10 will connect debtholders' control rights with their cash-flow rights).

By focusing on a simple model of credit rationing, we do not do full justice to the corporate finance literature, which has developed a wide array of models with a similar flavor. For the sake of completeness, the supplementary section studies three broad classes of models that, through more sophisticated modeling, have aimed at deriving an interpretation of the "leftover claim" of outside investors as a standard debt claim.

3.2 The Role of Net Worth: A Simple Model of Credit Rationing

3.2.1 The Fixed-Investment Model

Variants of the following entrepreneurial model[2] will be used in the following: an entrepreneur (also called the "insider" or the "borrower," "she") has a project. This project requires a fixed investment I. The entrepreneur initially has "assets" ("cash on hand" or "net worth") $A < I$. For the moment we interpret these assets as being cash or liquid securities that can be used toward covering the cost of investment. (We will later explore the possibility that these assets be illiquid. For example, they might be equipment or premises that are needed for the implementation of the project.) The entrepreneur's cash can either be invested in the project or used for consumption. To implement the project the entrepreneur must borrow $I - A$ from lenders. (We will later observe that we can ignore the possibility that the entrepreneur consumes some of the cash and borrows more than $I - A$.)

Project. If undertaken, the project either succeeds, that is, yields verifiable income $R > 0$, or fails, that is, yields no income. The probability of success is denoted by p. The project is subject to moral hazard. The entrepreneur can "behave" ("work," "exert effort," "take no private benefit") or "misbehave" ("shirk," "take a private benefit");[3] or, equivalently, the entrepreneur chooses between a project with a high probability of success and another project which ceteris paribus she prefers (is easier to implement, is more fun, has greater spinoffs in the future for the entrepreneur, benefits a friend, delivers perks, is more "glamorous," etc.) but has a lower probability of success.[4]

Behaving yields probability $p = p_H$ of success and no private benefit to the entrepreneur, and misbehaving results in probability $p = p_L < p_H$ of success and private benefit $B > 0$ (measured in units of account) to the entrepreneur.[5] In the "effort interpretation," B can also be interpreted as a disutility of effort saved by the entrepreneur when shirking. Let $\Delta p \equiv p_H - p_L$.

Preferences and the loan agreement. Both the borrower and the potential lenders (or "investors") are risk neutral.[6] For notational simplicity, there is no time preference; the rate of return expected by investors (which is also the riskless rate, due to risk neutrality) is taken to be 0.[7] The borrower is protected by limited liability, and so her income cannot take negative values.

Lenders behave competitively in the sense that the loan, if any, makes zero profit. That is, we have in mind that several prospective lenders compete for issuing a loan to the borrower, and that, if the most attractive loan offer made a positive profit, the borrower could turn to an alternative lender and offer

2. This specific model is taken from Holmström and Tirole (1997). But its main idea can be found in various forms in many anterior papers.

3. See Exercise 3.20 for the continuous-effort version of the model.

4. Note that, for simplicity, we treat the entrepreneur as a unitary actor. There is an interesting question as to how moral hazard and incentives propagate down within the corporate hierarchy. Pagano and Volpin (2005) assume that benefits accrue to all company insiders, and not only to managers; in their model, managers need workers' cooperation to produce and therefore share benefits with employees.

5. For example, in the biotechnology alliance financing discussed in Section 2.4.2, the private benefit might be the entrepreneur's benefit from working on other projects (with other partners or on her own). The shift in attention then reduces the probability of success of the project under consideration.

6. Exercise 3.2 generalizes this analysis to allow for entrepreneurial risk aversion. Exercise 3.12 considers risk-averse investors.

7. The investors have rate of time preference equal to 0, which is also the market rate of interest.

to switch for a slightly lower interest rate.[8] We use the plural "lenders" even though a single lender may turn out to finance the entire loan, because we want to emphasize that lending is a passive and anonymous activity in the theories reviewed in Part II.

Let us turn to the loan contract. A contract first stipulates whether the project is financed.[9] If so, it further specifies how the profit is shared between the lenders and the borrower. The borrower's limited liability will imply that both sides receive 0 in the case of failure (the gross payoffs are the *ex post* monetary payoffs and take no account of past investments and private benefit). Intuitively, there is no point in specifying a positive transfer from the lenders to the borrower, as such a transfer can only weaken incentives, while it has no insurance benefit under risk neutrality. This property will be proved more rigorously and is here taken for granted. In the case of success, the two parties share the profit R; R_b goes to the borrower and R_l to the lenders.[10] To sum up, we posit an incentive scheme for the entrepreneur of the following form: R_b in the case of success, 0 in the case of failure.

The zero-profit constraint for the lenders can be written as

$$p_\mathrm{H} R_\mathrm{l} = I - A,$$

assuming that the loan agreement induces the borrower to behave (which under our assumptions will be the case). The rate of interest ι is given by

$$R_\mathrm{l} = (1+\iota)(I-A) \quad \text{or} \quad 1+\iota = 1/p_\mathrm{H}.$$

So, unless $p_\mathrm{H} = 1$, the nominal rate of interest ι reflects a default premium and exceeds the expected rate of return (called r in Part VI and here normalized to 0) demanded by investors.

We summarize the timing in Figure 3.1.

We assume that the project is viable only in the absence of moral hazard. That is, the project has positive NPV if the entrepreneur behaves,

$$p_\mathrm{H} R - I > 0, \tag{3.1}$$

but negative NPV, even if one includes the borrower's private benefit, if she does not,

$$p_\mathrm{L} R - I + B < 0. \tag{3.2}$$

It is easy to see that inequality (3.2) implies that no loan that gives an incentive to the borrower to misbehave will be granted. Indeed, rewrite (3.2) as

$$[p_\mathrm{L} R_\mathrm{l} - (I-A)] + [p_\mathrm{L} R_\mathrm{b} + B - A] < 0.$$

So, in the case of misbehavior, either the lenders must lose money in expectation, or the borrower would be better off using her cash for consumption, or both.

Figure 3.1

3.2.2 The Lenders' Credit Analysis

Because the project has negative NPV in the case of misbehavior, the loan agreement must be careful to preserve enough of a stake for the borrower in the enterprise. The borrower faces the following tradeoff once the financing has been secured: by misbehaving, she obtains private benefit B, but she reduces the probability of success from p_H to p_L. Because she has stake R_b in the firm's income (she receives R_b in the case of success and 0 in the case of failure), the borrower will therefore behave if the following "incentive compatibility constraint" is satisfied:

$$p_\mathrm{H} R_\mathrm{b} \geqslant p_\mathrm{L} R_\mathrm{b} + B \quad \text{or} \quad (\Delta p) R_\mathrm{b} \geqslant B. \tag{IC$_\mathrm{b}$}$$

From this incentive compatibility constraint we infer that the highest income in the case of success that can be pledged to the lenders without jeopardizing the borrower's incentives is

$$R - \frac{B}{\Delta p}.$$

The *(expected) pledgeable income* is then

$$\mathcal{P} = p_\mathrm{H}\left(R - \frac{B}{\Delta p}\right).$$

8. See Exercise 3.13 for the extension of the model to lender market power.

9. "Random financing" contracts, in which the borrower brings equity in exchange for a probability between 0 and 1 of being financed may in some cases be optimal when the investment size is fixed (as it is here) or more generally in the presence of indivisibilities or increasing returns to scale (see Exercise 3.1). For simplicity, we focus on deterministic contracts.

10. The lenders' net payoff is thus $R_\mathrm{l} - (I-A)$ in the case of success, and $-(I-A)$ in the case of failure. The borrower's net payoff is thus $R_\mathrm{b} - A$ in the case of success, and $-A$ in the case of failure, to which, in both cases, must be added a private benefit B if shirking occurs.

Because the lenders must break even in order to be willing to finance the project, a necessary condition for the borrower to receive a loan is that the expected pledgeable income exceed the lenders' initial outlay:

$$\mathcal{P} \equiv p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) \geqslant I - A, \qquad (\mathrm{IR_l})$$

where "$\mathrm{IR_l}$" stands for the lenders' individual rationality constraint (which we will also often call the "breakeven constraint" or the "participation constraint"). Thus a necessary condition for financing to be arranged is

$$A \geqslant \bar{A} = p_{\mathrm{H}}\frac{B}{\Delta p} - (p_{\mathrm{H}}R - I). \qquad (3.3)$$

To make things interesting, we will assume that

$$\bar{A} > 0 \quad \Longleftrightarrow \quad p_{\mathrm{H}}R - I < p_{\mathrm{H}}\frac{B}{\Delta p}, \qquad (3.4)$$

otherwise even a borrower with no wealth of her own would be able to obtain credit. Condition (3.4) says that the NPV is smaller than the minimum expected rent that must be left to the borrower to provide her with an incentive to behave.

Thus, *the borrower must have enough assets in order to be granted a loan.* Note that, if $A < \bar{A}$, the project has positive NPV and yet is not funded. With insufficient assets, the entrepreneur must borrow a large amount and therefore pledge a large fraction of the return in the case of success. The entrepreneur then keeps only a small fraction of the monetary gain and is demotivated. The two parties cannot find a loan agreement that both induces effort (which requires a high compensation for the borrower in the case of success) and allows the lenders to recoup their investment. There is *credit rationing.* A rationed borrower may be willing to give a high fraction of the return to the lenders,[11] which here is equivalent to be willing to pay a high interest rate. But the lenders do not want to grant such a loan.

Conversely, if $A \geqslant \bar{A}$, the entrepreneur is able to secure financing, and so condition (3.3) is both a necessary and a sufficient condition for financing. The entrepreneur offers claim R_{l} to competitive investors so as not to leave them with a surplus:

$$p_{\mathrm{H}}R_{\mathrm{l}} = I - A.$$

Her stake,

$$R_{\mathrm{b}} = R - R_{\mathrm{l}} = R - \frac{I - A}{p_{\mathrm{H}}} \geqslant R - \frac{I - \bar{A}}{p_{\mathrm{H}}} = \frac{B}{\Delta p}, \qquad (3.5)$$

then induces her to behave.

As the conventional wisdom goes, "one only lends to the rich." The threshold $\bar{A}$ has a natural interpretation. As noted earlier, the term $p_{\mathrm{H}}B/\Delta p$ is nothing but the minimum expected monetary payoff to be left to the borrower to preserve incentives; it will be called the *agency rent.* The borrower must make an initial contribution at least equal to $\bar{A}$ so as to reduce the agency rent net of the initial downpayment A to at most the monetary profit $p_{\mathrm{H}}R - I$ of the project.

Using the breakeven condition for the lenders ($p_{\mathrm{H}}R_{\mathrm{l}} = I - A$), the borrower obtains net utility or payoff (where "net" means that we subtract the consumption utility, A, that the entrepreneur would get by not undertaking the project):

$$U_{\mathrm{b}} = \begin{cases} 0 & \text{if } A < \bar{A}, \\ p_{\mathrm{H}}R_{\mathrm{b}} - A = p_{\mathrm{H}}(R - R_{\mathrm{l}}) - A & \\ \qquad\qquad = p_{\mathrm{H}}R - I & \text{if } A \geqslant \bar{A}. \end{cases} \qquad (3.6)$$

As could have been expected from the zero-profit condition for the lenders, *the borrower receives the entire social surplus or net present value if the project is funded.*[12]

So, the borrower's utility jumps up at $A = \bar{A}$. While the discontinuity is an artefact of the rigidity of the level of investment, the fact that 1 unit of assets may be worth more than 1 to the borrower in a situation of asymmetric information is quite general. Indeed, in the continuous investment version of this model to be developed in Section 3.4, we will see that for the borrower *assets or net worth have a shadow value exceeding* 1.

Determinants of credit rationing. To sum up, two factors may make a firm credit-constrained in this model:[13]

(i) low amount of cash on hand (low A);

11. This will be the case if A is small.

12. This property holds only in equilibrium. Were the entrepreneur to deviate and misbehave, the entrepreneur's (off-the-equilibrium-path) utility would exceed the smaller (off-the-equilibrium-path) NPV (at least for A close to $\bar{A}$), since the lenders would lose money.

13. The market interest rate, here normalized at 0, is another determinant of the strength of the balance sheet. More generally, the pledgeable income must exceed the investors' outlay times $(1 + r)$,

(ii) high agency cost, where the agency cost can be measured, fixing the project's NPV, $p_H R$, by the combination of the private benefit B and the likelihood ratio $\Delta p/p_H$.

The entrepreneur's ability to borrow is limited by the nonpledgeability of some $(p_H B/\Delta p)$ of the value to investors. Here moral hazard is determined by two factors: the private benefit B that the entrepreneur can enjoy by misbehaving, and the extent to which the verifiable performance reveals such misbehavior. The informativeness of the performance variable regarding effort is defined by the likelihood ratio $(\Delta p/p_H) = (p_H - p_L)/p_H$.[14] This ratio measures the proportional reduction in the probability of success when the entrepreneur misbehaves and is therefore also a measure of the marginal productivity of effort by the borrower. The higher the likelihood ratio, the more informative about effort choice the outcome is ("the better the performance measurement"), and the easier the access to outside financing (in the sense that the minimum net worth $\bar{A}$ decreases). In the model of this section, the pledgeable income never exceeds $p_H R - B$, since the entrepreneur can always take her private benefit B, but may be much smaller when performance measurement is poor, i.e., the likelihood ratio is low.

In practice, the agency cost is influenced not only by the project's and the entrepreneur's characteristics, but also by the surrounding legal, regulatory, and corporate environment. Countries with strong investor protection limit the managers' ability to squander investor money and thereby exhibit lower agency costs; relatedly, the firms' ability to cross-list in jurisdictions with good shareholder protection is expected to reduce their agency cost and therefore to facilitate financing.[15]

Remark (full investment of entrepreneurial assets). We have assumed that the borrower invests her entire wealth. However, it is easy to see that this is an optimal choice for the borrower. Would the borrower want to consume $c \leqslant A$ and invest only $A - c$? If the project is still funded, the borrower still obtains the entire social surplus $p_H R - I$. On the other hand, it becomes more difficult to obtain a loan. Now, the entrepreneur's initial assets must exceed $\bar{A} + c$ in order for the project to be funded. Therefore the entrepreneur cannot gain by not investing her entire wealth in the project.[16]

Remark (high-powered incentive scheme). Earlier we claimed that risk neutrality implies that the absence of reward for the entrepreneur in the case of failure involves no loss of generality. Suppose, more generally, that the entrepreneur receives R_b^S in the case of success and R_b^F in the case of failure, where

$$p_H R_b^S + (1 - p_H) R_b^F \geqslant p_L R_b^S + (1 - p_L) R_b^F + B$$
$$\iff \quad (\Delta p)(R_b^S - R_b^F) \geqslant B$$

in order to discourage the entrepreneur from misbehaving. The investors' income is then

$$p_H(R - R_b^S) + (1 - p_H)(-R_b^F) \leqslant p_H\left(R - \frac{B}{\Delta p}\right) - R_b^F \leqslant \mathcal{P}.$$

Rewarding the entrepreneur in the case of failure implies a uniform upward shift in her minimum incentive-compatible pay structure and an overall reduction in what can be pledged to investors (note the analogy with the previously considered case of an initial consumption c). By contrast, the entrepreneur's utility, provided that she can secure funding, is not affected: because the investors break even, the entire surplus goes to the entrepreneur, who receives

$$U_b = p_H R - I,$$

regardless of the choice of R_b^F. We thus conclude that rewarding the entrepreneur in the case of failure cannot raise her utility, but can compromise financing.

where r is the rate of interest:

$$p_H\left(R - \frac{B}{\Delta p}\right) \geqslant (1 + r)(I - A).$$

Thus, keeping the investment cost I fixed, an increase in the rate of interest r is equivalent to a decrease in the cash on hand.

14. The likelihood ratio is often defined as p_H/p_L. The two notions are obviously equivalent.

15. See, for example, the empirical confirmations by Doidge et al. (2004), Miller (1999), Pagano et al. (2001), and Reese and Weisbach (2002).

16. This reasoning relies, of course, on the borrower's putting equal weight on current and future consumption. If the borrower had immediate consumption needs, she would put some of A aside for consumption. We invite the reader to extend the analysis to the more general specification in which the borrower consumes c_0 at the start and c_1 after the outcome is realized, and has utility the expectation of $u_0(c_0) + u_1(c_1)$, where the functions $u_0(\cdot)$ and $u_1(\cdot)$ are increasing and concave. (The basic insights are unaltered. See also Exercise 3.2.)

Remark (value and investor value). Because the essence of corporate finance is that investors cannot appropriate the full benefit attached to the investments they enable, we must distinguish two slices in the overall cake: that for the insiders and the rest for the outsiders (the decomposition must be finer if there are multiple categories of each). In this book, "value" or "total value" refers to the total cake, while "investor value" refers to the investors' slice; in the barebones model of this section, these two values are pR and pR_l for probability of success p once the investment has been sunk (of course, one needs to subtract I and $I - A$, respectively, if one wants to obtain the corresponding net or *ex ante* magnitudes). The empirical literature often uses the phrase "value" for what we call here "investor value," but this should not create confusion.

Remark (risk taking). Moral hazard here refers to the possibility that the borrower takes an action that reduces investor value (and total value as well). There is no risk taking. We will come back to risk taking in subsequent chapters, but the reader may want to consult Exercises 3.15, 3.16, and 4.15 for three simple ways of introducing risk taking in the context of this simple model.

3.2.3 Do Investors Hold Debt or Equity?

We interpreted the loan agreement as a profit-sharing contract. It turns out that with two levels of profit, 0 and R, the lenders' claim can be thought of as being either debt or equity: put differently, there is here no difference between risky debt and equity. The debt interpretation goes as follows: the borrower must reimburse R_l or else go bankrupt. In the case of reimbursement the borrower keeps the residual $R - R_l$. Alternatively, the two parties can define shares in an all-equity venture. The entrepreneur and the investors hold fractions R_b/R and R_l/R, respectively, of equity. These are called "inside equity" and "outside equity."

This feature of the two-outcome model is both a weakness and a strength. A serious weakness is that it cannot, as it stands, account for the richness of existing securities; but we will show how to extend it in order to generate a more realistic diversity of claims. A strength of this modeling is that it will enable us to analyze a number of key ideas without being held back by the need to specify whether one is analyzing debt, equity, or an alternative claim. Some readers may find it surprising that a lack of predictive power relative to the structure of outside claims may constitute a strength. To clarify this point, it is worth pointing out that many phenomena in corporate finance have wider scope than that defined by the context in which they were discovered. Let us provide some illustrations in support of this view:

(a) As we will study in Chapter 5, Easterbrook (1984) and Jensen (1986) have argued that it is optimal to require cash-rich firms to pay out income on a regular basis, thereby forcing them to return to the capital market. The payment takes the form of a dividend in Easterbrook and of a short-term debt obligation in Jensen. The starting point for both analyses, namely, the desire to pump free cash flow out of the firm, is the same.

(b) The foundations for the soft-budget-constraint problem, also studied in Chapter 5, do not rely on outside claims being debt or equity. While it is usually analyzed in the context of specific assumptions on the financial structure, its logic is quite general.

(c) The literatures on monitoring of a firm by a large shareholder and by a bank holding debt claims have much in common. They are both concerned with the monitor's incentive to supervise and with the impact of monitoring on the firm's behavior.

(d) The idea of using dispersed claimholders to extract rents from third parties (see Chapters 7 and 11) has been developed in separate literatures on debt and on equity.

Thus, abstracting in a first step from the complex issues associated with the diversity of outside claims may generate a better focus on, and a more rigorous analysis of the fundamentals of such phenomena. A richer analysis can then be obtained from the introduction of further modeling features that motivate a diversity of outside claims.

3.2.4 Dilution and Overborrowing

Recall from Section 2.3.3 (see also Fama and Miller 1972) that debt contracts include negative covenants

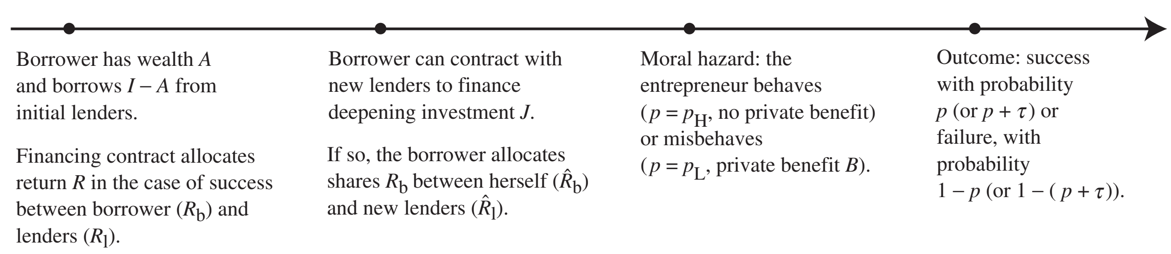

Figure 3.2

prohibiting the dilution of creditors' claims through the issue of new securities, especially ones with equal or higher seniority. There are two basic reasons for such covenants. First, creditors obviously do not want the borrower to issue claims that have a higher or the same seniority as theirs, as this reduces the amount they can collect if the firm defaults. Second, and more subtly, the issue of new securities may alter managerial incentives and the size of the pie.

