PART VII

Answers to Selected Exercises,
and Review Problems
Exercise 3.1 (random financing). (i) The investors’ breakeven condition is
\[ \frac{xI}{R} - A \leq \frac{xpH(R - B)}{R} - c. \]
Because the NPV is negative if the entrepreneur has an incentive to shirk, \( R_b \) must satisfy
\[ (\Delta p)R_b \geq B. \]
The investors’ breakeven condition (which will be satisfied with equality under a competitive capital market) is then
\[ \frac{xH(R - B)}{R} - c \leq \frac{xI}{R} - A \]
or
\[ xA \leq A. \]
(ii) The NPV is equal to
\[ U_b = \frac{xpH(R - I)}{R} - pH(R - B)\Delta p \]
and so maximizing \( U_b \) is tantamount to maximizing \( x \). Hence,
\[ x^* = \frac{A}{R} \]
The probability that the project is undertaken grows from 0 to 1 as the borrower’s net worth grows from 0 to \( R \).

Exercise 3.2 (impact of entrepreneurial risk aversion). (i) When \( pH < 1 \), the entrepreneur must receive at least \( c_0 \), the case of failure, because the probability of failure is positive even in the case of good behavior. Because of risk neutrality above \( c_0 \), it is optimal to give the entrepreneur exactly \( c_0 \) in the case of failure. Let \( R_b \) denote the reward in the case of success. The incentive constraint is
\[ (\Delta p)(R_b - c_0) \geq B. \]
\[ \text{(IC)} \]
The pledgeable income is
\[ pHR - (1 - p)u(c_0) - pH(R - B)\Delta p \]
To allow financing, this pledgeable income must exceed \( I - A \). Hence, \( A = I + c_0 - pH(R - B)/\Delta p \).
When \( pH = 1 \), the pledgeable income is then \( pHR \) (if \( c_0 > 0 \), deviations can be punished harshly by giving the entrepreneur, say, 0 in the case of failure).
(ii) Let \( R_b^c \) and \( R_b^f \) denote the rewards in the cases of success and failure, respectively. The incentive constraint is
\[ (\Delta p)[u(R_b^c) - u(R_b^f)] \geq B. \]
The optimal contract solves
\[ \max U_b = \frac{xpH(R - I)}{R} - pH(R - B)\Delta p \]
s.t.
\[ pHR - pHR_b^c - (1 - p)HR_b^f \geq I - A, \]
\[ (\Delta p)[u(R_b^c) - u(R_b^f)] \geq B, \]
and (if limited liability is imposed)
\[ R_b^f \geq 0. \]
It must also be the case that the solution to this program exceeds the utility, \( u(A) \), obtained by the entrepreneur if the project is not financed. The entrepreneur’s incentive compatibility constraint is binding; otherwise, the solution to this program would give full insurance to the entrepreneur, which would violate the incentive compatibility condition. We refer to Holmström\(^1\) and Shavell\(^2\) for general considerations on this moral-hazard problem.

Exercise 3.3 (random private benefits). (i) \( B^* = pHR - r \).
or, equivalently,

\[ B^* = \frac{1}{4} p_0 R, \]

The borrowing capacity is maximized for

\[ B^* = \frac{1}{2} p_0 R, \]

or, equivalently,

\[ n_1 = \frac{1}{2} p_0 R. \]

(iii) Using the fact that investors break even, the entrepreneur’s expected utility is

\[
\left( p_0 B^* + \frac{R}{2} \left( B^* \right)^2 \right) I = \frac{p_0 B^* + \frac{1}{2} R + \left( B^* \right)^2 / 2 R}{1 - B^* / R}. \]

At the optimum,

\[ \frac{1}{2} p_0 R < B^* < p_0 R. \]

Recall that \( B^* = p_0 R \) maximizes the return per unit of investment as it eliminates shrinking, while \( B^* = \frac{1}{2} p_0 R \) maximizes borrowing capacity.

(iv) When \( B \) is verifiable, the entrepreneur’s expected utility is still

\[
\left( p_0 B^* + R \left( B^* \right)^2 \right) I, \]

For a given \( B^* \), the contract should specify

\[ r_i(B) = R - B / p_0 \]

(since \( p_0 \) is constant), if \( B < B^* \), and

\[ p_0 \]

if \( B > B^* \).

The maximal investment is then

\[ I = 1 - p_0 B^* + \left( B^* \right)^2 / 2 R. \]

Borrowing capacity is maximized at \( B^* = p_0 R \). Because this threshold also maximizes per-unit expected income, it is clearly optimal overall.

**Exercise 3.4 (product-market competition and financing)**. (i) Because the two projects are statistically independent, there is no point making an entrepreneur’s reward contingent on the outcome of the other firm’s performance. (Technically, this result is a special case of the “statistical significance” results of Holmstrom and Shavell.) This result states that an agent’s reward should be contingent only on variables that the agent can control—a sufficient statistic for the vector of observable variables relative to effort—and not on extraneous noise.) So, let \( R_0 \) and \( R_1 \) denote an entrepreneur’s reward in the cases of success and failure. As usual,

\[ (\Delta p_0)(R_0 - R_1) \geq B \]

and \( R_1 = 0 \).

Let \( x \in [0,1] \) denote the probability that the rival firm invests. Then the expected income is

\[ p_0 \left[ x p_0 D + (1 - x p_0) M \right]. \]

The pledgeable income is equal to this expression minus \( p_0 B / \Delta p \).