Let us illustrate the second reason in our simple context. Consider the borrowing contract above in which the lenders take claim R_l in the case of success and the borrower an incentive-compatibility claim $R_b \geqslant B/\Delta p$. Now suppose that there is an opportunity for a "deepening investment." This investment costs an extra J and increases the probability of success uniformly by τ. That is, the probability of success becomes $p_H + \tau$ if the entrepreneur behaves and $p_L + \tau$ if the entrepreneur misbehaves.[17] Assume that this deepening investment is inefficient in that its net cost C_1 is positive, or put differently the expected increase in profit is smaller than J:

$$C_1 \equiv J - \tau R > 0.$$

The timing goes as in Figure 3.2.

We assume away any negative covenant prohibiting further borrowing and so the borrower can contract with new lenders.[18] However, in the case of new financing, initial lenders are not formally diluted in that they keep their stake R_l in success when the borrower contracts with new lenders. So the first motivation for inserting a covenant that prohibits the issuing of new securities is absent.

Note first that it is not in the interest of the borrower to contract with new investors if this results in the same effort, i.e., in no taking of private benefit. Intuitively, the new investment reduces total value by C_1, and so someone must lose in the process. Because the value of the initial investors' claim is increased (to $(p_H + \tau)R_l$) if the borrower still behaves, either the entrepreneur or the new investors must lose, which is impossible because the losing party would refuse to write the second financing contract. So assume that the new financing contract disincentivizes the borrower. This reduced incentive results in a second cost:

$$C_2 \equiv (\Delta p)R - B > 0.$$

As described in the timing, let $\hat{R}_b$ and $\hat{R}_l$ denote the new stake of the borrower and the stake of the new lenders, with

$$\hat{R}_b + \hat{R}_l = R_b.$$

Assuming that the new lenders are competitive, then

$$(p_L + \tau)\hat{R}_l = J.$$

The entrepreneur gains from overborrowing if and only if

$$(p_L + \tau)\hat{R}_b + B > p_H R_b,$$

or, using the breakeven condition for the new investors,

$$[(p_L + \tau)R_b - J] + B > p_H R_b.$$

After some manipulations, this condition becomes

$$[p_H - (p_L + \tau)]R_l > C_1 + C_2.$$

This necessary and sufficient condition for the deepening investment to be financed has a simple

17. This additivity property is convenient because it separates the incentive compatibility constraint from the impact of the new investment.

18. More generally, the division of the pie ($R_l + R_b = R$) is not made contingent on the event of a deepening investment.

interpretation. The right-hand side is the total cost of refinancing: direct cost plus incentive cost. The left-hand side of the inequality is the *externality* on the initial investors. Thus the total cost must be smaller than the loss of value for the initial investors.

When the borrower's balance sheet (as measured by A, say) improves, R_{b} increases, R_{l} decreases, and so this inequality is less likely to be satisfied. Put differently, in the absence of negative covenant, *overborrowing is more likely to happen with weak borrowers.*

Let us conclude this analysis of overborrowing with a few remarks. First, overborrowing in this situation can alternatively be avoided by forcing the entrepreneur not to dilute her own claim; this requirement is usually included in compensation contracts, although there have been attempts to evade it through derivative contracts (see Section 1.2.2). Second, the financing contracts need not be signed sequentially: simultaneous contracts also give rise to an overborrowing problem (see Bizer and DeMarzo 1992; Segal 1999). Third, the overborrowing problem arises with a vengeance in the context of sovereign borrowing, in which it is hard to specify a limit on indebtedness of the sovereign, if only because there are many different ways for a government to add new liabilities (see Bolton and Jeanne (2004) for an analysis of sovereign borrowing with the possibility of dilution). Finally, *in a multi-period financing context, uncoordinated lending further leads to excessively short maturity structures of debt*, as investors scramble to obtain priority over other investors (see Exercise 5.9).

3.2.5 Boosting the Ability to Borrow: Reputational Capital and Capability

Recall from Chapter 2 that lenders do not only look at tangible assets such as cash, land, and equipment. Ceteris paribus, they are more likely to issue a loan if the borrower has a good reputation, as was stressed in particular by Diamond (1991). The role of this intangible capital is easily analyzed in the credit rationing model.

Suppose, for example, that the borrower has less attractive opportunities for misbehavior, in that the private benefit B from misbehaving is reduced to $b < B$.[19] This may have several interpretations. Along the lines of the "effort interpretation" of moral hazard, one might imagine that the project falls well within the core competency of the entrepreneur and therefore demands less attention or supervision of the subordinates: the task is just easier for the entrepreneur. Alternatively, one could imagine that the entrepreneur has less attractive outside options (focusing on other, separate projects of her own) or opportunities for fraud and embezzlement (e.g., it is harder to buy inputs at an inflated price from a friend or family).

With reduced scope for moral hazard, the asset threshold is accordingly lower: from equation (3.3),

$$\bar{A}(b) < \bar{A}(B),$$

where

$$\bar{A}(\beta) \equiv p_{\mathrm{H}}\frac{\beta}{\Delta p} - (p_{\mathrm{H}}R - I),$$

and thus

$$\bar{A}(B) - \bar{A}(b) = \frac{p_{\mathrm{H}}}{\Delta p}(B - b) > 0.$$

In this sense, a "more reliable borrower" (that is, a borrower who has a lower private benefit from misbehaving) is more likely to obtain a loan.

How does this fit with the idea that a good reputation helps raise external finance? Suppose now that the private benefit (B or b) is not directly observed by the lenders, who only have the borrower's track record at their disposal. That is, the lenders know whether the borrower's past projects have been successful or whether past loans have been reimbursed. They use this information to update their beliefs about the reliability of the borrower. A better track record is an (imperfect) indicator of good reliability, that is, in our example, of a low private benefit from misbehaving.

Consider an entrepreneur who got a loan for a first project, and may in the future have new projects that will also call for outside financing. Let us further assume that these future projects are not yet well-defined, and focus on short-term finance. (Chapter 5 will analyze long-term loans.) In this situation, the entrepreneur should adopt a long-term perspective.

19. We could alternatively analyze the impact of a higher probability of success or of changes in other variables, with similar insights. The focus on the private benefit allows a cleaner analysis because changes in the private benefit keep the NPV of the project constant.

That is, she should not content herself with comparing the private benefit and the monetary payoff attached to the first project; she should also take into account the fact that a current success will bring two further benefits:

A *retained-earnings benefit*: even under symmetric information between the parties about the entrepreneur's reliability, a current success helps the entrepreneur build up net worth. This net worth has a shadow value; a unit of income is valued above 1 by the entrepreneur if there is a probability of credit rationing in the future. This benefit is studied in Exercise 3.11.

A *reputational benefit*: if, furthermore, the lenders have incomplete information about the entrepreneur's reliability, their updating of beliefs about this reliability confers an extra benefit on the entrepreneur in the case of success. Reputation complements net worth in reducing the probability of future credit rationing.[20]

An implication of the existence of this reputational benefit is that an unreliable borrower who would have no incentive to behave were her unreliability known to the lenders may have an incentive to behave today in order to get a loan tomorrow. The analysis of the situation becomes more complex once we realize that lenders are unlikely to be fools and understand that unreliable borrowers may have an incentive to masquerade as reliable ones. A proper study of reputational capital requires some (at least intuitive) understanding of dynamic games with incomplete information (see Exercise 6.3). We hope that the idea that reputational capital can substitute for net worth to thwart credit rationing is clear enough. There is indeed empirical evidence that reputation helps borrowers to obtain credit as well as better terms (see, for instance, Banerjee and Duflo's (2000) study of the Indian software industry).

Remark (information sharing). The impact of reputational capital is stronger, the more widely the information about borrower performance is disseminated. Padilla and Pagano (2000) observe that information sharing among lenders reinforces the borrowers' incentives to perform and argue that this may account for the fact that lenders (banks, finance companies, and retailers) spontaneously provide information about past defaults, delays in payment, current debt exposure, and riskiness of their borrowers to credit bureaus and credit-rating agencies, and therefore to their competitors. They develop a model in which lenders may share information even when this may encourage consumer poaching and thus enhanced *ex post* competition.

3.2.6 Making Efficient Use of Information to Reduce the Agency Cost

A basic theoretical result in the economics of agency, due to Holmström (1979), states that making economic agents accountable for events over which they have no control does not help with moral-hazard problems and generally worsens incentives. Roughly speaking, one should try to use the most informative or precise measurement of the agent's economic activity, or what is called in statistics a "summary" or "sufficient statistic."[21] This result underlies much of the thinking about managerial compensation, for example, the quest for good metrics to reward employees (based on customer satisfaction, reduction of unit costs, sales, etc.) or division managers (like EVA (economic value added) or balanced scorecard methods). More to the point for our context, it offers theoretical foundations for the use of benchmarking. Benchmarking, also called relative performance evaluation, consists in comparing the performance of, say, a firm with that of similar firms, to better assess managerial accomplishments. For example, a car producer's good financial performance is less indicative of good management if other car producers also do well than if the automobile industry is in a

20. Things are actually a bit more complicated than this dual benefit suggests: the reputational benefit depends on the borrower's equilibrium behavior (which itself depends on the retained-earnings benefit) and not only on the reputational one. Technically, if the retained-earnings benefit is strong enough to induce a high-private-benefit entrepreneur to behave, then success brings no reputational benefit.

21. A good introduction to sufficient statistics is Chapter 9 of DeGroot (1970). Suppose that one observes two variables x and y, and that one is trying to infer a third, unobservable variable z. The joint distribution of x and y, given z, is $f(x, y \mid z)$. The variable x is a sufficient statistic for (x, y) if the posterior distribution of z conditional on the observation of x and y depends only on x. To recognize sufficient statistics, a necessary and sufficient condition is the factorization criterion, that is, the existence of functions g and h such that $f(x, y \mid z) = g(x, y)h(x, z)$. A simple computation then shows that the distribution of z conditional on x and y does not depend on y.

recession. Or, a high price fetched by the stock of a software or biotechnology start-up company in an initial public offering (IPO) is not foolproof evidence of good entrepreneurship and careful venture capital monitoring if this price is reached during a stock price bubble.

We will come back a few times in this book to the issue of the quality of performance measurement and how it affects the ability to receive financing.[22] Let us just observe that in our context, the ability to raise financing is enhanced by conditioning entrepreneurial compensation on the performance measure with the highest available likelihood ratio.[23]

Let us provide a first illustration of this principle.

Benchmarking. A possible reinterpretation of our model is that there are three states of nature.

(i) Favorable state (probability p_L). The environment is sufficiently favorable that the project will succeed regardless of the entrepreneur's effort.

(ii) Unfavorable state (probability $1 - p_H$). The environment is harsh and the project will fail even if the entrepreneur does her best.

(iii) Intermediate state (probability $\Delta p = p_H - p_L$). Success is not guaranteed, but is reached provided the entrepreneur exerts effort.

Of course, no one *ex ante* knows which state prevails. The financing and effort decisions are chosen in the ignorance of the state of nature.[24]

Suppose now that one will learn *ex post* whether the state was favorable or not (i.e., intermediate or unfavorable), say, by looking at a less promising firm in the same industry that succeeds only if circumstances are favorable. Consider the following compensation scheme:

- the entrepreneur receives 0 if the state is favorable;
- the entrepreneur otherwise receives R_b in the case of success and 0 in the case of failure.

The incentive constraint is still

$$(\Delta p)R_b \geqslant B \qquad (\mathrm{IC_b})$$

since the entrepreneur's stake is still R_b in the state of nature in which she affects profit. The pledgeable income, however, has increased since one no longer pays the entrepreneur for being lucky: now the maximal pledgeable income is

$$p_H R - (\Delta p)\Big[\min_{\{\mathrm{IC_b}\}} R_b\Big] = p_H R - B,$$

where $[\min_{\{\mathrm{IC_b}\}} R_b]$ denotes the smallest reward R_b that ensures incentive compatibility.

Next, let us assume that the firm's performance can be compared with that of an identical firm facing the same state of nature. Assuming that the entrepreneur in the other firm behaves, then "success" in the other firm provides information that the state is either favorable or intermediate, while "failure" in the other firm reveals an unfavorable state. Then, conditional on the entrepreneur failing, one learns either that she was unlucky or that she failed because she misbehaved. In this case, the pledgeable income cannot be increased by benchmarking if one abides by the entrepreneur's limited liability:[25] when

22. See, for example, Section 4.4 and Chapter 9.

23. For example, it would never come to one's mind to condition the entrepreneur's compensation on the weather in Bali, on the outcome of the soccer World Cup, or on other "irrelevant" variables. Why? Let us be a bit more technical here. In the notation of footnote 21, let (x, y) denote the verifiable state of nature (which includes, but is not limited to, the firm's profit $x \in \{0, R\}$), on which the entrepreneur's reward R_b can be conditioned. Thus, let $R_b(x, y)$ denote the state-contingent compensation specified by the financing contract. Suppose that the firm's profit x is a sufficient statistic for (x, y) when assessing the entrepreneur's effort, which we will here call $z \in \{L, H\}$ (see footnote 21 for the definition of a sufficient statistic). The density of the verifiable state (x, y) for a given effort z can be factorized: $f(x, y \mid z) = g(x, y)h(x, z)$. Thus for a choice of effort $z \in \{L, H\}$, the entrepreneur's expected reward is

$$\int_x \int_y R_b(x, y)g(x, y)h(x, z)\,dx\,dy = \int_x \hat{R}_b(x)h(x, z)\,dx,$$

where $\hat{R}_b(x) \equiv \int_y R_b(x, y)g(x, y)\,dy$.

So, a contract that rewards the entrepreneur solely as a function for profit $(\hat{R}_b(x))$ can do at least as well as a more general contract. And, in general, it can do better (in our context, it does strictly better in particular if $\int_y R_b(0, y)g(0, y) > 0$ and if a strictly positive borrower payoff in the case of success jeopardizes financing). Added risk is bad when the limited liability constraint is binding (and would be bad if the agent were risk averse even if she is not protected by limited liability).

24. Information accrues *ex post* through profit realization. Still the state is not learned *ex post* in the basic model.

25. Benchmarking could become relevant again in this example if we relaxed the limited liability constraint by introducing reputational concerns such as a stigma that affects future borrowing or other future relationships of the borrower or else costly nonmonetary penalties (jail or costly collateral pledging as in Chapter 4). Then, the observation that both entrepreneurs fail implies that the state of nature was unfavorable and so stigmas and/or nonmonetary penalties are not in order, unlike the situation in which only one entrepreneur fails (which implies that she misbehaved).

the entrepreneur fails, she already receives 0. On the other hand, it would be optimal to punish the entrepreneur harshly when she fails and the benchmark firm succeeds.[26]

3.2.7 Sensitivity of Investment to Cash Flow: A First Look

Recall from Section 2.5 the empirical finding that investment is sensitive to cash flow. An interesting issue is whether this "investment–cash flow sensitivity" increases with the extent to which the firm is financially constrained. Fazzari et al. (1988) use *a priori* measures of financial constraints and find that the sensitivity of investment to cash flow is particularly large for firms that have trouble raising external funds (for example, firms facing high agency costs). Kaplan and Zingales (1997) argue that there is no theoretical basis for this relationship and present empirical evidence that differs from that of Fazzari et al.

Although the model in this chapter is static while the empirical evidence relates to ongoing concerns (multistage financing is studied in Chapter 5), it can shed some light on the debate. We can imagine that cash on hand A includes the cash flow accruing from the firm's previous activity and see how investment reacts to a small change in the cash flow.[27]

There is a sense in which Fazzari et al. (1988) are right on the theoretical front: the firms whose investment is boosted by a small increase in cash flow are the marginal firms, i.e., those whose cash on hand A lies just below $\bar{A} = I - \rho_0$, where ρ_0 denotes the pledgeable income:

$$\rho_0 \equiv p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) = p_{\mathrm{H}}R - \frac{p_{\mathrm{H}}B}{\Delta p}.$$

Firms with more cash or a lower agency cost do not modify their investment behavior as their investment was already unconstrained.

Suppose, however, that firms are heterogeneous in the two dimensions: cash A and pledgeable income ρ_0 (we normalize the investment I to be the same for all). Assume for simplicity that these two variables are independently distributed (there is no reason for this to be the case: for example, firms with higher pledgeable income may have been able to invest more in the past and be richer today). Let $G(A)$ denote the (continuous) cumulative distribution of cash among firms in the economy, with density $g(A)$. Because only firms with cash on hand A satisfying $\rho_0 \geqslant I - A$ receive financing, aggregate investment among firms with pledgeable income ρ_0 is

$$\mathcal{I}(\rho_0) \equiv [1 - G(I - \rho_0)]I.$$

Now, consider a small, uniform increase in cash δA for all firms. Then, investment among firms characterized by ρ_0 increases by

$$\delta\mathcal{I}(\rho_0) = g(I - \rho_0)I\delta A.$$

And so

$$\frac{\partial}{\partial\rho_0}(\delta\mathcal{I}(\rho_0)) = -g'(I - \rho_0)I\delta A.$$

If the density is decreasing ($g' < 0$), the sensitivity of investment to cash flow is higher for firms with a *low* agency cost (a high ρ_0) as in Kaplan and Zingales; intuitively, the cutoff $\bar{A}$ for firms with a low agency cost is low, and so with a decreasing density there are a lot of marginal firms. With an increasing density ($g' > 0$), the sensitivity of investment to cash flow is higher for firms with a *high* agency cost (a low ρ_0), as in Fazzari et al. Thus, unless one has more precise information about the actual heterogeneity of firms, it is difficult to predict how the sensitivity of investment to cash flow varies with an *a priori* measure of financial constraints (a proxy for (minus) ρ_0).

26. This is one aspect in which a "limited liability model" differs from a "risk-aversion model." In the rest of Section 3.2, we might as well have assumed that the entrepreneur is very risk averse at her subsistence level, normalized at zero consumption, that is, her utility falls very quickly at that level. Suppose at the extreme that the entrepreneur gets $-\infty$ when receiving a negative income. Then, provided that $p_{\mathrm{H}} < 1$ (the entrepreneur may behave and be unlucky), it would not be optimal to set rewards below the subsistence level.

27. This thought experiment in a sense consists in looking at a single period of an ongoing firm that engages in short-term borrowing from investors. There are two reasons why this is only a first step toward an understanding of the sensitivity of investment to cash flow. First, if the firm anticipates that it may be credit-constrained tomorrow, the shadow value of 1 unit of profit at the end of the period exceeds 1, as it may help overcome financing problems in the future. More importantly, this shadow value may vary with current investment. Second, the description of the financial arrangements as a sequence of short-term borrowing contracts misses important long-term financing features (credit lines, debt–equity ratio, maturity structure of debt, etc.) that have an important impact on financial constraints (see Chapter 5).

3.3 Debt Overhang

Following Myers (1977), a number of contributions have studied situations in which a borrower is debt-ridden and unable to raise funds for an otherwise profitable project. The borrower is then said to suffer from debt overhang. The framework just developed suggests two possible interpretations of debt overhang. The first interpretation pursued below is a mere reinterpretation of the credit rationing analysis above: previous investors' collateral claim on the firm's assets reduces the net worth to below the threshold asset level for financing the new investment. Furthermore, the new project overall produces too little pledgeable income and so investment does not take place even if previous investors are willing to renegotiate their claim. The second and more interesting interpretation, and that stressed by the literature, emphasizes the need for renegotiating past liabilities in order to enable new investments.

3.3.1 Decrease in Net Worth

First, the borrower may have a positive-NPV project that would be financed in the absence of any previous debt obligation, but is denied financing due to such an obligation. Namely, suppose that (i) the entrepreneur has A in cash or collateral, but owes D from previous borrowing to a group of investors whom we will call the "initial investors," (ii) the initial investors have insisted on a covenant specifying that the borrower cannot raise more funds without their consent, and (iii) the borrower's assets A are pledged to the initial investors as collateral in case of default. If[28]

$$A > \bar{A} > A - D \geqslant 0,$$

the project would have been financed in the absence of previous borrowing but is not undertaken, since investors as a whole, that is, the initial investors and new investors (who can, of course, be the initial investors themselves), cannot recoup the cost of their investment $(I - A)$ plus the previous debt obligation (D) while they can receive D by seizing the collateral. More precisely, suppose the borrowers, the initial investors, and the new investors enter an agreement so as to finance the project. Because initial investors can secure themselves D by seizing the collateral, they must receive an expected payment at least equal to D under this agreement. Because the pledgeable income net of the investment cost is equal to

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) - I,$$

new investors obtain at most

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) - I - D + A = A - D - \bar{A} < 0,$$

which contradicts the fact that rational investors must at least break even.

3.3.2 Lack of Renegotiation[29]

Second, and more interestingly, suppose that

(i) the project is sufficiently profitable to attract funds even if the borrower has zero net worth,

$$\bar{A} < 0;$$

(ii) the borrower has previously been granted a long-term loan and is due to reimburse D "at the end," that is, when the outcome of the project (if financed) occurs;

(iii) this long-term debt obligation is contractually *senior* to any claim that the borrower might issue (a senior claim is a claim that must be paid before the borrower or any other claimholder receives any money);

(iv) the borrower has no cash ($A = 0$); and

(v) the debt overhang problem is sufficiently serious as not to be overcome by the expected profitability of the new project, or, put differently, the "slack" in pledgeable income, $-\bar{A}$, is smaller than what has to be paid back to previous investors, $p_{\mathrm{H}}D$, if the project is funded:

$$\bar{A} + p_{\mathrm{H}}D > 0.$$

28. Recall that

$$\bar{A} = p_{\mathrm{H}}\frac{B}{\Delta p} - (p_{\mathrm{H}}R - I)$$

is the minimum net worth to obtain financing.

29. The notion that renegotiation breakdowns generate debt overhang is central to Myers's (1977) original analysis, and also underlies that in Hart and Moore (1995) and Bhattacharya and Faure-Grimaud (2001). We will describe the debt overhang situation as one in which a new investment cannot be financed solely because renegotiation with previous debtholders proves infeasible. Debt overhang is generally described in the literature as a situation in which a firm may not be able to continue because it cannot renegotiate with its creditors. It is clear that the two situations are formally equivalent. The act of spending money to let a distressed firm continue is equivalent to an investment.

Because the borrower has no cash, initial investors receive nothing if the project is not financed. So, they are willing to participate in the financing of the project as long as they break even on this investment. For example, they can forgive existing debt, finance the investment I, and demand the entire cash-flow rights attached to external shares, that is, $R - B/\Delta p$ in the case of success. The initial investors then obtain

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) - I = -\bar{A} > 0.$$

The borrower is willing to go along with this arrangement, which allows her to continue and obtain rent $p_{\mathrm{H}}B/\Delta p$ in expectation rather than 0 if the project is not financed.

Suppose next that the initial investors have no cash and thus cannot directly finance the investment I. The borrower then needs to turn to new investors. Are the latter willing to finance the project? Because the initial debt is senior, and because the borrower needs to keep a minimum stake in the firm in order to commit to behave, at most

$$R - \frac{B}{\Delta p} - D$$

can be pledged to new investors in the case of success (and 0 in the case of failure). New investors are willing to enter an agreement to finance the project if and only if

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p} - D\right) \geqslant I$$

or

$$\bar{A} + p_{\mathrm{H}}D \leqslant 0,$$

which contradicts assumption (v).

To sum up, *the borrower cannot raise funds from new investors if she does not renegotiate some debt forgiveness from initial investors.* If renegotiation with initial investors is infeasible, gains from trade between the borrower and the community of investors may not be realized. Renegotiation breakdown creates debt overhang.

The possibility of debt overhang is often invoked in contexts in which "initial investors" stand for "corporate bondholders." It is often thought that because they are dispersed, and despite the existence of some coordinating mechanisms (nomination of a bond trustee, possibility for the firm to offer new securities in exchange for the bonds), bondholders have trouble renegotiating their claim when the borrower faces distress and requires some debt forgiveness.

In contrast, let us assume that initial investors are able to act collectively and renegotiate their initial claims. Because $\bar{A} < 0$, we know that there exists some renegotiated arrangement that is agreeable to all parties (borrower, initial investors, new investors), who would all get nothing if they failed to reach an agreement. Suppose, for example, that the initial investors accept a reduction in the face value of the debt from D to $d < D$, where

$$\bar{A} + p_{\mathrm{H}}d = 0.$$

Then new investors receive

$$\left(R - \frac{B}{\Delta p} - d\right)$$

in the case of success and are therefore willing to invest, since

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p} - d\right) = I$$

is equivalent to their breakeven constraint (3.3):

$$p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) = I - \bar{A}.$$

Initial investors benefit from forgiving some of their claim as they now get

$$p_{\mathrm{H}}d = -\bar{A} > 0.$$

Lastly, the borrower can undertake the project and obtains rent $p_{\mathrm{H}}B/\Delta p > 0$.

Debt renegotiation thus allows the project to be undertaken and all parties to share the resulting gains from trade. How these gains from trade are actually shared depends, of course, on the relative bargaining power of the borrower and the initial investors (the new investors being assumed to be competitive and thus to just break even). The arrangement described above corresponds to the renegotiation that is most favorable to the initial investors. But, by varying continuously the relative bargaining power of the borrower and initial investors, one can generate any level of debt forgiveness from $D-d$ (the most favorable to initial investors) to D (the least favorable to them).

3.4 Borrowing Capacity: The Equity Multiplier

3.4.1 The Continuous-Investment Model

The continuous-investment model of this section is the polar opposite of the fixed-investment one. The fixed-investment model depicts a situation in which returns are sharply decreasing beyond some investment level. In contrast, we now assume that there are constant returns to scale in the investment technology. An investment $I \in [0, \infty)$ yields income RI, proportional to I, in the case of success, and 0 in the case of failure. The borrower's private benefit from misbehaving is also taken to be proportional to investment. As before, the borrower has a choice between behaving, in which case she derives no private benefit and the probability of success is p_H, and misbehaving, that is, enjoying private benefit BI, and reducing the probability of success to $p_L = p_H - \Delta p < p_H$. (One can also analyze the intermediate case of a continuous investment with general decreasing returns to scale (see Exercise 3.5).)

The borrower initially has cash A, and must therefore borrow $I - A$ to finance a project of size I. A loan agreement specifies that the lenders (who as before are assumed to make no profit) and the borrower receive 0 each in the case of failure, and R_l and R_b, respectively, in the case of success, where $R_l + R_b = RI$.

As in Section 3.2, we assume that investment has positive NPV (net present value), here per unit of investment, if the borrower behaves,

$$p_H R > 1, \tag{3.7}$$

but negative NPV otherwise,

$$1 > p_L R + B, \tag{3.8}$$

so that unless one can control the agency problem the investment cannot be funded. We also make an assumption that guarantees that the equilibrium investment is finite:

$$p_H R < 1 + \frac{p_H B}{\Delta p}. \tag{3.9}$$

Like inequality (3.5) in Section 3.2, inequality (3.9) has a simple interpretation: the expected net revenue per unit of investment, $p_H R - 1$, is lower than the per-unit agency cost, $p_H B / \Delta p$.

Finally, we keep assuming that the capital market is competitive. The analysis is very similar when the borrower faces a lender with market power, except that the resulting investment scale is smaller (see Exercise 3.13).

3.4.2 The Lenders' Credit Analysis

Following the steps of Section 3.2, the incentive compatibility and the breakeven conditions are

$$(\Delta p) R_b \geqslant BI \tag{IC$_b$}$$

and

$$p_H(RI - R_b) \geqslant I - A. \tag{IR$_l$}$$

In equilibrium, competitive lenders make no profit on the contract that is most advantageous for the borrower; the borrower's net utility is therefore equal to the social surplus brought about by the investment:

$$U_b = (p_H R - 1) I. \tag{3.10}$$

From (3.10) it is optimal for the borrower to invest as much as possible. The upper bound on investment and in turn her borrowing capacity ("outside financing capacity" or "debt capacity") are determined by constraints (IC$_b$) and (IR$_l$). Substituting (IC$_b$) into (IR$_l$), we obtain

$$I \leqslant kA, \tag{3.11}$$

where

$$k = \frac{1}{1 - p_H(R - B/\Delta p)} > 1. \tag{3.12}$$

The denominator of k is positive from (3.9). Furthermore, conditions (3.7) and (3.8) imply that $(\Delta p) R > B$, and therefore that the denominator of k is smaller than 1. This is important: the fact that $k > 1$ shows that the borrower can lever her wealth, k being the multiplier.

The multiplier is smaller, the higher the private benefit (B) and the lower the likelihood ratio ($\Delta p / p_H$, fixing p_H and thus the profitability of the investment), which are our two measures of the agency cost.

Conditions (3.7) and (3.10) furthermore imply that it is optimal for the borrower to invest k times her cash A, that is, to borrow $d = (k - 1)$ times her level of cash, where

$$d = \frac{p_H(R - B/\Delta p)}{1 - p_H(R - B/\Delta p)}. \tag{3.13}$$

The maximum loan, dA, is called "*borrowing capacity*."[30]

Another important concept (which will be used, for example, in computing the value of retained earnings in a dynamic context) is the *shadow value v of equity (here cash)*. The entrepreneur derives gross utility $v > 1$ from one more unit of equity. Letting $U_b^g \equiv A + U_b$ denote the borrower's gross utility, and using (3.10) and (3.11), we have

$$U_b^g \equiv vA, \tag{3.14}$$

where the shadow value of equity is

$$v = \frac{p_H B/\Delta p}{1 - p_H(R - B/\Delta p)} > 1. \tag{3.15}$$

As one would expect (in the relevant range defined by (3.7)–(3.9)), the borrowing capacity increases with per-unit income R and decreases with the extent of the moral-hazard problem (measured by the borrower's private benefit or the inverse of the likelihood ratio). The shadow value of equity increases with per-unit income R and also with the extent of the moral-hazard problem.[31]

Finally, let us introduce some notation that will be used repeatedly throughout the book. Let

$$\rho_1 \equiv p_H R$$

denote the expected payoff per unit of investment and

$$\rho_0 \equiv p_H\left(R - \frac{B}{\Delta p}\right)$$

denote the expected pledgeable income per unit of investment. Assumptions (3.7) and (3.9) can be rewritten as

$$\rho_1 > 1 > \rho_0.$$

The equity multiplier is then

$$k = \frac{1}{1 - \rho_0}, \tag{3.11$'$}$$

the debt capacity per unit of net worth

$$d = \frac{\rho_0}{1 - \rho_0}, \tag{3.12$'$}$$

and the borrower's gross utility

$$U_b^g = vA = \frac{\rho_1 - \rho_0}{1 - \rho_0} A. \tag{3.14$'$}$$

The borrower's net utility can then be written as

$$U_b^n = U_b = U_b^g - A = \frac{\rho_1 - 1}{1 - \rho_0} A = (\rho_1 - 1)I,$$

as one could have expected.

Remark (factors that keep the investment bounded). Condition (3.9) (the condition that the pledgeable income per unit of investment is smaller than 1) was needed in order to keep the investment finite in this constant return to scale environment. Such a condition is no longer needed if the price of output and therefore the revenue in the case of success is not fixed but rather depends on, say, industry investment. An increase in per-firm investment then lowers the market price, reducing both value and pledgeable income (see Exercise 3.17 for more detail).

Remark (sensitivity of investment to cash flow). Let us briefly return to the sensitivity of investment to cash flow. In the variable-investment model,

$$\frac{\partial}{\partial \rho_0}\left(\frac{\partial I}{\partial A}\right) = \frac{1}{(1 - \rho_0)^2} > 0,$$

and so firms with a low agency cost, which are therefore less financially constrained, exhibit a higher sensitivity. Intuitively, such firms have a high multiplier and their investment is therefore more sensitive to available cash.

3.4.3 Collateral Values: Outside Debt and the Maximal Incentives Principle

We now return to the indeterminacy of the financial structure (debt or equity) discussed earlier. It turns out that this indeterminacy was an artefact of the absence of profit in the case of failure.[32]

Thus, assume that, for investment size I, the profit is $R^S I$ in the case of success and $R^F I$ in the case of failure, where R^F is now positive. $R^F I$ can be thought of as the salvage value of assets and

$$RI \equiv (R^S - R^F)I$$

30. Note also that the "gearing ratio" $g = d/k = p_H R - p_H B/\Delta p$ is less than 1, and that the debt-over-inside-equity ratio is equal to d.

31. The shadow value is here constant with wealth A. With a decreasing-returns-to-scale technology, v depends on wealth and $v'(A) < 0$: the marginal wealth enables less and less profitable marginal investments as wealth increases (see Exercise 3.5).

32. The clearest illustration of this point is for the variable-investment model, which is why we treat this here. The same point can be made in a slightly different form (as some indeterminacy may remain) in the fixed-investment version (see Exercise 3.18).

as the increase in profit brought about by success. One would expect R^{F} to be larger when secondary asset markets are liquid.[33]

The model is otherwise the same as in the rest of Section 3.4: the private benefit (BI in the case of misbehavior, 0 otherwise) is also proportional to investment.

The generalization of the condition that the NPV per unit of investment is positive while the pledgeable income per unit of investment is negative ($p_{\mathrm{H}}R > 1 > p_{\mathrm{H}}(R - B/\Delta p)$) is

$$p_{\mathrm{H}}R + R^{\mathrm{F}} > 1 > p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right) + R^{\mathrm{F}}.$$

A contract specifies an investment level I and a sharing rule, or equivalently a reward for the entrepreneur for each performance level: $\{R_{\mathrm{b}}^{\mathrm{S}}, R_{\mathrm{b}}^{\mathrm{F}}\}$, with $R_{\mathrm{b}}^{\mathrm{S}}, R_{\mathrm{b}}^{\mathrm{F}} \geqslant 0$ due to limited liability.

The optimal contract maximizes the entrepreneur's expected compensation,

$$U_{\mathrm{b}} = \max_{\{R_{\mathrm{b}}^{\mathrm{S}}, R_{\mathrm{b}}^{\mathrm{F}}, I\}} \{p_{\mathrm{H}}R_{\mathrm{b}}^{\mathrm{S}} + (1 - p_{\mathrm{H}})R_{\mathrm{b}}^{\mathrm{F}} - A\},$$

subject to two constraints (that will turn out to be binding at the optimum): the entrepreneur's incentive constraint,

$$(\Delta p)(R_{\mathrm{b}}^{\mathrm{S}} - R_{\mathrm{b}}^{\mathrm{F}}) \geqslant BI,$$

and the investors' breakeven constraint,

$$p_{\mathrm{H}}(R^{\mathrm{S}}I - R_{\mathrm{b}}^{\mathrm{S}}) + (1 - p_{\mathrm{H}})(R^{\mathrm{F}}I - R_{\mathrm{b}}^{\mathrm{F}}) \geqslant I - A.$$

To show that the investors' breakeven constraint is binding, note that, if it were not, then the entrepreneur could increase $R_{\mathrm{b}}^{\mathrm{S}}$ and $R_{\mathrm{b}}^{\mathrm{F}}$ by an equal and small amount without affecting the incentive compatibility constraint. This uniform increase in compensation would raise the entrepreneur's payoff. As is now familiar, we conclude that the investors receive no surplus, and so (by substituting the breakeven constraint into the objective function) the entrepreneur's utility is equal to the NPV:

$$U_{\mathrm{b}} = (p_{\mathrm{H}}R + R^{\mathrm{F}} - 1)I.$$

Because the NPV per unit of investment is positive, the entrepreneur therefore chooses the highest possible investment.

33. Several chapters in this book (primarily Chapter 14) will investigate the determinants of asset prices in secondary markets.

Next, note that the incentive constraint is binding (otherwise, the optimal investment would be infinite, which would violate the two constraints combined).

Lastly, suppose that $R_{\mathrm{b}}^{\mathrm{F}} > 0$ at the optimum. And consider a small increase $\delta R_{\mathrm{b}}^{\mathrm{S}} > 0$ in managerial compensation in the case of success together with a small decrease $\delta R_{\mathrm{b}}^{\mathrm{F}} < 0$ in the case of failure that keeps the investors' profitability constant:

$$p_{\mathrm{H}}\delta R_{\mathrm{b}}^{\mathrm{S}} + (1 - p_{\mathrm{H}})\delta R_{\mathrm{b}}^{\mathrm{F}} = 0.$$

This small change (which is feasible only if $R_{\mathrm{b}}^{\mathrm{F}} > 0$) also keeps the objective function constant. But the incentive constraint is now slack, a contradiction. We thus conclude that at the optimum

$$R_{\mathrm{b}}^{\mathrm{F}} = 0.$$

Hence, an all-equity firm cannot be optimal: in the absence of debt, the entrepreneur would receive $R^{\mathrm{F}}I$ times her share of stocks in the firm, and therefore would be rewarded even in the case of failure. By contrast, investors' holding debt $D \geqslant R^{\mathrm{F}}I$ is an optimal financial structure. Using the fact that the two constraints are binding, the borrowing capacity is given by

$$R^{\mathrm{F}}I + p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right)I = I - A$$

or

$$I = \frac{A}{1 - [p_{\mathrm{H}}(R - B/\Delta p) + R^{\mathrm{F}}]}. \tag{3.16}$$

Predictions. The variable-investment model of this section is, of course, much too simplistic to provide even a stylized account of capital structure and investment. It, however, delivers three interesting preliminary insights.

• *Firms with lower agency costs borrow more.* As in Section 3.2.2, the firm's outside financing capacity is higher, the lower the agency cost as measured either by the private benefit B or by (the inverse of) the likelihood ratio $\Delta p/p_{\mathrm{H}}$ (keeping p_{H} and therefore profitability constant).

• *The investors' holding safe debt plus some equity maximizes the entrepreneur's stake in the project and thereby her incentives.* (We will investigate the generality of this insight in Section 3.5.)

Decomposing the investors' claim into safe debt (which repays $R^{\mathrm{F}}I$) and risky equity (which repays in expectation $p_H[R - B/\Delta p]I$), the leverage ratios,

$$\frac{\text{debt}}{\text{total equity}} = \frac{R^F I}{p_H R I} = \frac{R^F}{p_H R}$$

and

$$\frac{\text{debt}}{\text{outside equity}} = \frac{R^F I}{p_H(R - B/\Delta p)I} = \frac{R^F}{p_H(R - B/\Delta p)},$$

are both constant in this simple-minded model.

• *Credit rationing is more binding for firms with less tangible assets or assets that have a lower value in liquidation* (there is indeed substantial evidence in this direction: see Chapter 2). To see this, let us decrease the value of tangible assets while keeping the NPV per unit of investment constant: that is, keeping other parameters constant, let us consider a decrease from R^F to $\hat{R}^F$ ($\hat{R}^F < R^F$) of the per-unit salvage value and an increase from p_H to $p_H + \tau$ ($\tau > 0$) of the probability of success such that

$$\hat{R}^F + (p_H + \tau)R = R^F + p_H R.$$

In order to keep the agency problem invariant (so as not to interfere with the first prediction), let us assume that the probability of success in the case of misbehavior becomes $p_L + \tau$. The borrower's incentive compatibility constraint is then unchanged as

$$[(p_H + \tau) - (p_L + \tau)](R_b^S - R_b^F) = (\Delta p)(R_b^S - R_b^F).$$

The analysis is unchanged and the new investment becomes

$$\hat{I} = \frac{A}{1 - [(p_H + \tau)(R - B/\Delta p) + \hat{R}^F]} < I.$$

Thus, ceteris paribus, tangible assets facilitate financing.[34]

3.4.4 Going Forward

This chapter offered a first glance at the basic conflict between value and pledgeable income. When pressed to produce returns to attract investors, borrowers first offer them a large debt repayment or a higher share of profits (Section 3.2). This policy is, however, limited by entrepreneurial moral hazard and must be supplemented by costly "concessions."

34. Rather than increase the probability of success, we could have increased the payoff R in the case of success. Then investment would have been invariant (see equation (3.16)). As increases in the value of the risky component are in general associated with both types of changes, the conclusion that tangible assets facilitate financing is robust.

Technically, with competitive investors, the total value (NPV) goes to the entrepreneur, who aims at maximizing this value subject to the constraint that the pledgeable income be sufficient to enable the investors to recoup their investments. The resulting policy (charter, covenants, governance structure, etc.) therefore sacrifices value to generate enough pledgeable income if the breakeven constraint condition is binding.

The variable-investment model of Section 3.4 pointed at such an elementary concession: a limited investment size. Indeed, with constant returns to scale, it would be optimal for the firm to grow without bounds, but pleasing investors requires a limited size (all the more so, as we have seen, as the agency problem is important and as assets are intangible). The rest of the book will provide further illustrations of the idea that entrepreneurs must sometimes "bend over backwards" in order to attract investors: costly collateral pledging, restricted exit options, short maturity structures, enlisting of active and speculative monitors, allocations of control rights to equityholders and debtholders, limits on takeover defenses, and so forth.

Supplementary Sections

3.5 Related Models of Credit Rationing: Inside Equity and Outside Debt

This supplementary section reviews three classic, alternative models of credit rationing. These models are a bit more complex than the basic credit rationing model developed in this chapter and this supplementary section is accordingly more technical than the text. They are not relegated to the supplementary section because they are deemed "less important." Rather, the reader should recall from the introduction that we want to conduct controlled experiments throughout the book. Using the same simple and tractable model throughout allows us to concentrate on the key insights of the theory without getting bogged down by extraneous modeling changes. This is the motivation for setting these models aside. It should furthermore be borne in mind that these

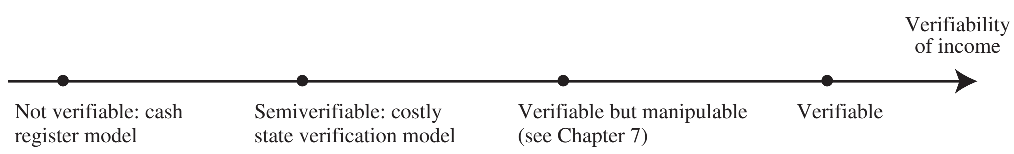

Figure 3.3

models yield pretty much the same insights as our basic model. While this supplementary section can be skipped without adverse consequences for the comprehension of the rest of the book, students intending to specialize in corporate finance should thoroughly learn these alternative models.

Two assumptions are shared by the three models reviewed in the supplementary section and by the moral-hazard model developed in the text.

(a) The entrepreneur can divert some of the income.[35] Hence, only part of the project's income can be pledged to investors, and so positive-NPV projects may not be financed.

(b) Investors are passive. Their claim is thus defined as a "leftover" once the entrepreneur's optimal incentive scheme is derived.

The point of departure between the models is the form of diversion that is presumed. The scope for diversion is determined by what is presumed with regard to the verifiability of income. This chapter has adopted a polar assumption, namely, that of a fully verifiable income. Figure 3.3 presents some alternative assumptions.

In the other polar case, the entrepreneur can divert money as she wants. One may then wonder why the entrepreneur would ever repay her loans and therefore why lenders would bring in money in the first place. For example, in the two-outcome model, the entrepreneur can appropriate R in the case of success and pretend that the project has failed, thus repaying nothing to the lenders. Anticipating this "strategic default," the lenders would not want to invest. Repayment must then be motivated by some other consideration. The lenders' foreclosing on the entrepreneur's assets (held as hostages) is an important but obvious example. A perhaps more interesting motivation for repayment in the context of unverifiable income, and a motivation that has been emphasized in the literature, is the threat that the entrepreneur's future projects not be financed.

In between these two polar cases lies the influential costly state verification (CSV) model, in which the borrower cannot steal money from the firm (unlike in the nonverifiable income model), but only a costly audit reveals the firm's income to the lenders. To economize on audits, the lenders and the borrower can agree to let the borrower report on the realized income. However, the lenders cannot just trust the borrower to report truthfully and must at least occasionally engage in the costly auditing process in order to verify that the borrower does not underreport income.

Lastly, one can maintain the assumption that the firm's income is verifiable (there is a reliable accounting structure), so that the firm's accounts truthfully reflect its cash position. However, the *significance* of this cash position is unclear if the entrepreneur can manipulate income, for example, by shifting income across accounting periods.

Little attention has been devoted to assessing the empirical relevance of the various assumptions on the verifiability of firm income. This is all the more unfortunate since, as we have seen, a wide range of hypotheses have been entertained. The nonverifiability of income is perhaps most plausible for a small enterprise. For example, a farmer or a shopkeeper who can arrange sales that are not recorded by invoices can divert money. They then literally steal money from the firm. Most firms, however, have proper accounts and it may then be difficult for insiders to steal from the cash register. On the other hand, lenders may not know exactly how much there is in the firm. While the firm's cash and investments in marketable assets are readily verifiable, the value

35. The moral-hazard model can be viewed as one in which the entrepreneur can divert money. Namely, the diversion activity involves a deadweight loss equal to $(\Delta p)R_l - [B - (\Delta p)R_b] = (\Delta p)R - B$, that is, the difference between the money lost by investors and the (monetary equivalent of the) net gain for the borrower when she misbehaves.

of the firm's other (tangible or intangible) assets in general is revealed to outsiders only after a costly audit. It has been argued in the literature that this audit ought to be interpreted as a bankruptcy process. Lastly, another useful paradigm is that of verifiable but manipulable income, reviewed in Chapter 7; while it seems very relevant for many firms, it has unfortunately been studied much less than the other three paradigms and little yet is known about its properties.

As we shall see, a key result that is common to all three models reviewed in the supplementary section is that they all structure the entrepreneur's incentive problem so that her claim optimally takes the form of an equity claim and the lenders' that of a fixed payment. In other words, these models predict a combination of *inside equity and outside (risky) debt*.

From principal–agent theory, we know that the agent's optimal incentive scheme in general does not take the form of "inside equity." Therefore, a fair amount of structure must be imposed on the agency relationship in order to generate a standard debt contract for the lenders. Consequently, the theories described below are often criticized for their lack of robustness; it is also pointed out that they do not account for the diversity of capital structures that characterize modern corporations, and that even small firms sometimes admit outside equity (for example, venture capital). Such criticisms are well-taken, but, left unqualified, they miss the point of these modeling exercises; for, the purpose of such exercises is not to show that the standard debt contract should be the unique outside claim in a wide range of circumstances, but rather to identify forces that make standard debt an appealing instrument, leaving the relaxation of the assumptions and the derivation of more realistic corporate financing modes to further modeling effort.

3.6 Verifiable Income

For continuity of exposition, we start with the least departure from the model in the text. The first approach to standard debt contracts employs the verifiable income paradigm and draws on the logic of *maximal insider incentives*. Namely, a standard debt contract for outsiders makes the borrower residual claimant for the marginal income above the debt repayment level and, *under some conditions*, provides the entrepreneur with maximal incentives to exert effort.

Two remarks are in order here. First, residual claimancy exposes the borrower with substantial risk, and so borrower risk neutrality must be assumed in order not to introduce a tradeoff between incentives and insurance.[36] Second, a standard result in incentive theory is that full incentives are provided when the agent receives at the margin one dollar whenever profit increases by one dollar, that is, when the agent pays a fixed amount to the principal and is "residual claimant" for the remaining profit. This is not quite so under a debt contract; under a standard debt contract, the borrower is residual claimant for income only when income exceeds the repayment level; she receives nothing at the margin as long as income lies below the repayment level. This is why we added the qualifier "under some conditions."

Innes (1990) analyzes the verifiable income model for a continuum of effort levels, and, more interestingly, for a continuum of outcomes. The firm's income R is now a random variable distributed over an interval $[0, \bar{R}]$ according to the distribution $p(R \mid e)$, where $e \geqslant 0$ is the entrepreneur's effort level. The borrower's disutility of effort function $g(e)$ satisfies the standard assumptions:

$$g' > 0, \quad g'' > 0,$$
$$g(0) = 0, \quad g'(0) = 0, \quad g'(\infty) = \infty.$$

In particular, this cost function is convex and the assumptions on its derivative guarantee that the borrower's optimal effort is strictly positive and finite.

We assume that a higher effort raises income in the sense of the monotone (log) likelihood ratio property (MLRP):

$$\frac{\partial}{\partial R}\left[\frac{\partial p(R \mid e)/\partial e}{p(R \mid e)}\right] > 0.$$

36. As is well-known, lenders in general should bear some of the risk faced by a risk-averse agent. See, for example, Mirrlees (1975), Holmström (1979), and Shavell (1979) for general considerations on the principal–agent model, and Lacker (1991) for an application to financing.

This condition says that a higher income "signals" a higher effort (see, for example, Holmström (1979) and Milgrom (1981) for more details on MLRP).

We maintain the assumptions of verifiable income, limited liability for the borrower and risk neutrality on both sides, and that the lenders demand a rate of return equal to 0. Let $w(R)$ denote the borrower's reward when the realized income is R. Let us make the following assumption.

Assumption (monotonic reimbursement):

$$R - w(R) \quad \textit{is nondecreasing for all } R. \qquad \text{(M)}$$

Innes motivates this assumption by the possibility that the borrower secretly adds cash into the firm's accounts. Suppose that $R_1 < R_2$, but $R_1 - w(R_1) > R_2 - w(R_2)$. Then when the realized income is R_1, the borrower could borrow $(R_2 - R_1)$ from a third party and increase her reward by $w(R_2) - w(R_1) > R_2 - R_1$; and so the borrower could repay the third party and make a surplus from the transaction. The reimbursement would then be the same, namely, $R_2 - w(R_2)$, for both realizations of income and would thus be nondecreasing.

Let us now consider the problem of maximizing the borrower's utility (i.e., the NPV under a competitive capital market) subject to the incentive compatibility constraint (as depicted by the borrower's first-order condition with respect to her effort choice), the lenders' breakeven condition and the monotonicity constraint.

Program I:

$$\max_{\{w(\cdot),e\}} \left\{ \int_0^{\bar{R}} w(R) p(R \mid e)\, \mathrm{d}R - g(e) \right\}$$

s.t.

$$\int_0^{\bar{R}} w(R) \frac{\partial p(R \mid e)}{\partial e}\, \mathrm{d}R = g'(e), \qquad (\mathrm{IC_b})$$

$$\int_0^{\bar{R}} [R - w(R)] p(R \mid e)\, \mathrm{d}R = I - A, \qquad (\mathrm{IR_l})$$

$$R - w(R) \quad \text{is nondecreasing for all } R. \qquad \text{(M)}$$

As is usual in principal–agent models, most of the interesting insights are derived from the maximization with respect to the managerial compensation schedule $w(\cdot)$. Letting μ and λ denote the (nonnegative) multipliers of the constraints $(\mathrm{IC_b})$ and $(\mathrm{IR_l})$, and *ignoring in a first step the monotonicity constraint*, the Lagrangian of Program I is

$$\mathcal{L} = \int_0^{\bar{R}} w(R) \left[1 + \mu \frac{\partial p(R \mid e)/\partial e}{p(R \mid e)} - \lambda \right] p(R \mid e)\, \mathrm{d}R - g(e) - \mu g'(e) + \lambda \left[\int_0^{\bar{R}} R p(R \mid e)\, \mathrm{d}R - I + A \right].$$

It is therefore linear in $w(R)$ for all R (this is, of course, due to risk neutrality).

Let us begin with a thought experiment and impose the extra constraint that lenders have limited liability, $w(R) \leqslant R$ for all R. This assumption, which we will later dispense with, is less natural than borrower's limited liability since investors could at the contracting date put assets (e.g., Treasury bonds) into escrow and therefore credibly commit to pay rewards exceeding the firm's income. Under this lenders-limited-liability assumption, the solution would be

$$w(R) = \begin{cases} R & \text{if } 1 + \mu \dfrac{\partial p(R \mid e)/\partial e}{p(R \mid e)} > \lambda, \\[2ex] 0 & \text{if } 1 + \mu \dfrac{\partial p(R \mid e)/\partial e}{p(R \mid e)} < \lambda. \end{cases}$$

Assume that the shadow price μ of the incentive constraint is strictly positive.[37] Then, MLRP implies that there exists a threshold level of income R^* such that

$$w(R) = \begin{cases} R & \text{if } R > R^*, \\ 0 & \text{if } R < R^*. \end{cases}$$

The borrower's reward and the reimbursement are depicted in Figure 3.4.[38]

The solution thus generalizes the maximal insider incentive principle: the borrower receives nothing for $R < R^*$ and the firm's entire income for $R > R^*$.[39] Note, though, that the reimbursement pattern is unfamiliar in that the lenders' claim is valueless in good states of nature.

After this thought experiment, let us come back to Program I. We leave it to the reader to check

37. If the incentive constraint is not binding, the optimal effort in Program I is then the first-best effort, given by

$$g'(e) = \int_0^{\bar{R}} R \frac{\partial p(R \mid e)}{\partial e}\, \mathrm{d}R.$$

38. If rewards in excess of income were allowed, the solution would be degenerate with $w(R) = 0$ for $R < \bar{R}$ and a spike at $\bar{R}$ (or with a discrete number of outcomes, a reward only when the income is the highest possible one).

39. Such contracts are called "live or die" contracts in the literature.

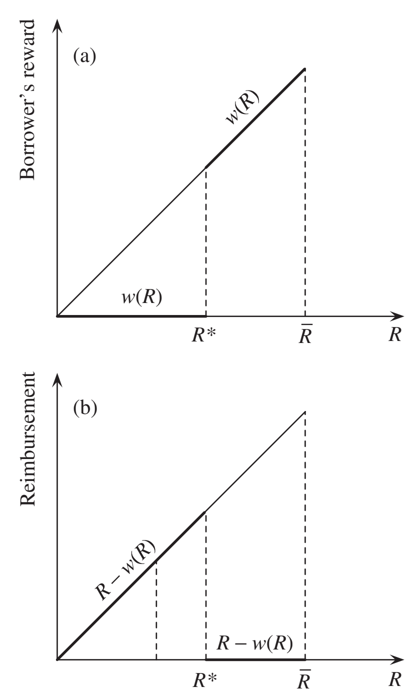

Figure 3.4

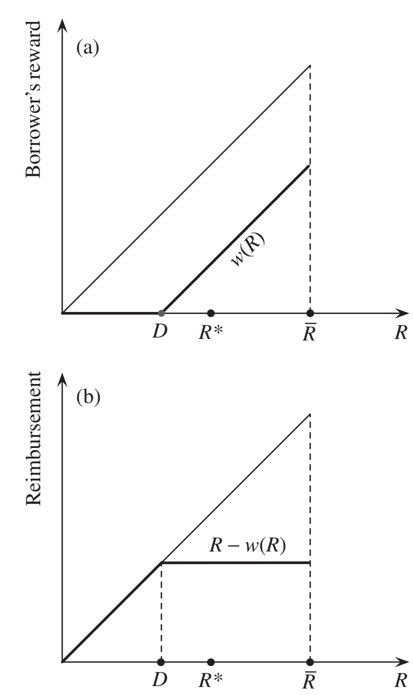

Figure 3.5

that adding the monotonic reimbursement constraint to Program I[40] yields the solution depicted in Figure 3.5.

Intuitively, the optimal reimbursement scheme subject to the monotonicity constraint (depicted in Figure 3.5(b)) approximates as closely as possible the optimal reimbursement scheme (depicted in Figure 3.4(b)) in the absence of this constraint. Note also that under the monotonicity constraint the assumption of limited liability on the lenders' side no longer bites: the borrower receives nothing for low incomes, and since the reward cannot grow faster than the firm's income from the monotonicity constraint, the reward can never exceed the firm's income. The assumption of limited liability on the lenders' side thus need not be made if monotonicity of reimbursement is imposed.

The Innes derivation of a standard debt contract relies on strong assumptions (risk neutrality, monotonic reimbursement), but it illustrates nicely the fact that debt contracts have good incentives properties, provided that the borrower's discretion consists in raising or decreasing income. Leaving aside borrower risk aversion, to which we turn next, there are, however, several caveats. First, a debt contract is less appropriate when the borrower's discretion also involves a choice of riskiness, a case which we will discuss in Chapter 7. Second, a debt contract may not be optimal if the borrower learns information after the contract is signed and before the borrower chooses her effort: a debt contract offers poor incentives to work in bad states of nature, as shown by Chiesa (1992). (Chiesa's point also applies to the other models reviewed in this supplementary section.)

Risk aversion. We have assumed that the entrepreneur and the lenders are both risk neutral. Does the debt optimality result carry over to, say, entrepreneurial risk aversion?[41] When the entrepreneur is risk averse, the optimal contract must, besides satisfying the lenders' breakeven constraint, aim at two targets: effort inducement and insurance.

40. Monotonicity implies that the function $R \to R - w(R)$ is differentiable almost everywhere, and so $(d/dR)[R - w(R)] \geqslant 0$ almost everywhere. It also implies that the reimbursement schedule has no downward jump (unlike that depicted in Figure 3.4(b)).

41. Some of the results reviewed next carry over to investor risk aversion as well. See the papers cited below.

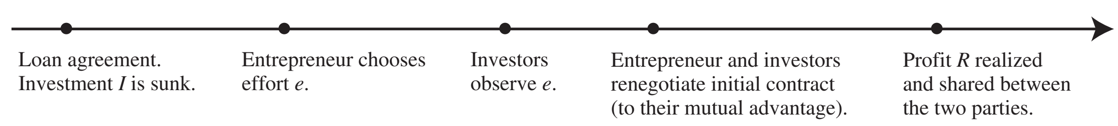

Figure 3.6

As is well-known (see, for example, Holmström 1979), these two goals are, in general, in conflict. Insuring the entrepreneur against variations in profit makes her unaccountable, and results in a low level of effort.

The literature, though, has identified a case in which there is no conflict between the two targets. Namely, this literature assumes that the investors observe the entrepreneur's effort before the profit is realized and that renegotiation then takes place.[42]

The investors' observing the entrepreneur's effort turns out to substantially improve what incentive schemes can achieve.[43]

Hermalin and Katz (1991). Let us begin with the work of Hermalin and Katz. Assume, for simplicity, that investors are risk neutral, while the entrepreneur is risk averse, with a separable utility function:

$$U_{\mathrm{b}} = \int_0^{\bar{R}} u(w(R))p(R \mid e)\,\mathrm{d}R - g(e),$$

where u is increasing and concave.

42. "Renegotiation" means that both parties agree to alter the initial contract to their mutual advantage; the initial contract is perfectly enforceable if any party wants it to be enforced.

43. Two points here for the more technically inclined reader.

First, the original as well as general result in this line of research is due to Maskin (1977). He shows that, under very weak assumptions, the prospect of sharing information about the noncontractible dimensions (here effort) enables parties to achieve what they could have achieved if this shared information were also received by an impartial judge. In a nutshell, courts do not need to observe what the parties observe. It suffices that the parties be given proper incentives to reveal what they know, in a sort of "*adversarial hearing*." The contribution of the papers to be discussed shortly is, among other things, to link Maskin's so-called "Nash implementation" literature with the principal–agent model with renegotiation, and to derive some concrete implications relative to debt contracts.

Second, it is crucial that the parties observe effort and renegotiate *before* the profit is realized. In particular, if the entrepreneur's utility function is separable in effort and reward (as we will assume here), then effort no longer affects the entrepreneur's von Neumann–Morgenstern utility function once profit is realized, and so the level of effort can no longer be elicited through a Maskin adversarial hearing scheme.

For further discussion of these and related issues, see, for example, Hart and Moore (1999), Maskin and Tirole (1999), and Tirole (1999).

Unlike Innes, Hermalin and Katz do not need to assume that the likelihood ratio is monotone or that the investors' payoff is monotonic in profit. But they make the following two assumptions.

Assumption (entrepreneur's unlimited liability):

$$w(R) \gtrless 0 \quad \textit{for all } R.$$

Assumption (entrepreneur-offer renegotiation). *At the renegotiation stage (see Figure 3.6), the entrepreneur makes a take-it-or-leave-it contract offer* $(\tilde{w}(\cdot))$*. If the investors (who at that point have observed effort) accept, the new contract is in force. Otherwise, the initial contract* $(w(\cdot))$ *still prevails.*[44]

It is simple to see that the first-best outcome can then be implemented through a *debt contract*.[45]

The first-best outcome refers to the hypothetical situation in which effort would be observed, and so there is no incentive compatibility constraint.

Program II:

$$\max_{\{w(\cdot),e\}} \left\{ \int_0^{\bar{R}} u(w(R))p(R \mid e)\,\mathrm{d}R - g(e) \right\}$$

s.t.

$$\int_0^{\bar{R}} [R - w(R)]p(R \mid e)\,\mathrm{d}R = I - A. \qquad (\mathrm{IR_l})$$

The solution to this program yields full insurance (all the risk is borne by risk-neutral investors, none by the risk-averse entrepreneur),

$$w(R) = w^* = E(R \mid e^*) + A - I \quad \text{for all } R,$$

44. See Edlin and Hermalin (2000, 2001) for extensions of the Hermalin and Katz analysis to situations with shared bargaining power.

45. Again, for the more technically inclined reader, note that we did not allow for a more general message space. We look at a particular form of initial contract (a wage schedule) that does not involve general messages from both parties, as more general contracts would call for. Here the *ex post* messages are created by the renegotiation process: the offer of a new wage schedule by the entrepreneur, and the acceptance or refusal decision of the investors. This is, of course, inconsequential: because the optimal allocation is attained, more general contracts could not do better.

where $E(R \mid e) = \int_0^{\bar{R}} Rp(R \mid e)\,\mathrm{d}R$ is the expected profit, and a first-best effort level e^* given by

$$e \text{ maximizes } \{u(w^*) - g(e)\}$$
$$\text{s.t.} \quad E(R \mid e) - w^* = I - A,$$

or, equivalently,

$$e^* \text{ maximizes } \{u(E(R \mid e) + A - I) - g(e)\}.$$

Now consider the case in which effort is not verifiable by a court, but is observed by the investors before the profit accrues. At the renegotiation stage, for an arbitrary effort e chosen by the entrepreneur, the entrepreneur will offer a contract $\tilde{w}(\cdot) = \{w(R)\}_{R \in [0,\bar{R}]}$ so as to solve the following program.

Program III:

$$\max_{\{\tilde{w}(\cdot)\}} \left\{ \int_0^{\bar{R}} u(\tilde{w}(R))p(R \mid e)\,\mathrm{d}R - g(e) \right\}$$

s.t.

$$\int_0^{\bar{R}} [R - \tilde{w}(R)]p(R \mid e)\,\mathrm{d}R \geqslant \hat{V}(e),$$

where

$$\hat{V}(e) \equiv E(R \mid e) - \int_0^{\bar{R}} w(R)p(R \mid e)\,\mathrm{d}R$$

is the investors' expected income under the initial contract.

Note that Program III coincides with Program II provided that

$$\hat{V}(e) = I - A.$$

It therefore suffices to find an initial contract such that, regardless of the effort choice, the investors' expected income is equal to $I - A$. This is achieved by a riskless debt contract in which the entrepreneur must reimburse

$$D \equiv I - A.$$

(The risk-free character of this form of debt is due to the entrepreneur's unlimited liability. With limited liability, a debt contract is risky for the lender: it pays only R whenever $R < D$. And so a low effort reduces the investors' status quo utility $\hat{V}(e)$ in the renegotiation process.)

We thus derive Hermalin and Katz's result: the incentive and insurance problems separate. A debt contract makes the entrepreneur residual claimant (i.e., eliminates any externality of effort choice on the investors' welfare), and therefore provides her with optimal incentives. The debt contract is, however, very risky for the borrower; but renegotiation shifts the entire risk to the risk-neutral investors.[46]

Remark (varying the bargaining power in renegotiation). That a debt contract cum renegotiation results in the first-best outcome does not generalize to arbitrary renegotiation processes. Suppose, for example, that the investors, rather than the entrepreneur, make a take-it-or-leave-it renegotiation offer. The entrepreneur's reservation value in renegotiation is

$$\hat{U}(e) = \int_0^{\bar{R}} u(R - D)p(R \mid e)\,\mathrm{d}R - g(e).$$

Because the entrepreneur obtains no surplus from the renegotiation, she chooses effort so as to maximize $\hat{U}(e)$, rather than $[u(E(R \mid e) - I + A) - g(e)]$. On the other hand, renegotiation still results in full insurance for the entrepreneur.

Dewatripont, Legros, and Matthews (2003). In a sense Dewatripont et al. combine the models of Innes and Hermalin and Katz. Like the latter, they allow risk aversion and confer upon renegotiation the task of creating efficient risk sharing (full insurance if the investors are risk neutral). But they share with Innes the presumption that the entrepreneur does not have unlimited liability and so a debt contract does not insulate investors against risk and therefore against externalities induced by the entrepreneur's effort choice.

Dewatripont et al. make the following assumptions (the first three are borrowed from Innes and

46. It is crucial that the investors observe the effort. Were the investors not to observe effort, then renegotiation would potentially take place under asymmetric information about the effort choice. Indeed, equilibrium behavior results in an asymmetry of information at the renegotiation stage and in inefficient renegotiation. To see this, suppose, for example, that the entrepreneur in equilibrium selects the efficient effort e^* for certain. Then the investors agree to fully insure the entrepreneur at wage equal to $E(R \mid e^*) - \hat{V}(e^*)$. But full insurance then induces the entrepreneur to select the lowest possible effort. The equilibrium is then in mixed strategies (at least for the optimal contract).

For more detail about contract renegotiation when the effort is not observed by the investors, see Fudenberg and Tirole (1990), Ma (1991, 1994), and Matthews (1995). Matthews (2001) analyzes this asymmetric-information renegotiation under the limited liability and monotonicity assumptions made here.

General results on contract design with renegotiation under *symmetric* information can be found in Maskin and Moore (1999) and Segal and Whinston (2002).

the fourth from Hermalin and Katz):

(i) Entrepreneur's limited liability.
(ii) Monotonicity of the investors' claim.
(iii) Monotone likelihood ratio property.
(iv) Entrepreneurial risk aversion (let us assume for simplicity that investors are risk neutral).

For these assumptions, a central result of their paper[47] is that under entrepreneur-offer renegotiation (the entrepreneur makes a take-it-or-leave-it offer at the renegotiation stage), *the optimal contract is again a debt contract.*

Renegotiation clearly leads to full insurance. Hence, we only need to worry about the equilibrium level of effort. The first point to note is that there is always underprovision of effort: the entrepreneur does not internalize the impact of her effort on the investors' pre-renegotiation (equal to post-renegotiation) utility,

$$\hat{V}(e) = \int_0^{\bar{R}} [R - w(R)]p(R \mid e)\,\mathrm{d}R$$
$$\equiv \int_0^{\bar{R}} R_1(R)p(R \mid e)\,\mathrm{d}R.$$

Now

$$\hat{V}'(e) = \int_0^{\bar{R}} \left[R_1(R)\frac{p_e(R \mid e)}{p(R \mid e)}\right] p(R \mid e)\,\mathrm{d}R$$
$$= \operatorname{cov}\left(R_1(R), \frac{p_e(R \mid e)}{p(R \mid e)}\right),$$

using the well-known property of the likelihood ratio that its mean is equal to 0.[48] Because p_e/p is increasing and has mean 0, its covariance with a nondecreasing function is positive, and so

$$\hat{V}'(e) \geqslant 0.$$

47. Dewatripont et al. also show that there is no loss of generality in considering contracts in which the investors exercise an option after observing the entrepreneur's effort (this, of course, does not imply that only contracts in this class can implement the optimum. Indeed, the renegotiated debt contract studied below involves post-effort "messages" from both parties and does not belong to this class. By contrast, convertible debt does). The intuition is that, by not allowing a post-effort message by the entrepreneur, one minimizes the size of the set of her possible deviations. By contrast, including a message (an option since this is the only message) sent by the investors is important, because it keeps the entrepreneur on her toes.

48. Let $P(\cdot)$ denote the cumulative distribution of the density $p(\cdot)$:

$$\int_0^{\bar{R}} \frac{p_e}{p}\,p\,\mathrm{d}R = \int_0^{\bar{R}} p_e\,\mathrm{d}R = \frac{\mathrm{d}}{\mathrm{d}e}(P(\bar{R}) - P(0))$$
$$= \frac{\mathrm{d}}{\mathrm{d}e}(1 - 0) = 0.$$

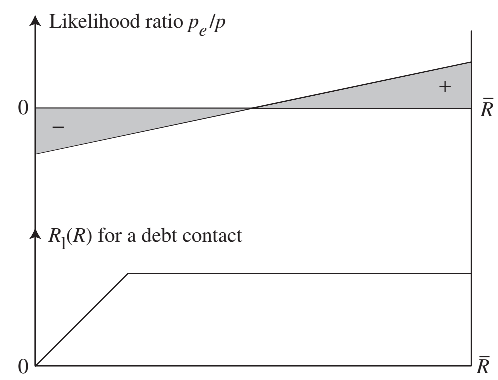

Figure 3.7

Actually, as p_e/p is strictly increasing and $R_1(\cdot)$ cannot in general be constant without violating the entrepreneur's limited liability constraint,[49]

$$V'(\hat{e}) > 0.$$

This means that at the margin, entrepreneurial effort exerts a strictly positive externality on the investors. Because the equilibrium effort is necessarily privately optimal for the entrepreneur, the effort is socially suboptimal.

In order to *minimize the externality* of the entrepreneur's effort choice on investors' welfare (that is, in order to make the entrepreneur as accountable as possible), one must give as much income as possible to investors for low profit and as little as possible for high profit, subject to $R_1(\cdot)$ being nondecreasing. Simple computations show that this is obtained for a debt contract. The intuition is provided in Figure 3.7.

That is, a debt contract maximizes entrepreneurial incentives, although it in general results in inefficiently low effort relative to the first best. Hence, it yields the preferred outcome, given that renegotiation results in efficient risk sharing.

Like Hermalin and Katz's, Dewatripont et al.'s result relies on the entrepreneur's having the full bargaining power in the renegotiation process. Dewatripont et al. show that the entrepreneur may exert an effort above the first-best level under a debt contract when the investors have bargaining power

49. The investors would receive a constant $R_1 = I - A$; and so, if $I > A$ and the minimum profit is 0, the entrepreneur must have a negative income for low profits.

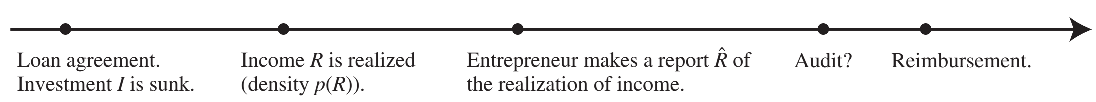

Figure 3.8

in the renegotiation process.[50] Interestingly and relatedly, the investors may be made worse off by a higher effort choice by the entrepreneur. While they are made better off by such a choice in the absence of renegotiation (from the monotonicity of their claim), a higher effort also strengthens the entrepreneur's status quo point in the renegotiation, which may hurt the investors if the latter have the bargaining power.

3.7 Semiverifiable Income

This section reviews the costly state verification (CSV) model of Townsend (1979), Diamond (1984), and Gale and Hellwig (1985).[51] While the earlier literature posited, rather than derived, specific financial structures, Townsend's contribution was the first to obtain a financial structure from an optimization problem, and therefore from primitive assumptions.

As we discussed earlier, the CSV model presumes that diversion of income takes the form of hiding income rather than enjoying a private benefit or reducing one's effort. The lenders can perfectly verify income, but only by incurring an audit cost K.[52] This cost is borne by the lenders, since the borrower (optimally) invests her net worth A in the project—as in the moral-hazard, verifiable-income model—and no longer has money to pay for the audit cost. Given borrower's net worth A and investment cost I, the lenders must invest $I - A$ in the project. This investment yields a random income R distributed on $[0, \infty)$, say, according to density $p(R)$. This income is costlessly observed by the borrower. Note that we do not introduce any moral-hazard conditioning the distribution of R; for, the semiverifiability of income already creates scope for diversion. The timing of the CSV model is described in Figure 3.8.

The revelation principle[53] states that, in designing the loan agreement, there is no loss of generality involved in focusing on contracts that require the entrepreneur to report income, and, furthermore, that the contract can be structured, again without loss of generality, so that the entrepreneur has an incentive to report the true realized income ($\hat{R} = R$).

A *contract* specifies for each report $\hat{R}$ a probability $y(\hat{R}) \in [0, 1]$ of no audit, and nonnegative rewards $w_0(\hat{R}, R)$ and $w_1(\hat{R}, R)$ in the absence and presence of audit; note that the investors' return R_l can depend only on the report in the absence of audit: $w_0(\hat{R}, R) = R - R_l(\hat{R})$, and can be made contingent on the true income as well when an audit takes place.

For an arbitrary contract, let

$$w(R) \equiv y(R)w_0(R, R) + (1 - y(R))w_1(R, R)$$

denote the borrower's expected reward when realized income is R.

A *standard debt contract* specifies a debt level D, no audit if D is repaid, and an audit and no reward

50. It is still the case that debt provides the greatest incentives. Debt may induce the entrepreneur to work too hard in order to lower the probability that the realized output is low.

On the other hand, the first best can often be achieved through a different type of contract in conformity with the general results of Maskin (1977) in the absence of renegotiation, and of Maskin and Moore (1999) and Segal and Whinston (2002) in the presence of renegotiation. The limited liability, monotonicity, and no-third-party assumptions, however, put a limit on what can be achieved through elicitation schemes. Dewatripont et al. show that either the first best is implementable, or, if it is not, debt is an optimal contract.

51. See also Williamson (1986). As in the rest of this chapter, this section presumes "universal risk neutrality." In Townsend (1979), the borrower may be risk averse. Two-sided risk aversion is studied in Krasa and Villamil (1994) and Winton (1995).

Winton (1995) also introduces multiple investors by assuming that (a) each investor can invest less than the total funding need $(I - A)$ and (b) investors conduct separate audits. One of his main results is that an absolute-priority rule is optimal even with symmetric lenders, in particular, because it avoids a complete duplication of verification costs.

52. Diamond (1984) interprets K as a nonpecuniary penalty imposed on the borrower rather than as an audit cost. One possible interpretation is that the debtor goes to jail if she does not repay her debt. Lacker (1992) provides a different interpretation of the nonmonetary cost. In his model, the optimal contract is a debt contract in which the borrower transfers collateral which she values more than the lenders (see Section 4.3) in the case of default. We will stick to the audit cost interpretation for the purpose of the exposition.

53. See, for example, Fudenberg and Tirole (1991, Chapter 7) for a presentation of the revelation principle and of mechanism design.

if it is not. So, $y(R) = 1$ if $R \geqslant D$ and $y(R) = 0$ if $R < D$, and $w(R) = \max(R - D, 0)$.

The optimal contract maximizes the borrower's expected income subject to the incentive constraint that the borrower reports the truth and the break-even constraint for the investors.

Program IV:

$$\max_{\{y(\cdot),w_0(\cdot),w_1(\cdot,\cdot)\}} \left\{ \int_0^\infty w(R)p(R)\,\mathrm{d}R \right\}$$

s.t.

$$w(R) = \max_{\hat{R}}\{y(\hat{R})w_0(\hat{R},R) + (1 - y(\hat{R}))w_1(\hat{R},R)\}, \tag{IC$_\mathrm{b}$}$$

$$\int_0^\infty [R - w(R) - [1 - y(R)]K]p(R)\,\mathrm{d}R \geqslant I - A. \tag{IR$_\mathrm{l}$}$$

Note that, since ($\mathrm{IR_l}$) will be binding at the optimum and thus can be added to the objective function, Program IV is equivalent to that of minimizing the expected audit cost

$$\left[\int_0^\infty [1 - y(R)]p(R)\,\mathrm{d}R\right]K$$

subject to ($\mathrm{IC_b}$) and ($\mathrm{IR_l}$).

The following assumption substantially simplifies the analysis and, as we will see, underlies the optimality of a standard debt contract.

Assumption (deterministic audit):

$$y(R) = 0 \textit{ or } 1 \textit{ for all } R.$$

The deterministic audit assumption divides the set of feasible incomes into two regions $\mathcal{R}_0$ and $\mathcal{R}_1$ (such that $\mathcal{R}_0 \cap \mathcal{R}_1 = \varnothing$ and $\mathcal{R}_0 \cup \mathcal{R}_1 = [0, \infty)$), labeled respectively the no-audit and the audit regions. The assumption further implies that the reimbursement, $R - w(R)$, is constant over the no-audit region; indeed, suppose that the reimbursement is higher for R' than for R, where R' and R both belong to $\mathcal{R}_0$. For income R', the borrower would be better off pretending income is R and reimbursing less. The lenders, who do not audit when reported income is R, are then unable to detect misreporting. So, the reimbursement, D say, is constant over $\mathcal{R}_0$. And $\mathcal{R}_0 \subseteq [D, \infty)$. The same reasoning also implies that the reimbursement for an R in $\mathcal{R}_1$ cannot exceed D: if it did, then $R - w(R) > D$ and the borrower would be better off reporting an income in $\mathcal{R}_0$.

Let us now show that *for any contract satisfying* ($\mathrm{IC_b}$) *and* ($\mathrm{IR_l}$)*, there exists a standard debt contract that does at least as well for the borrower.* The proof is in two steps. First, we show that for an arbitrary contract, there exists a first debt contract that pays out more to lenders at a smaller audit cost. Second, we show that there exists a second debt contract for which the lenders break even and which involves an even smaller audit cost. These two steps imply that, comparing the second debt contract to the initial contract, both the audit cost and the lenders' payoff are (weakly) smaller in the second debt contract and therefore the borrower is (weakly) better off under the second debt contract than under the initial contract.

So consider an arbitrary contract (which is incentive compatible and individually rational for the lenders). Let $\mathcal{R}_0$ and $\mathcal{R}_1$ denote the no-audit and audit regions and let D denote the repayment in the no-audit region. We know that $\mathcal{R}_0 \subseteq [D, \infty)$. Construct a first debt contract, in which the repayment is D as well. Its no-audit and audit regions are defined by $\mathcal{R}_0^* = [D, \infty)$ and $\mathcal{R}_1^* = [0, D)$. The borrower receives nothing in the latter, no-audit region. Because $\mathcal{R}_0 \subseteq \mathcal{R}_0^*$, the expected audit cost is smaller under this first debt contract. Let us next show that repayment to lenders is (weakly) larger under the new debt contract. For $R \in \mathcal{R}_0$, this repayment is the same, namely, D. For $R \in \mathcal{R}_1 \cap \mathcal{R}_0^*$, the repayment is at most D under the initial contract and equal to D under the new debt contract. For $R \in \mathcal{R}_1 \cap \mathcal{R}_1^*$, the lenders' payoff is $R - K$ under the new debt contract and therefore cannot be larger under the initial contract. This concludes the first step of the proof.

The second step is straightforward. Suppose that the first debt contract leaves a strictly positive surplus to the lenders (it cannot leave a negative surplus from the first step of the proof and from the fact that they at least break even under the initial contract). Then, there exists $D' < D$ such that the lenders' expected net payoff

$$[1 - P(D')]D' + \int_0^{D'} Rp(R)\,\mathrm{d}R - P(D')K - (I - A)$$

is equal to 0 (where $P(\cdot)$ denotes the cumulative distribution corresponding to density $p(\cdot)$). This second debt contract, with nominal debt D', involves a lower audit cost than the first debt contract

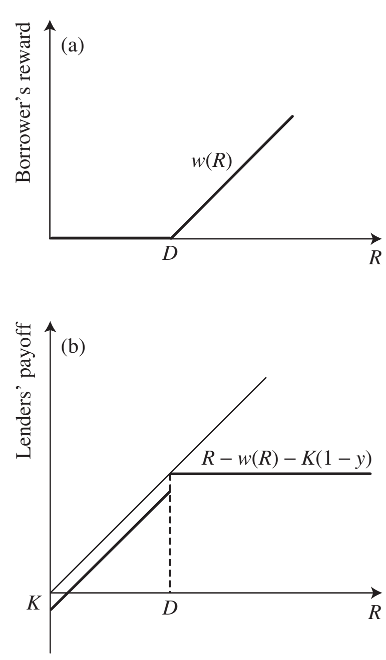

Figure 3.9

$(P(D')K < P(D)K)$ and leaves no surplus to the lenders. It is therefore preferred by the borrower to the initial contract. This concludes the proof of Townsend's classic result.

The state-contingent payoffs under a standard debt contract with debt D are depicted in Figure 3.9.

Random audits. Townsend (1979) pointed out that a debt contract is in general no longer optimal when random audits (a standard feature of taxation and insurance institutions) are allowed. We refer the reader to Mookherjee and P'ng (1989) for a general analysis of random audits (see also Border and Sobel 1987). We here content ourselves with an illustration of the benefit of random audits for the two-outcome case. Suppose that the project yields $R^{\rm S}$ (in the case of success) or $R^{\rm F}$ (in the case of failure), where $R^{\rm S} > R^{\rm F} > 0$. The pledgeable income is maximized, and the probability of an audit minimized, if the full income in the case of failure goes to the lenders when the borrower reports a failure and there turns out to be no audit (this can be seen most clearly from condition (3.18) below). For a given debt level D such that $R^{\rm F} < D < R^{\rm S}$, incentive compatibility is ensured by a probability of no audit $y^{\rm F}$ in the case of a report of the low income, where

$$R^{\rm S} - D = y^{\rm F}(R^{\rm S} - R^{\rm F}). \tag{3.17}$$

Thus, to the extent that the debt level is smaller than the higher income, there is no need to audit with probability 1. Since the optimal deterministic audit would have $y^{\rm F} = 0$, we conclude that a random audit economizes on audit costs.

If p denotes the probability of $R^{\rm S}$, then the break-even condition for the lenders is

$$pD + (1-p)[R^{\rm F} - (1-y^{\rm F})K] = I - A. \tag{3.18}$$

Renegotiation. Gale and Hellwig (1989) observe that, to the extent that audit is not a mechanical exercise triggered by the report, the threat to audit in the case of a small report may not be credible. They conclude that the possibility of renegotiation undoes the optimality of standard debt contracts and reduces welfare.

The basic insight is that the audit's raison d'être is to induce truthful reporting and therefore that the audit no longer serves a purpose once the borrower has reported her income. The borrower and the lenders are then tempted to renegotiate in order to economize on the audit cost if the contract specifies that the firm is audited for the report made by the borrower. However, the anticipation of the absence of audit after renegotiation undermines the borrower's incentive to report truthfully.[54]

To obtain some intuition as to why renegotiation is an issue, consider a standard debt contract with debt level D and suppose that the borrower is expected to pay back D whenever $R \geqslant D$. Suppose that the borrower says that she is not able to repay D but offers to repay $D - K$. The lenders should then be happy to forgo the audit and receive $D - K$ because they will never receive more if they audit. On the other hand, such debt forgiveness cannot be equilibrium behavior either, since the borrower then has an incentive to ask for debt forgiveness even when $R > D$. As this rather loose reasoning suggests, the equilibrium analysis is complex and requires a good knowledge of the theory of dynamic

54. Renegotiation is always welfare-reducing when the initial contract is complete, as is the case in Townsend's analysis. Renegotiation only adds further constraints to the mechanism design.

games with incomplete information. A full analysis thus lies outside the scope of these notes.[55]

Interpretation of the CSV model. Although the CSV model is elementary, its interpretation requires some thinking through. An implicit assumption is that the borrower can withdraw nothing from the cash register before the audit, but can fully withdraw the residual income after repayment if there has been no audit. One interpretation of the model is that the borrower can actually steal the income, but cannot consume it and must refund it if an audit takes place. An alternative interpretation is that the entrepreneur can, over time, transform the hidden income into (utility-equivalent) perks; the entrepreneur can enjoy these perks only if the firm is not shut down. The audit decision is then interpreted as a bankruptcy process, in which the lenders recoup the value of the assets in the firm.[56]

3.8 Nonverifiable Income

Let us conclude this review of alternative models of credit rationing with the polar case in which the borrower's income cannot be observed even through an audit. That is, the borrower can consume this income with complete impunity. As we observed, the borrower's incentive to repay can then only result from a threat of termination or nonfinancing of future projects. Bolton and Scharfstein (1990) and Hart and Moore (1989) (with Bolton and Scharfstein (1996) and Gromb (1994) extending their analysis) have constructed such models in which the borrower repays under the threat of termination.

There are two dates. The date-1 investment I yields income R_1 with probability p and 0 with probability $1 - p$ (as for the CSV model, to which this model is somewhat akin as we will see, there is no need to introduce a dependence of p on entrepreneurial effort since the nonverifiability of income allows for strategic nonrepayment and therefore for moral hazard). At date 2, the initial investment, if not terminated, yields expected income R_2 to the entrepreneur. Since date 2 is the last period in this model, the entrepreneur repays nothing at date 2 (as long as return 0 belongs to the support of the distribution of the second-period income, which we will assume). Thus we may as well treat R_2 as if it were a deterministic private benefit of continuation for the entrepreneur. If the project is liquidated at the end of date 1, the lenders receive liquidation value L, $0 \leqslant L < I - A$, the entrepreneur receives nothing at date 2 (in some contributions, L is equivalently interpreted not as the liquidation value, but rather as the savings associated with not incurring a second-period investment yielding R_2). Assume $L < R_2$, so liquidation is inefficient. Lastly, we will assume for expositional simplicity that there is no discounting between dates 1 and 2.

Let us now look for an optimal contract, that is, the contract that maximizes the borrower's expected payoff subject to incentive compatibility and to the constraint that the investors break even. The entrepreneur obviously repays nothing when the first-period income is equal to 0. Let $y_0 \in [0, 1]$ denote the probability of continuation when there is no repayment at date 1 (so $1 - y_0$ is the probability of termination). Consider a contract that specifies a repayment equal to $D \leqslant R_1$ when the first-period income is R_1, together with a probability y_1 of continuation if D is repaid.

The payment of D when the first-period income is R_1 must be incentive compatible, or

$$R_1 - D + y_1 R_2 \geqslant R_1 + y_0 R_2 \iff (y_1 - y_0) R_2 \geqslant D.$$

In words, the increase in the probability of termination due to nonrepayment must offset the loss in income D for the entrepreneur.

55. Another relevant contribution is that of Krasa and Villamil (2000). They assume, among other things, that the investors cannot commit to spend the fixed inspection cost. The investors thus decide whether to enforce the initial contract after observing the entrepreneur's payment. The key result is that enforcement must then be deterministic and the optimal time-consistent contract is a simple debt contract.

56. Chang (1993) builds a model of payout policy that is closely related to both the moral-hazard model in the text and the CSV model. There are three dates rather than two. Investment and financing occur at date 0. Some random income accrues at date 1, which is observed solely by the manager. The manager selects an allocation of this income between a payout to investor and a low-yield reinvestment (called in the paper "over-consumption" or "on-the-job spending"). This reinvestment increases date-2 income, but by less than a date-1 dividend of the same magnitude; in contrast it yields a private benefit at date 1. It is then optimal to link compensation to the payout so as to avoid excessive reinvestment. Chang derives conditions under which the optimal contract can be implemented through a debt contract, according to which investors can seize control when the contractually specified payment to investors is not made at date 1 and have then incentives to pay an audit cost in order to measure the date-1 income.

The optimal contract thus solves the following program.

Program V:

$$\max_{\{y_0, y_1, D \leqslant R_1\}} \{p(R_1 - D + y_1 R_2) + (1-p)(y_0 R_2)\}$$

s.t.

$$(y_1 - y_0)R_2 \geqslant D, \qquad (\mathrm{IC_b})$$

$$p[D + (1-y_1)L] + (1-p)(1-y_0)L \geqslant I - A. \quad (\mathrm{IR_l})$$

To avoid considering multiple cases, let us assume that R_1 is "sufficiently large" so that the constraint $D \leqslant R_1$ is not binding. (We will later provide a condition for this to be the case.) We first note that the breakeven constraint ($\mathrm{IR_l}$) is binding. Otherwise, the debt D could be lowered while keeping the two constraints satisfied (and ($\mathrm{IR_l}$) implies that D cannot be equal to 0 since $L < I - A$).

Second, note that $y_1 = 1$ (there is no liquidation in the case of repayment); for, assume that $1 > y_1 > y_0$. Increase y_1 by a small amount $\varepsilon > 0$, and raise D by εL so as to keep ($\mathrm{IR_l}$) satisfied. Note that the incentive constraint remains satisfied as $R_2 > L$. The borrower's utility increases by $p(R_2 - L)\varepsilon > 0$. In words, liquidating in the case of repayment is bad both for efficiency (liquidation is always inefficient) and for incentives.

Third, the incentive constraint must be binding. Note that y_0 must be lower than 1 in order for it to be satisfied (there would never be a repayment if there were no threat of liquidation in the case of nonrepayment). If the incentive constraint is not binding, raise y_0 by a small $\varepsilon > 0$, and increase D by $\varepsilon L(1-p)/p$, so as to keep ($\mathrm{IR_l}$) satisfied. The borrower's welfare increases by

$$-p\left[\frac{\varepsilon L(1-p)}{p}\right] + (1-p)\varepsilon R_2 = (1-p)(R_2 - L)\varepsilon > 0.$$

Using these results, we conclude that $y_1 = 1$ and D and y_0 solve

$$(1 - y_0)R_2 = D \qquad (3.19)$$

and

$$pD + (1-p)(1-y_0)L = I - A. \qquad (3.20)$$

And so the probability of liquidation in the absence of repayment is

$$1 - y_0 = \frac{I - A}{pR_2 + (1-p)L}. \qquad (3.21)$$

Following Bolton and Scharfstein and Hart and Moore, we have thus formalized the idea that *the threat of termination provides incentives for repayment when income is nonverifiable.*

Some interesting comparative statics results emerge from (3.21). Termination is less likely in the case of nonrepayment if

- the value of continuing (R_2) increases (the borrower then has more to lose from being terminated and the probability of termination can be reduced),
- the liquidation value L increases (the lenders obtain more money when liquidating and therefore can liquidate less often and still recoup their investment),
- the probability p of first-period success increases (the lenders are then repaid often), and
- the borrower's net worth A increases.

Povel and Raith (2004) extend Bolton and Scharfstein's model by allowing for a noncontractible choice of investment level in the first period. In their model, the date-1 revenue is continuous and takes value $\theta z(J) + [I - J]$, where θ is a random variable, $J \leqslant I$ is the actual investment secretly chosen by the entrepreneur, $z(J)$ the concave production function, and $I - J$ the noninvested funds (which are not diverted). Because a debt contract maximizes the entrepreneur's incentive to take risk, the entrepreneur ends up investing all the funds that are made available to her by the investors ($J = I$). And so debt remains the optimal contract.[57]

Relationship to the CSV model. This model is closely related to the CSV model. In both cases lenders cannot be repaid (at least if the lowest possible income is 0) unless they undertake some wasteful action. The counterpart to the audit cost K in the CSV model is the waste in second-period value, $R_2 - L$, in the nonverifiable income model. Indeed, in the two-outcome case, the incentive constraints (3.17) (taken for $R^{\mathrm{S}} = R$ and $R^{\mathrm{F}} = 0$) and (3.19) are identical. There are some differences between the two models, though. The cost of the wasteful activity

57. Povel and Raith also consider various extensions in which entrepreneurial moral hazard takes different forms. For example, they show that a simple debt contract may no longer be optimal when the entrepreneur chooses how much effort to exert or the project's riskiness rather than how much of the funds to invest.

(audit, liquidation) is borne by the lenders in the CSV model and by the borrower in the nonverifiable income model. In a world in which some agent (here, the borrower) is cash constrained, who bears the cost matters, which accounts for a small discrepancy between the breakeven conditions (3.18) and (3.20). We should also point out that the CSV model is notoriously difficult to extend to a multiperiod context (see Chang 1990; Snyder 1994; Webb 1992), while the nonverifiable income model can be more straightforwardly extended (see Gromb 1994).

Relationship to costly collateral pledging. The next chapter will argue that firms can boost pledgeable income and facilitate financing by pledging collateral in the case of default. Collateral pledging serves two purposes. First, it incentivizes management to repay investors. Second, it boosts pledgeable income. But collateral pledging is costly to the extent that lenders may value the collateral less than the borrower and so transferring it to lenders involves a deadweight loss. The Bolton–Scharfstein model can be viewed as a special case of costly collateral pledging. The collateral is the date-2 project. The lenders' gain, L from "seizing the collateral," i.e., taking the control over the decision to continue away from the borrower, is lower than the value, R_2, accruing to the borrower when continuing at date 2.

Renegotiation. As for the CSV model, there has been some discussion of the impact of renegotiation in the literature on nonverifiable income.

Consider first renegotiation *after* "liquidation" has taken place. For such renegotiation to make sense, one must adopt the interpretation of "liquidation" as the "nonfinancing of a second-period investment $I_2 = L$ that allows the borrower to receive expected income R_2 in period 2," and not as a (possibly piecewise) resale of the firm's assets. Even though financing the second-period investment increases total surplus by $R_2 - L$, no such financing occurs unless it is specified by the initial contract. The lenders do not want to bring in money at date 2 since they will not be repaid anything. So, a contract that specifies liquidation is *renegotiation proof* in the two-period model. Incidentally, it is no longer renegotiation proof with more than two periods, as was shown by Gromb (1994). For example, at date 2 the lenders may anticipate to be repaid at the end of date 2 through the threat of noncontinuation at date 3. Gromb characterizes the equilibrium outcomes with renegotiation.[58]

Second, consider renegotiation after the termination decision has been made (the borrower has defaulted, and the draw of the random variable has indicated liquidation), but before it is implemented. Suppose that the borrower at that point in time offers to the lenders a bribe slightly above L for not liquidating. Although this offer demonstrates that the borrower has strategically defaulted (otherwise, she would have no money), the lenders should be eager to accept. This in turn encourages strategic default and undermines the efficiency of the debt contract.[59]

Notice again the analogy with the CSV model. In both cases, a wasteful action (audit, liquidation) by the lenders serves as an incentive device in order to induce the borrower to pay out income. Once this income has been paid, though, the wasteful action no longer serves a purpose and the parties are better off renegotiating to avoid the corresponding efficiency loss. The prospect of renegotiation, however, *ex ante* eliminates incentives to pay out income, and reduces welfare overall. We again refer the reader to the original articles for more details about the impact of renegotiation.

Relation to the sovereign debt literature. The strategic default literature is closely linked to that on sovereign borrowing in international finance. Repayment of debt by the sovereign responds to two incentives: international sanctions and the future cost of being shut down from the international capital market after default. A subliterature, starting with Bulow and Rogoff (1989a,b), assumes away sanctions and focuses on the incentives provided by exclusion. In this literature, future refinancing (or the lack

58. To do so, he rules out retained earnings by the borrower (an assumption labeled the "fresh tomato assumption," by reference to the hypothesis that the borrower is not able to carry over resources for investment in future periods). He shows that even a monopoly lender may make no profit when the horizon is long. The intuition for this result is that if the lender enjoys a rent from continuation the borrower can safely default as the lender will always be eager to renegotiate after termination.

59. The extent of renegotiation as well as the sharing of the *ex post* gains from trade may depend on the number of lenders. See Bolton and Scharfstein (1996) for an analysis of the impact of lender dispersion on the optimal contract.

thereof) must be self-sustaining rather than contracted upon. The basic mechanism is otherwise similar to the Bolton–Scharfstein mechanism, in that lenders cannot appropriate any of the current return and count solely on the nonrefinancing threat to recoup their investment. Bulow and Rogoff consider an infinite-horizon, symmetric-information model in which (a) the sovereign can decide not to reimburse and (b) the sovereign can save, and (c) the rate of growth of the economy is smaller than the rate of interest. They show that no lending is feasible as the borrower always prefers to default (and save some of the concomitant extra income).

Several contributions have shown that borrowing may be feasible in more general no-sanction environments. First, Hellwig and Lorenzoni (2004) show that when the rate of growth in the absence of sovereign borrowing exceeds the rate of interest, then sovereign debt borrowing is feasible, even though incentive-compatible repayments still require borrowing levels below the first-best level. Intuitively, exclusion from borrowing is a stronger threat when the rate of growth is large relative to the rate of interest. Second, an outright exclusion, in which the defaulting sovereign cannot even save, makes it particularly costly for the sovereign to repudiate its debt. Again, some sovereign debt may then be issued in equilibrium (Kehoe and Levine 1993; Kocherlakota 1996). Finally, standard "type-based" reputation models (see, for example, Kreps et al. 1982) would deliver some equilibrium borrowing.

3.9 Exercises

Exercise 3.1 (random financing). Consider the fixed-investment model of Section 3.2. We know that if $A \geqslant \bar{A}$, where

$$I - \bar{A} = p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right),$$

it is both optimal and feasible for the borrower to sign a contract in which the project is undertaken for certain. We also noted that for $A < \bar{A}$, the borrower cannot convince investors to undertake the project with probability 1. With $A > 0$, the entrepreneur benefits from signing a "random financing contract," though.

(i) Consider a contract in which the borrower invests $\hat{A} \in [0, A]$ of her own money, the project is financed with probability x, and the borrower receives R_{b} in the case of success and 0 otherwise. Write the investors' breakeven condition.

(ii) Show that (provided the NPV, $p_{\mathrm{H}}R - I$, is positive) it is optimal for the borrower to invest

$$\hat{A} = A.$$

How does the probability that the project is undertaken vary with A?

Exercise 3.2 (impact of entrepreneurial risk aversion). Consider the fixed-investment model developed in this chapter: an entrepreneur has cash amount A and wants to invest $I > A$ into a project. The project yields $R > 0$ with probability p and 0 with probability $1 - p$. The probability of success is p_{H} if the entrepreneur works and $p_{\mathrm{L}} = p_{\mathrm{H}} - \Delta p$ ($\Delta p > 0$) if she shirks. The entrepreneur obtains private benefit B if she shirks and 0 otherwise. Assume that

$$I > p_{\mathrm{H}}\left(R - \frac{B}{\Delta p}\right).$$

(Suppose that $p_{\mathrm{L}}R + B < I$; so the project is not financed if the entrepreneur shirks.)

(i) In contrast with the risk-neutrality assumption of this chapter, assume that the entrepreneur has utility for consumption c:

$$u(c) = \begin{cases} c & \text{if } c \geqslant c_0, \\ -\infty & \text{otherwise.} \end{cases}$$

(Assume that $A \geqslant c_0$ to ensure that the entrepreneur is not in the "$-\infty$ range" in the absence of financing.)

Compute the minimum equity level $\bar{A}$ for which the project is financed by risk-neutral investors when the market rate of interest is 0. Discuss the difference between $p_{\mathrm{H}} = 1$ and $p_{\mathrm{H}} < 1$.

(ii) Generalize the analysis to risk aversion. Let $u(c)$ denote the entrepreneur's utility from consumption with $u' > 0$, $u'' < 0$. Conduct the analysis assuming either limited liability or the absence of limited liability.

Exercise 3.3 (random private benefits). Consider the variable-investment model: an entrepreneur ini-

Figure 3.10

tially has cash A. For investment I, the project yields RI in the case of success and 0 in the case of failure.

The probability of success is equal to $p_{\mathrm{H}} \in (0,1)$ if the entrepreneur works and $p_{\mathrm{L}} = 0$ if the entrepreneur shirks. The entrepreneur obtains private benefit BI when shirking and 0 when working. The per-unit private benefit B is unknown to all *ex ante* and is drawn from (common knowledge) uniform distribution F:

$$\Pr(B < \hat{B}) = F(\hat{B}) = \hat{B}/R \quad \text{for } \hat{B} \leqslant R,$$

with density $f(\hat{B}) = 1/R$. The entrepreneur borrows $I - A$ and pays back $R_{\mathrm{l}} = r_{\mathrm{l}} I$ in the case of success. The timing is described in Figure 3.10.

(i) For a given contract (I, r_{l}), what is the threshold B^*, i.e., the value of the private per-unit benefit above which the entrepreneur shirks?

(ii) For a given B^* (or equivalently r_{l}, which determines B^*), what is the debt capacity? For which value of B^* (or r_{l}) is this debt capacity highest?

(iii) Determine the entrepreneur's expected utility for a given B^*. Show that the contract that is optimal for the entrepreneur (subject to the investors breaking even) satisfies

$$\tfrac{1}{2} p_{\mathrm{H}} R < B^* < p_{\mathrm{H}} R.$$

Interpret this result.

(iv) Suppose now that the private benefit B is observable and verifiable. Determine the optimal contract between the entrepreneur and the investors (note that the reimbursement can now be made contingent on the level of private benefits: $R_{\mathrm{l}} = r_{\mathrm{l}}(B) I$).

Exercise 3.4 (product-market competition and financing). Two firms, $i = 1, 2$, compete for a new market. To enter the market, a firm must develop a new technology. It must invest (a fixed amount) I. Each firm is run by an entrepreneur. Entrepreneur i has initial cash $A_i < I$. The entrepreneurs must borrow from investors at expected rate of interest 0. As in the single-firm model, an entrepreneur enjoys private benefit B from shirking and 0 when working. The probability of success is p_{H} and $p_{\mathrm{L}} = p_{\mathrm{H}} - \Delta p$ when working and shirking.

The return for a firm is

$$R = \begin{cases} D & \text{if both firms succeed in developing the} \\ & \text{technology (which results in a duopoly),} \\ M & \text{if only this firm succeeds (and therefore} \\ & \text{enjoys a monopoly situation),} \\ 0 & \text{if the firm fails,} \end{cases}$$

where $M > D > 0$.

Assume that $p_{\mathrm{H}}(M - B/\Delta p) < I$. We look for a Nash equilibrium in contracts (when an entrepreneur negotiates with investors, both parties correctly anticipate whether the other entrepreneur obtains funding). In a first step, assume that the two firms' projects or research technologies are independent, so that nothing is learned from the success or failure of the other firm concerning the behavior of the borrower.

(i) Show that there is a cutoff $\underline{A}$ such that if $A_i < \underline{A}$, entrepreneur i obtains no funding.

(ii) Show that there is a cutoff $\bar{A}$ such that if $A_i > \bar{A}$ for $i = 1, 2$, both firms receive funding.

(iii) Show that if $\underline{A} < A_i < \bar{A}$ for $i = 1, 2$, then there exist two (pure-strategy) equilibria.

(iv) The previous questions have shown that when investment projects are independent, product-market competition makes it *more difficult* for an entrepreneur to obtain financing. Let us now show that when projects are correlated, product-market competition may *facilitate financing* by allowing financiers to benchmark the entrepreneur's performance on that of competing firms.

Let us change the entrepreneur's preferences slightly:

$$u(c) = \begin{cases} c & \text{if } c \geqslant c_0, \\ -\infty & \text{otherwise.} \end{cases}$$

That is, the entrepreneur is infinitely risk averse below c_0 (this assumption is stronger than needed, but it simplifies the computations).

Suppose, first, that only one firm can invest. Show that the necessary and sufficient condition for investment to take place is

$$p_{\mathrm{H}}\left(M - \frac{B}{\Delta p}\right) - c_0 \geqslant I - A.$$

(v) Continuing on from question (iv), suppose now that there are two firms and that their technologies are *perfectly correlated* in that if both invest and both entrepreneurs work, then they both succeed or both fail. (For the technically oriented reader, there exists an underlying state variable ω distributed uniformly on $[0, 1]$ and common to both firms such that a firm always succeeds if $\omega < p_L$, always fails if $\omega > p_H$, and succeeds if and only if the entrepreneur works when $p_L < \omega < p_H$.)

Show that if

$$p_H D - c_0 \geqslant I - A,$$

then it is an equilibrium for both entrepreneurs to receive finance. Conclude that product-market competition may facilitate financing.

Exercise 3.5 (continuous investment and decreasing returns to scale). Consider the continuous-investment model, with one modification: investment I yields return $R(I)$ in the case of success, and 0 in the case of failure, where $R' > 0$, $R'' < 0$, $R'(0) > 1/p_H$, $R'(\infty) < 1/p_H$. The rest of the model is unchanged. (The entrepreneur starts with cash A. The probability of success is p_H if the entrepreneur behaves and $p_L = p_H - \Delta p$ if she misbehaves. The entrepreneur obtains private benefit BI if she misbehaves and 0 otherwise. Only the final outcome is observable.) Let I^* denote the level of investment that maximizes total surplus: $p_H R'(I^*) = 1$.

(i) How does investment $I(A)$ vary with assets?

(ii) How does the shadow value v of assets (the derivative of the borrower's gross utility with respect to assets) vary with the level of assets?

Exercise 3.6 (renegotiation and debt forgiveness). When computing the multiplier k (given by equation (3.12)), we have assumed that it is optimal to specify a stake for the borrower large enough that the incentive constraint (IC_b) is satisfied. Because condition (3.8) implies that the project has negative NPV in the case of misbehavior, such a specification is clearly optimal when the contract cannot be renegotiated. The purpose of this exercise is to check in a rather mechanical way that the borrower cannot gain by offering a loan agreement in which (IC_b) is not satisfied, and which is potentially renegotiated before the borrower chooses her effort. While there is a more direct way to prove this result, some insights are gleaned from this pedestrian approach. Indeed, the exercise provides conditions under which the lender is willing to forgive debt in order to boost incentives (the analysis will bear some resemblance to that of liquidity shocks in Chapter 5, except that the lender's concession takes the form of debt forgiveness rather than cash infusion).[60]

(i) Consider a loan agreement specifying investment I and stake $R_b < BI/\Delta p$ for the borrower. Suppose that the loan agreement can be renegotiated after it is signed and the investment is sunk and before the borrower chooses her effort. Renegotiation takes place if and only if it is mutually advantageous. Show that the loan agreement is renegotiated if and only if

$$(\Delta p)RI - \frac{p_H BI}{\Delta p} + p_L R_b \geqslant 0.$$

(ii) Interpret the previous condition. In particular, show that it can be obtained directly from the general theory. Hint: consider a fictitious, "fixed-investment" project with income $(\Delta p)RI$, investment 0, and cash on hand $p_L R_b$.

(iii) Assume for instance that the entrepreneur makes a take-it-or-leave-it offer in the renegotiation (that is, the entrepreneur has the bargaining power). Compute the borrowing capacity when $R_b < BI/\Delta p$ and the loan agreement is renegotiated.

(iv) Use a direct, rational expectations argument to point out in a different way that there is no loss of generality in assuming $R_b \geqslant BI/\Delta p$ (and therefore no renegotiation).

Exercise 3.7 (strategic leverage). (i) A borrower has assets A and must find financing for an investment $I(\tau) > A$. As usual, the project yields R (success) or 0 (failure). The borrower is protected by limited liability. The probability of success is $p_H + \tau$ or $p_L + \tau$, depending on whether the borrower works or shirks, with $\Delta p = p_H - p_L > 0$. There is no private benefit when working and private benefit B when shirking. The financial market is competitive and the expected rate of return demanded by investors is equal to 0. It is never optimal to give incentives to shirk.

60. The phenomenon of debt renegotiation has been analyzed in a number of settings: see, for example, Bulow and Rogoff (1989a,b), Eaton and Gersovitz (1981), Fernandez and Rosenthal (1990), Gale and Hellwig (1989), Gromb (1994), Hart and Moore (1989, 1995), and Snyder (1994).

The investment cost I is an increasing and convex function of τ (it will be further assumed that $p_H R > I(0)$, that in the relevant range $p_H + \tau < 1$, and that $I'(0)$ is "small enough" so as to guarantee an interior solution). Let τ^*, A^*, and τ^{**} be defined by

$$I'(\tau^*) = R,$$
$$[p_H + \tau^*]\left[R - \frac{B}{\Delta p}\right] = I(\tau^*) - A^*,$$
$$I'(\tau^{**}) = R - \frac{B}{\Delta p}.$$

Can the borrower raise funds? If so, what is the equilibrium level τ of "quality of investment"?

(ii) Suppose now that there are two firms (that is, two borrowers) competing on this product market. If only firm i succeeds in its project, its income is (as in question (i)), equal to R (and firm j's income is 0). If the two firms succeed (both get hold of "the technology"), they compete *à la* Bertrand in the product market and get 0 each. For simplicity, assume that the lenders observe only whether the borrower's income is R or 0, rather than whether the borrower has succeeded in developing the technology (showoffs: you can discuss what would happen if the lenders observed "success/failure"!).

So, if $q_i \equiv p_i + \tau_i$ denotes the probability that firm i develops the technology (with $p_i = p_H$ or p_L), the probability that firm i makes R is $q_i(1 - q_j)$. (This assumes implicitly that projects are independent.)

Consider the following timing. (1) Each borrower simultaneously and secretly arranges financing (if feasible). A borrower's leverage (or quality of investment) is *not* observed by the other borrower. (2) Borrowers choose whether to work or shirk. (3) Projects succeed or fail.

- Let $\hat{\tau}$ be defined by
$$I'(\hat{\tau}) = [1 - (p_H + \hat{\tau})]R.$$
Interpret $\hat{\tau}$.
- Suppose that the two borrowers have the same initial net worth A. Find the lower bound $\hat{A}$ on A such that $(\hat{\tau}, \hat{\tau})$ is the (symmetric) Nash outcome.
- Derive a sufficient condition on A under which it is an equilibrium for a single firm to raise funds.

(iii) Consider the set up of question (ii), except that borrower 1 moves first and publicly chooses τ_1. Borrower 2 may then try to raise funds (one will assume either that τ_2 is secret or that borrower 1 is rewarded on the basis of her success/failure performance; this is in order to avoid strategic choices by borrower 2 that would try to induce borrower 1 to shirk). Suppose that each has net worth $\tilde{A}$ given by

$$\tilde{q}\left[(1 - \tilde{q})R - \frac{B}{\Delta p}\right] = I(\tilde{q} - p_H) - \tilde{A},$$

where $\tilde{q}$ satisfies

$$I'(\tilde{q} - p_H) = (1 - \tilde{q})R - \frac{B}{\Delta p}.$$

- Interpret $\tilde{q}$.
- Show that it is optimal for borrower 1 to choose $\tau_1 > \tilde{q} - p_H$.

Exercise 3.8 (equity multiplier and active monitoring). (i) Derive the equity multiplier in the variable-investment model. (Reminder: the investment $I \in [0, \infty)$ yields income RI in the case of success and 0 in the case of failure. The borrower's private benefit from misbehaving is equal to BI. Misbehaving reduces the probability of success from p_H to $p_L = p_H - \Delta p$. The borrower has cash A and is protected by limited liability. Assume that $\rho_1 = p_H R > 1$, $\rho_0 = p_H(R - B/\Delta p) < 1$ and $1 > p_L R + B$. The investors' rate of time preference is equal to 0.) Show that the equity multiplier is equal to $1/(1 - \rho_0)$.

(ii) Derive the equity multiplier with active monitoring: the entrepreneur can hire a monitor, who, at private cost cI, reduces the entrepreneur's private benefit from shirking from BI to $b(c)I$, where $b(0) = B$, $b' < 0$. The monitor must be given incentives to monitor (denote by R_m his income in the case of success). The monitor wants to break even, taking into account his private monitoring cost (so, there is "no shortage of monitoring capital").

- Suppose that the entrepreneur wants to induce level of monitoring c. Write the two incentive constraints to be satisfied by R_m and R_b (where R_b is the borrower's reward in the case of success).
- What is the equity multiplier?
- Show that the entrepreneur chooses c so as to maximize
$$\max_c \left\{\frac{\rho_1 - 1 - c}{1 - \rho_0 + (p_H/\Delta p)[b(c) + c - B]}\right\}.$$

Exercise 3.9 (concave private benefit). Consider the variable-investment model with a concave private benefit. The entrepreneur obtains $B(I)$ when shirking and 0 when behaving, where $B(0) = 0$, $B' > 0$, $B'' < 0$ (and $B'(0)$ large, $\lim_{I\to\infty} B'(I) = B$, where $p_{\rm H}(R - B/\Delta p) < 1$).

(i) Compute the borrowing capacity.

(ii) How does the shadow price v of the entrepreneur's cash on hand vary with A?

Exercise 3.10 (congruence, pledgeable income, and power of incentive scheme). The credit rationing model developed in this chapter assumes that the entrepreneur's and investors' interests are *a priori* dissonant, and that incentives must be aligned by giving the entrepreneur enough of a stake in the case of success.

Suppose that the entrepreneur and the investors have indeed dissonant preferences with probability x, but have naturally aligned interests with probability $1-x$. Which prevails is unknown to both sides at the financing stage and is discovered (only) by the entrepreneur just before the moral-hazard stage.

More precisely, consider the fixed-investment model of Section 3.2. The investors's outlay is $I - A$ and they demand an expected rate of return equal to 0. The entrepreneur is risk neutral and protected by limited liability. With probability x, interests are dissonant: the entrepreneur obtains private benefit B by misbehaving (the probability of success is $p_{\rm L}$) and 0 by behaving (probability of success $p_{\rm H}$). With probability $1 - x$, interests are aligned: the entrepreneur's taking her private benefit B coincides with choosing probability of success $p_{\rm H}$.

(i) Consider a "simple incentive scheme" in which the entrepreneur receives $R_{\rm b}$ in the case of success and 0 in the case of failure. $R_{\rm b}$ thus measures the "power of the incentive scheme."

Show that it may be optimal to choose a low-powered incentive scheme if preferences are rather congruent (x low) and that the incentive scheme is necessarily high-powered if preferences are rather dissonant (x high).

(ii) Show that one cannot improve on simple incentive schemes by presenting the entrepreneur with a menu of two options (two outcome-contingent incentive schemes) from which she will choose once she learns whether preferences are congruent or dissonant.

Exercise 3.11 (retained-earnings benefit). An entrepreneur has at date 1 a project of fixed size with characteristics $\{I^1, R^1, p_{\rm H}^1, p_{\rm L}^1, B^1\}$ (see Section 3.2). This entrepreneur will at date 2 have a different fixed size project with characteristics $\{I^2, R^2, p_{\rm H}^2, p_{\rm L}^2, B^2\}$, which will then require new financing. So, we are considering only a short-term loan for the first project. Retained earnings from the first project can, however, be used to defray part of the investment cost of the second project. Assume that all the characteristics of the second project are known at date 1 except B^2, which is distributed on $[\underline{B}^2, \bar{B}^2]$ according to the cumulative distribution $F(B^2)$. Assume for simplicity that $\underline{B}^2 > \Delta p^2(p_{\rm H}^2 R^2 - I^2)/p_{\rm H}^2$. The characteristics of the second project become common knowledge at the beginning of date 2.

(i) Compute the shadow value of retained earnings. (Hint: what is the entrepreneur's gross utility in period 2?)

(ii) Show that it is possible that the first project is funded even though it would not be funded if the second project did not exist and even though the entrepreneur cannot pledge at date 1 income resulting from the second project.

Exercise 3.12 (investor risk aversion and risk premia). One of the key developments in the theory of *market* finance has been to find methods to price claims held by investors. Market finance emphasizes state-contingent pricing, the fact that 1 unit of income does not have a uniform value across states of nature. This book assumes that investors are risk neutral, and so it does not matter how the pledgeable income is spread across states of nature. This assumption is made only for the sake of computational simplicity, and can easily be relaxed.

Consider a two-date model of market finance with a representative consumer/investor. This consumer has utility of consumption $u(c_0)$ at date 0, the date at which he lends to the firm, and utility of consumption $u(c(\omega))$ at date 1, date at which he receives the return from investment. There is macroeconomic uncertainty in that the representative consumer's date-1 consumption depends on the state of nature ω. The state of nature describes both what happens in this particular firm and in the rest of the economy (even though aggregate consumption is independent of the outcome in this particular firm to the extent that the firm is atomistic, which we will assume).

Suppose that the entrepreneur works. Let S denote the event "the project succeeds" and F the event "the project fails." Let

$$q_S = E\left[\frac{u'(c(\omega))}{u'(c_0)} \;\middle|\; \omega \in S\right]$$

and

$$q_F = E\left[\frac{u'(c(\omega))}{u'(c_0)} \;\middle|\; \omega \in F\right].$$

The firm's activity is said to covary positively with the economy (be "procyclical") if $q_S < q_F$, and negatively (be "countercyclical") if $q_F < q_S$.

Suppose that

$$p_H q_S + (1 - p_H) q_F = 1.$$

(i) Interpret this assumption.

(ii) In the fixed-investment model of Section 3.2 (and still assuming that the entrepreneur is risk neutral), derive the necessary and sufficient condition for the project to receive financing.

(iii) What is the optimal contract between the investors and the entrepreneur? Does it involve maximum punishment ($R_b = 0$) in the case of failure? How would your answer change if the entrepreneur were risk averse? (For simplicity, assume that her only claim is in the firm. She does not hold any of the market portfolio.)

Exercise 3.13 (lender market power). (i) *Fixed investment.* An entrepreneur has cash amount A and wants to invest $I > A$ into a (fixed-size) project. The project yields $R > 0$ with probability p and 0 with probability $1 - p$. The probability of success is p_H if the entrepreneur works and $p_L = p_H - \Delta p$ ($\Delta p > 0$) if she shirks. The entrepreneur obtains private benefit B if she shirks and 0 otherwise. The borrower is protected by limited liability and everyone is risk neutral. The project is worthwhile only if the entrepreneur behaves.

There is a single lender. This lender has access to funds that command an expected rate of return equal to 0 (so the lender would content himself with a 0 rate of return, but he will use his market power to obtain a superior rate of return). Assume

$$V \equiv p_H R - I > 0$$

and let $\bar{A}$ and $\hat{A}$ be defined by

$$p_H\left[R - \frac{B}{\Delta p}\right] = I - \bar{A}$$

and

$$p_H \frac{B}{\Delta p} - \hat{A} = 0.$$

Assume that $\bar{A} > 0$ and that the *lender* makes a take-it-or-leave-it offer to the borrower (i.e., the lender chooses R_b, the borrower's compensation in the case of success).

- What contract is optimal for the lender?
- Is the financing decision affected by lender market power (i.e., compared with the case of competitive lenders solved in Section 3.2)?
- Draw the borrower's *net* utility (i.e., net of A) as a function of A and note that it is nonmonotonic (distinguish four regions: $(-\infty, \bar{A})$, $[\bar{A}, \hat{A})$, $[\hat{A}, I)$, $[I, \infty)$). Explain.

(ii) *Variable investment.* Answer the first two bullets in question (i) (lender's optimal contract and impact of lender market power on the investment decision) in the variable-investment version. In particular, show that lender market power reduces the scale of investment. (Reminder: I is chosen in $[0, \infty)$. The project yields RI if successful and 0 if it fails. Shirking, which reduces the probability of success from p_H to p_L, yields private benefit BI. Assume that $p_H R > 1 > p_H(R - B/\Delta p)$. Hint: show that the two constraints in the lender's program are binding.)

Exercise 3.14 (liquidation incentives). This exercise extends the fixed-investment model of Section 3.2 by adding a signal on the profitability of the project that (a) accrues after effort has been chosen, and (b) is privately observed. (The following model is used as a building block in a broader context by Dessi (2005).)

An entrepreneur has cash A and wants to invest $I > A$ into a project. The project yields R (success) or 0 (failure) at the end. An intermediate signal reveals the probability y that the project will succeed, with $y = \bar{y}$ or $\underline{y}$ ($\bar{y} = \underline{y} + \Delta y$ and $\Delta y > 0$). The probability, p, that $y = \bar{y}$ depends on the entrepreneur's effort. If the entrepreneur behaves, then $p = p_H$ and the entrepreneur receives no private benefit. If the entrepreneur misbehaves, then $p = p_L$

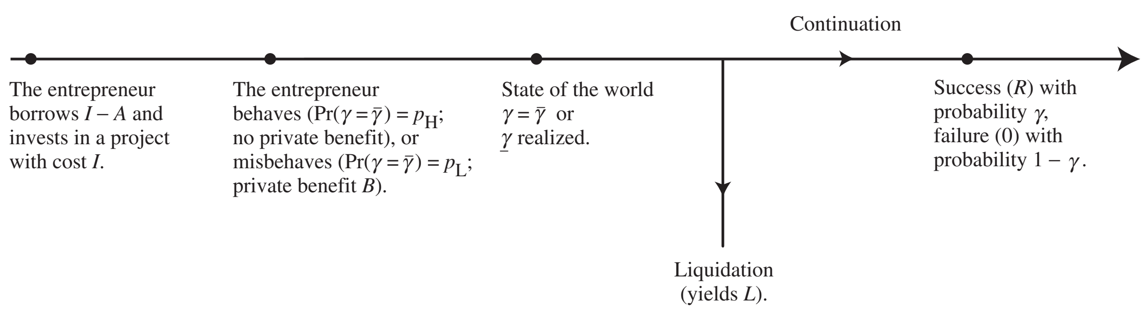

Figure 3.11

and the entrepreneur receives private benefit B. Investors and entrepreneur are risk neutral and the latter is protected by limited liability. The competitive rate of return is equal to 0.

Introduce further an option to liquidate after the signal is realized but before the final profit accrues. Liquidation yields L, and L is entirely pledgeable to investors.

One will assume that

$$\bar{\gamma}R > L > \underline{\gamma}R,$$

so that it is efficient to liquidate if and only if the signal is bad; and that

$$p_{\mathrm{H}}\bar{\gamma}R + (1 - p_{\mathrm{H}})L > I$$

(which will imply that the NPV is positive).

Figure 3.11 summarizes the timing.

(i) Suppose first that γ is verifiable. Argue that the entrepreneur should be rewarded solely as a function of the realization of γ. What is the pledgeable income? Show that the project is financed if and only if $A \geqslant \bar{A}$, where

$$p_{\mathrm{H}}\left(\bar{\gamma}R - \frac{B}{\Delta p}\right) + (1 - p_{\mathrm{H}})L = I - \bar{A}.$$

(ii) Suppose now that γ is observed only by the entrepreneur. This implies that the entrepreneur must be induced to tell the truth about γ. Without loss of generality, consider an incentive scheme in which the entrepreneur receives R_{b} in the case she announces $\gamma = \bar{\gamma}$ (and therefore the project continues) *and* the final profit is R, L_{b} if she announces $\gamma = \underline{\gamma}$ (and therefore the project is liquidated), and 0 otherwise.

Show that the project is funded if and only if

$$A \geqslant \bar{A} + \underline{\gamma}\frac{B}{(\Delta p)(\Delta\gamma)}.$$

Exercise 3.15 (project riskiness and credit rationing). Consider the basic, fixed-investment model (the investment is I, the entrepreneur borrows $I - A$; the probability of success is p_{H} (no private benefit) or $p_{\mathrm{L}} = p_{\mathrm{H}} - \Delta p$ (private benefit B), success (failure) yields verifiable profit R (respectively 0)). There are two variants, "A" and "B," of the projects, which differ only with respect to "riskiness":

$$p_{\mathrm{H}}^{\mathrm{A}}R^{\mathrm{A}} = p_{\mathrm{H}}^{\mathrm{B}}R^{\mathrm{B}}, \quad \text{but } p_{\mathrm{H}}^{\mathrm{A}} > p_{\mathrm{H}}^{\mathrm{B}};$$

so project B is "riskier." The investment cost is the same for both variants and, furthermore,

$$p_{\mathrm{H}}^{\mathrm{A}} - p_{\mathrm{L}}^{\mathrm{A}} = p_{\mathrm{H}}^{\mathrm{B}} - p_{\mathrm{L}}^{\mathrm{B}}.$$

Which variant is less prone to credit rationing?

Exercise 3.16 (scale versus riskiness tradeoff). Consider an entrepreneur with a project of variable investment I. The entrepreneur has initial wealth A, is risk neutral, and is protected by limited liability. Investors are risk neutral and demand a rate of return equal to 0.

The project comes in two versions:

Risky. The project costs I and ends up (potentially) productive only with probability $x < 1$. The timing goes as follows. (a) The scale of investment I is selected. (b) After the investment has been sunk, news accrues as to the profitability of the project. With probability $1 - x$, the project stops and yields 0. With probability x, the project continues (without any need for reinvestment). In the latter case, (c) the entrepreneur chooses an effort; good behavior confers no private benefit on the entrepreneur and yields subsequent probability of success p_H; misbehavior confers private benefit BI and yields probability of success p_L. Finally, (d) the outcome accrues: success yields RI and failure 0.

Safe. The investment cost, XI with $X > 1$, is higher for a given size I. But the project is always productive ("$x = 1$"). The moral hazard and outcome stages are as in the case of a risky choice.

We will assume that the contract aims at inducing good behavior. Letting

$$\rho_1 \equiv p_H R \quad \text{and} \quad \rho_0 \equiv p_H\left(R - \frac{B}{\Delta p}\right),$$

one will further assume that $x > 1/\rho_1$ and $X < \rho_1$.

Assume that entrepreneur and investors contract on which version will be selected.

(i) Show that the risky version is chosen if and only if

$$xX \geqslant 1.$$

(ii) Interpret this condition in terms of a "cost of bringing 1 unit of investment to completion."

Exercise 3.17 (competitive product market interactions). There is a mass 1 of identical entrepreneurs with the variable-investment technology described in Section 3.4. The representative entrepreneur has wealth A, is risk neutral, and is protected by limited liability.

Denote the average investment by I and the individual investment i (in equilibrium $i = I$ by symmetry but we need to distinguish the two in a first step in order to compute the competitive equilibrium). A project produces Ri units of goods when successful and 0 when it fails. The probability of success is p_H in the case of good behavior (the entrepreneur receives no private benefit) and $p_L = p_H - \Delta p$ in the case of misbehavior (the entrepreneur then receives private benefit Bi). Assume that it is optimal to induce the entrepreneur to behave.

The market price of output is $P = P(Q)$, with $P' < 0$, where Q is aggregate production (with $P(Q)$ tending to 0 as Q goes to infinity, to ensure that aggregate investment is finite). Finally, the shocks faced by the firms are independent (there is no industry-wide uncertainty) and the risk-neutral investors demand a rate of return equal to 0.

Show that the equilibrium is unique. Compute the equilibrium level of investment. (Hint: distinguish two cases, depending on whether A is large or small.)

Exercise 3.18 (maximal incentives principle in the fixed-investment model). Pursue the analysis of Section 3.4.3, but for the fixed-investment model of Section 3.2: the investment cost I is given and the income is either R^S or R^F (instead of R or 0), where $R^S > R^F > 0$. We assume that

$$R^F < I - A,$$

so the project cannot be straightforwardly financed by bringing in net worth A and pledging the lower income R^F to lenders. Let

$$R \equiv R^S - R^F$$

denote the increase in income from the low to the high level. Show that the debt contract is optimal, but unlike in the variable-investment case it may not be uniquely optimal.

Exercise 3.19 (balanced-budget investment subsidy and profit tax). This exercise shows that a balanced-budget public policy that is not based on information that is superior to investors' does not boost pledgeable income and therefore outside financing capacity (unless there are externalities among firms: see Exercise 3.17). This general point is illustrated in the context of the variable-investment model: an entrepreneur has cash amount A and wants to invest $I > A$ into a (variable size) project. The project yields $RI > 0$ with probability p and 0 with probability $1 - p$. The probability of success is p_H if the entrepreneur works and $p_L = p_H - \Delta p$ ($\Delta p > 0$) if she shirks. The entrepreneur obtains private benefit BI if she shirks and 0 otherwise. The borrower is protected by limited liability and everyone is risk neutral. The project is worthwhile only if the entrepreneur behaves. Competitive lenders demand a zero expected rate of return. Assume that the NPV is positive:

$$\rho_1 \equiv p_H R > 1,$$

but

$$\rho_0 \equiv p_H\left(R - \frac{B}{\Delta p}\right) < 1.$$

The government has two instruments: a subsidy s per unit of investment, and a proportional tax t on the final profit.

The government must set (s, t) so as to balance its budget. Show that the government's policy is neutral:

$$I = \frac{A}{1 - \rho_0} \quad \text{and} \quad U_b = (\rho_1 - 1)I$$

for any (s, t), where U_b is the entrepreneur's utility.

Exercise 3.20 (variable effort, the marginal value of net worth, and the pooling of equity). In the fixed-investment model, the shadow price of entrepreneurial net worth is equal to 0 almost everywhere and is infinite at the threshold $A = \bar{A}$. A more continuous response arises when the entrepreneur's effort is continuous rather than discrete. The object of this exercise is to show that the shadow price is positive and *decreasing in* A in the range in which the entrepreneur is able to finance her project but must borrow from investors. It then applies the analysis to the internal allocation of funds between two divisions.

An entrepreneur has cash A and wants to invest $I > A$ into a fixed-size project. The project yields R with probability p and 0 with probability $1 - p$. Reaching a probability of success p requires the entrepreneur to sink (unobservable) effort cost $\frac{1}{2}p^2$ (there is no private benefit in this version). The borrower is risk neutral and is protected by limited liability. Investors are risk neutral and the market rate of interest is 0. Assume that $\sqrt{2I} < R < 1$.

(i) Note that, had the borrower no need to borrow ($A \geqslant I$), the borrower's net utility would be

$$U_b = V^* = \tfrac{1}{2}R^2 - I,$$

independently of A.

(ii) Find the threshold $\bar{A}$ under which the project is not funded. (Hint: write the pledgeable income as a function of the entrepreneur's reward R_b in the case of success. Argue that one can focus attention on the values of R_b that exceed $\frac{1}{2}R$. Do not forget that the NPV must be nonnegative.)

Letting $V(A)$ denote the NPV in the region in which the entrepreneur's project is financed. Show that the shadow price of net worth, $V'(A)$, satisfies

$$V'(A) > 0,$$
$$V'(I) = 0,$$
$$V''(A) < 0.$$

(iii) Following Cestone and Fumagalli (2005), consider two entrepreneurs, each with net worth A. They will each have a project described as above, but with random investment cost. For simplicity, one of them will face investment cost I_H and the other I_L, where

$$I_L - A < \tfrac{1}{4}R^2 < I_H - A,$$

but it is not known in advance who will face which investment cost (each is equally likely to be the lucky entrepreneur). Investment costs, however, will become publicly known before the investments are sunk. Assume that

$$\tfrac{3}{8}R^2 > I_H,$$

so that the only binding constraint for financing in question (ii) is the investors' breakeven constraint; and that

$$\tfrac{1}{2}R^2 > (I_L + I_H) - 2A,$$

and so both projects can be financed by pooling resources. Do the entrepreneurs, behind the veil of ignorance, want to pool their resources and commit to force the lucky firm to cross-subsidize the unlucky one? (Hint: show that under pooling, and, if both invest, the net worth is split so that both entrepreneurs have the same stake in success.)

Exercise 3.21 (hedging or gambling on net worth?). Froot et al. (1993) analyze an entrepreneur's risk preferences with respect to net worth. In the notation of this book, the situation they consider is summarized in Figure 3.12.

The entrepreneur is risk neutral and protected by limited liability. The investors are risk neutral and demand a rate of return equal to 0.

At date 0, the entrepreneur decides whether to insure against a date-1 income risk

$$r = A_0 + \varepsilon,$$

where $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$, $E[\varepsilon] = 0$, and $A_0 + \underline{\varepsilon} \geqslant 0$.

For simplicity, we allow only a choice between full hedging and no hedging (the theory extends straightforwardly to arbitrary degrees of hedging). Hedging (which wipes out the noise and thereby guarantees that the entrepreneur has cash on hand A_0 at date 1) is costless.

After receiving income, the entrepreneur uses her cash to finance investment I and must borrow $I - A$ from investors, with $A = A_0$ in the case of hedging and $A = A_0 + \varepsilon$ in the absence of hedging (provided that $A \leqslant I$; otherwise there is no need to borrow).

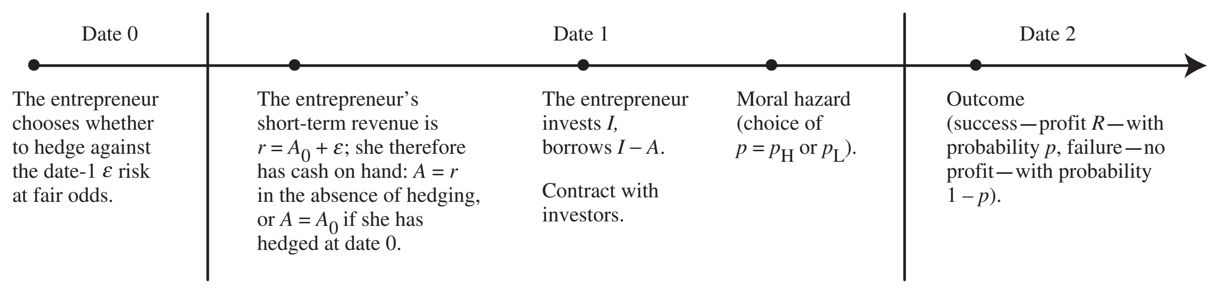

Figure 3.12

Note that there is no overall liquidity management as there is no contract at date 0 with the financiers as to the future investment.

This exercise investigates a variety of situations under which the entrepreneur may prefer either hedging or "gambling" (here defined as "no hedging").

(i) *Fixed investment, binary effort.* Suppose that the investment size is fixed (as in Section 3.2), and that the entrepreneur at date 1, provided that she receives funding, either behaves (probability of success p_H, no private benefit) or misbehaves (probability of success p_L, private benefit B). As usual, the project is not viable if it induces misbehavior and has a positive NPV ($p_H R > I > p_L R + B$, where R is the profit in the case of success). Let $\bar{A}$ be defined (as in Section 3.2) by

$$p_H\left(R - \frac{B}{\Delta p}\right) = I - \bar{A}.$$

Suppose that ε has a wide support.

Show that the entrepreneur

- hedges if $A_0 \geqslant \bar{A}$,
- gambles if $A_0 < \bar{A}$.

(ii) *Fixed investment, continuous effort.* Suppose, as in Exercise 3.20, that succeeding with probability p involves an unverifiable private cost $\frac{1}{2}p^2$ for the entrepreneur (so, effort in this subquestion involves a cost rather than the loss of a private benefit). (Assume $R < 1$ to ensure that probabilities are smaller than 1.)

Write the investors' breakeven condition as well as the NPV as functions of the entrepreneur's stake, R_b, in success. Note that one can focus without loss of generality on $R_b \in [\frac{1}{2}R, R]$. Assume that $I - A_0 < \frac{1}{4}R^2$ and that the support of ε is sufficiently small that the entrepreneur always receives funding when she does not hedge (and *a fortiori* when she hedges). This assumption eliminates the concerns about financing of investment that were crucial in question (i).

Show that the entrepreneur hedges.

(iii) *Variable investment.* Return to the binary effort case ($p = p_H$ or p_L), but assume that the investment I is variable (as in Section 3.4). The income is RI in the case of success and 0 in the case of failure. The private benefit of misbehaving is $B(I)$ with $B' > 0$. Assume that the size of investment is always constrained by the pledgeable income and that the optimal contract induces good behavior.

Show that the entrepreneur

- hedges if $B(\cdot)$ is convex;
- is indifferent between hedging and gambling if $B(\cdot)$ is linear;
- gambles if $B(\cdot)$ is concave.

(iv) *Variable investment and unobservable income.* Suppose that the investment size is variable and that the income from investment $R(I)$ is unobservable by investors (fully appropriated by the entrepreneur) and is concave. Suppose that it is always optimal for the entrepreneur to invest her cash on hand.

Show that the entrepreneur hedges.

(v) *Liquidity and risk management.* Suppose, in contrast with Froot et al.'s analysis, that the entrepreneur can sign a contract with investors at date 0. Show that the entrepreneur's utility can be maximized by insulating the date-1 volume of investment from the realization of ε, i.e., with full hedging, even in situations where gambling was optimal when funding was secured only at date 1.

References

Banerjee, A. and E. Duflo. 2000. Reputation effects and the limits of contracting: a study of the Indian software industry. *Quarterly Journal of Economics* 115:989–1017.

Bester, H. and M. Hellwig. 1987. Moral hazard and credit rationing: an overview of the issues. In *Agency Theory, Information, and Incentives* (ed. G. Bamberg and K. Spremann). Heidelberg: Springer.

Bhattacharya, S. and A. Faure-Grimaud. 2001. The debt hangover: renegotiation with noncontratible investment. *Economics Letters* 70:413–419.

Bizer, D. and DeMarzo, P. 1992. Sequential banking. *Journal of Political Economy* 100:41–61.

Bolton, P. and O. Jeanne. 2004. Structuring and restructuring sovereign debt: the role of seniority. Mimeo, Princeton University.

Bolton, P. and D. Scharfstein. 1990. A theory of predation based on agency problems in financial contracting. *American Economic Review* 80:93–106.

——. 1996. Optimal debt structure with multiple creditors. *Journal of Political Economy* 104:1–26.

Border, K. and J. Sobel. 1987. Samurai accountant: a theory of auditing and plunder. *Review of Economic Studies* 54: 525–540.

Bulow, J. and K. Rogoff. 1989a. A constant recontracting model of sovereign debt. *Journal of Political Economy* 97: 155–178.

——. 1989b. Multilateral negotiations for rescheduling developing country debt: a bargaining theoretic framework. In *Analytical Issues in Debt* (ed. J. Frenkel, M. Dooley, and P. Wickham). Washington, D.C.: IMF.

Cestone, G. and C. Fumagalli. 2005. The strategic impact of resource flexibility in business groups. *RAND Journal of Economics* 36:193–214.

Chang, C. 1990. The dynamic structure of optimal debt contracts. *Journal of Economic Theory* 52:68–86.

——. 1993. Payout policy, capital structure, and compensation contracts when managers value control. *Review of Financial Studies* 6:911–933.

Chiesa, G. 1992. Debt and warrants: agency problems and mechanism design. *Journal of Financial Intermediation* 2: 237–254.

DeGroot, M. 1970. *Optimal Statistical Decisions.* New York: McGraw-Hill.

Dessi, R. 2005. Start-up finance, monitoring and collusion. *RAND Journal of Economics* 36:255–274.

Dewatripont, M., P. Legros, and S. Matthews. 2003. Moral hazard and capital structure dynamics. *Journal of the European Economic Association* 1:890–930.

Diamond, D. 1984. Financial intermediation and delegated monitoring. *Review of Economic Studies* 51:393–414.

Diamond, D. 1991. Monitoring and reputation: the choice between bank loans and directly placed debt. *Journal of Political Economy* 99:689–721.

Doidge, C., A. Karolyi, and R. Stulz. 2004. Why are foreign firms listed in the U.S. worth more? *Journal of Financial Economics* 71:205–238.

Easterbrook, F. 1984. Two-agency-cost explanations of dividends. *American Economic Review* 74:650–659.

Eaton, J. and M. Gersovitz. 1981. Debt with potential repudiations: theoretical and empirical analysis. *Review of Economic Studies* 48:289–309.

Edlin, A. S. and B. Hermalin. 2000. Contract renegotiation and options in agency. *Journal of Law, Economics, & Organization* 16:395–423.

——. 2001. Implementing the first best in an agency relationship with renegotiation: a corrigendum. *Econometrica* 69:1391–1395.

Fama, E. and M. Miller. 1972. *The Theory of Finance.* New York: Holt, Rinehart and Winton.

Fazzari, S., R. G. Hubbard, and B. C. Petersen. 1988. Financing constraints and corporate investment. *Brookings Papers on Economic Activity* 1:141–195.

Fernandez, R. and R. Rosenthal. 1990. Strategic models of sovereign-debt renegotiations. *Review of Economic Studies* 57:331–350.

Froot, K., D. Scharfstein, and J. Stein. 1993. Risk management: coordinating corporate investment and financing policies. *Journal of Finance* 48:1629–1658.

Fudenberg, D. and J. Tirole. 1990. Moral hazard and renegotiation in agency contracts. *Econometrica* 58:1279–1320.

——. 1991. *Game Theory.* Cambridge, MA: MIT Press.

Gale, D. and M. Hellwig. 1985. Incentive-compatible debt contracts: the one-period problem. *Review of Economic Studies* 52:647–663.

——. 1989. Repudiation and renegotiation: the case of sovereign debt. *International Economic Review* 30:3–31.

——. 1994. Renegotiation in debt contracts. PhD thesis, Ecole Polytechnique, Paris.

Hart, O. 1985. A comment on Stiglitz and Weiss. Mimeo, MIT.

Hart, O. and J. Moore. 1989. Default and renegotiation: a dynamic model of debt. Mimeo, MIT and LSE. (Published in *Quarterly Journal of Economics* (1998) 113:1–42.)

——. 1995. Debt and seniority: an analysis of the role of hard claims in constraining management. *American Economic Review* 85:567–585.

——. 1999. Foundations of incomplete contracts. *Review of Economic Studies* 66:115–138.

Hellwig, C. and G. Lorenzoni. 2004. Bubbles and private liquidity. Mimeo, UCLA and MIT.

Hermalin, B. and M. Katz. 1991. Moral hazard and verifiability: the effects of renegotiation in agency. *Econometrica* 59:1735–1753.

Holmström, B. 1979. Moral hazard and observability. *Bell Journal of Economics* 10:74–91.

Holmström, B. and J. Tirole. 1997. Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics* 112:663–692.

Innes, R. 1990. Limited liability and incentive contracting with ex ante action choices. *Journal of Economic Theory* 52:45–67.

Jaffee, D. and T. Russell. 1976. Imperfect information, uncertainty, and credit rationing. *Quarterly Journal of Economics* 90:651–666.

Jensen, M. 1986. Agency costs of free cash flow, corporate finance and takeovers. *American Economic Review* 76:323–329.

Kaplan, S. N. and L. Zingales. 1997. Do investment-cash flow sensitivities provide useful measures of financing constraints? *Quarterly Journal of Economics* 112:169–216.

Keeton, W. R. 1979. *Equilibrium Credit Rationing.* New York and London: Garland.

Kehoe, T. and D. Levine. 1993. Debt-constrained asset markets. *Review of Economic Studies* 60:865–888.

Kocherlakota, N. 1996. Implications of efficient risk sharing without commitment. *Review of Economic Studies* 63:595–609.

Krasa, S. and A. Villamil. 1994. Optimal contracts with costly state verification: the multilateral case. *Economic Theory* 4:167–187.

———. 2000. Optimal contracts when enforcement is a decision variable. *Econometrica* 68:119–134.

Kreps, D., P. Milgrom, J. Roberts, and R. Wilson. 1982. Rational cooperation in the finitely repeated prisoner's dilemma. *Journal of Economic Theory* 27:245–252.

Lacker, J. 1991. Why is there debt? *Economic Review, Federal Reserve Bank of Richmond* 77(4):3–19.

———. 1992. Collateralized debt as the optimal contract. Mimeo, Federal Reserve Bank of Richmond.

Lacker, J. and J. Weinberg. 1989. Optimal contracts under costly state verification. *Journal of Political Economy* 97:1345–1363.

Ma, C. A. 1991. Adverse selection in a dynamic moral hazard. *Quarterly Journal of Economics* 106:255–275.

———. 1994. Renegotiation and optimality in agency contracts. *Review of Economic Studies* 61:109–130.

Maskin, E. 1977. Nash equilibrium and welfare optimality. Mimeo, MIT. (Published in *Review of Economic Studies* (1999) 66:23–38.)

Maskin, E. and J. Moore. 1999. Implementation and renegotiation. *Review of Economic Studies* 66:39–56.

Maskin, E. and J. Tirole. 1999. Unforeseen contingencies and incomplete contracts. *Review of Economic Studies* 66:83–114.

Matthews, S. 1995. Renegotiation of sales contracts. *Econometrica* 63:567–590.

———. 2001. Renegotiating moral hazard contracts under limited liability and monotonicity. *Journal of Economic Theory* 97:1–29.

Milgrom, P. 1981. Good news and bad news: representation theorems and applications. *Bell Journal of Economics* 12:380–391.

Miller, D. 1999. The market reaction to international cross-listings: evidence from depositary receipts. *Journal of Financial Economics* 51:103–123.

Mirrlees, J. 1975. The theory of moral hazard and unobservable behaviour. Part I. Mimeo, University of Oxford. (Published in *Review of Economic Studies* (1999) 66:3–21.)

Mookherjee, D. and I. P'ng. 1989. Optimal auditing, insurance and redistribution. *Quarterly Journal of Economics* 104:399–415.

Myers, S. 1977. Determinants of corporate borrowing. *Journal of Financial Economics* 5:147–175.

Padilla, J. and M. Pagano. 2000. Sharing default information as a borrower discipline device. *European Economic Review* 44:1951–1980.

Pagano, M. and P. Volpin. 2005. Shareholder protection, stock market development, and politics. Presented as the Marshall Lecture European Economic Association meeting, Amsterdam, August 27, 2005.

Pagano, M., O. Randl, A. A. Roell, and J. Zechner. 2001. What makes stock exchanges succeed? Evidence from cross-listing decisions. *European Economic Review* 45:770–782.

Povel, P. and M. Raith. 2004. Optimal debt with unobservable investments. *RAND Journal of Economics* 35:599–616.

Reese, W. and M. Weisbach. 2002. Protection of minority shareholder interests, cross-listings in the United States, and subsequent equity offerings. *Journal of Financial Economics* 68:65–104.

Segal, I. 1999. Contracting with externalities. *Quarterly Journal of Economics* 114:337–388.

Segal, I. and M. Whinston. 2002. The Mirrlees approach to mechanism design with renegotiation (with applications to hold-up and risk-sharing). *Econometrica* 70:1–45.

Shavell, S. 1979. Risk sharing and incentives in the principal and agent relationship. *Bell Journal of Economics* 10:55–73.

Snyder, C. 1994. Income shifting, slack constraints, and long-term financial contracts. In "Buyers, suppliers, competitors: the interaction between a firm's horizontal and vertical relationships," Chapter 2. PhD thesis, MIT.

Stiglitz, J. and A. Weiss. 1981. Credit rationing in markets with imperfect information. *American Economic Review* 71:393–410.

Tirole, J. 1999. Incomplete contracts: where do we stand? *Econometrica* 67:741–781.

Townsend, R. 1979. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory* 21:417–425.

Webb, D. 1992. Two-period financial contracts with private information and costly state verification. *Quarterly Journal of Economics* 107:1113–1123.

Williamson, O. 1986. Costly monitoring, financial intermediation, and equilibrium credit rationing. *Journal of Monetary Economics* 18:159–179.

Winton, A. 1995. Costly state verification and multiple investors: the role of seniority. *Review of Financial Studies* 8:91–123.