At best, the other firm is not financed, and \( R = M \) in the case of success. The threshold \( I \) is given by

\[ I - \Delta = p_0 \left( M - \frac{R}{\Delta p} \right). \]

(ii) At worst, the rival firm is financed. So, the expected return in the case of success is

\[ p_0 D + (1 - p_0) M. \]

So,

\[ I - \Delta = p_0 \left( p_0 D + (1 - p_0) M - \frac{B}{\Delta p} \right). \]

(iii) One of the firms gets funding while the other does not (obviously). There also exists a third, mixed-strategy equilibrium, in which each firm gets funded with positive probability.

(iv) If only one firm receives financing, then

\[ R_0^* = c_0 \]

(as long as \( p_0 < 1 \), so that there is always a probability of failing even when the entrepreneur works), and

\[ R_1^* = c_0 + \frac{B}{\Delta p}, \]

which yields the minimum net worth given in the statement of the question.

(v) Suppose now that both entrepreneurs receive financing. Consider the following reward scheme for

the entrepreneur:

- \( R_0 < c_0 \) if the firm fails and
- \( R_0 = c_0 \) otherwise.

There is no longer moral hazard: as long as the other entrepreneur works, shirking yields probability \( \Delta p \) that the other entrepreneur succeeds while this entrepreneur fails (recall that the two technologies are perfectly correlated), resulting in a large (infinite) punishment. If

\[
D > M - \frac{B}{\Delta p}
\]

then product-market competition facilitates financing! Correlation enables benchmarking provided that both firms secure financing.

**Exercise 3.5 (continuous investment and decreasing returns to scale).**

(i) The incentive constraint is,

\[
\left( \Delta p \right) R_b \geq BI.
\]

Thus the entrepreneur selects \( I \) to solve

\[
\text{max NPV} = \max U_b = p_H R(I) - I \quad \text{s.t.} \quad p_H \left[ R(I) - BI \Delta p \right] \geq I - A.
\]

Clearly, if \( I = \hat{I}^* \) satisfies (BB) \((A) \) is high), then it solves this program. The shadow price of the budget constraint is then \( \mu = 0 \).

So suppose \( A \) is small enough that (BB) is not satisfied at \( I = \hat{I}^* \). Then \( I \) is determined by (BB) (since the objective function is concave). In that region, by the envelope theorem

\[
\frac{dU_b}{dA} = v = p_H R(\hat{I}) - 1 - \left( \frac{dI}{dA} \right) \frac{B}{\Delta p}.
\]

So \( v \) decreases with \( A \).

**Exercise 3.6 (renegotiation and debt forgiveness).**

(i) Suppose that \( R_b < BI / (\Delta p) \).

In the absence of renegotiation, the entrepreneur will shirk and obtain utility

\[
BI + p_H R_b,
\]

and the lender’s expected revenue is

\[
p_L(RI - R_b).
\]

Renegotiation must be mutually advantageous. So a necessary condition for renegotiation is that total surplus increases. A renegotiation toward a stake \( \hat{R}_0 < BI / (\Delta p) \) does not affect surplus and thus is a mere redistribution of wealth between the investors and the entrepreneur. So renegotiation, if it happens, must yield stake

\[
\hat{R}_0 \geq \frac{BI}{\Delta p}
\]

for the entrepreneur. It constitutes a Pareto-improvement if the following two conditions are satisfied:

- \( p_H \hat{R}_0 \geq BI + p_L R_b \)
- \( p_H (RI - \hat{R}_0) \geq p_L (RI - R_b) \).

The second inequality, together with the incentive constraint, implies that

\[
(\Delta p) (RI - \hat{R}_0) \geq I - A.
\]

Conversely, if this condition is satisfied, then the two parties can find an \( \hat{R}_0 \) that makes them both better off.

Note that the standard assumptions

\[
p_H \left[ RI - BI / \Delta p \right] \geq I - A
\]

and

\[
I \geq p_L RI + BI
\]

imply that

\[
(\Delta p) (RI - \hat{R}_0) / (\Delta p) + A - BI \geq 0.
\]

So, if \( A > BI \) and \( R_b \) is small enough, the condition for renegotiation may not be satisfied.

(ii) The “project” consists in creating incentives for the entrepreneur. It creates NPV equal to \( (\Delta p) RI \), does not involve any new investment, and the entrepreneur can bring an amount of money \( \hat{A} \equiv p_H R_b \) that is the forgone expected income.
For this fictitious project, the pledgeable income is
\[
(\Delta p)RI - pu_RI
\]
and the investors' outlay is
\[
0 - A
\]
Hence, it is “financed” if and only if
\[
(\Delta p)RI - pu_RI > -pu_R0
\]

Exercise 3.7 (strategic leverage). (i) The NPV, if the project is funded, is
\[
(p_u + \tau)R - I(\tau)
\]
So, if \( A > A^* \) and \( \tau = \tau^* \),
• For \( A < A^* \), the pledgeable income can be increased by reducing \( \tau \) below \( \tau^{**} \):
\[
\frac{d}{d\tau} \left( (p_u + \tau) \left( R - \frac{B}{\Delta p} \right) - I(\tau) - A \right) = R - \frac{B}{\Delta p} - I'(\tau).
\]
Let \( \tau^{**} \) be defined by
\[
I'(\tau^{**}) = R - \frac{B}{\Delta p}.
\]
The pledgeable income decreases with \( \tau \) for \( \tau > \tau^{**} \). The borrower can raise funds if and only if \( A > A^{**} \), with
\[
(p_u + \tau^{**}) \left( R - \frac{B}{\Delta p} \right) - I(\tau^{**}) - A^{**}.
\]
The quality of investment increases with \( A \) (for \( A > A^{**} \)) and is flat beyond \( A^* \). For \( A \in [A^*, A^+] \),
\[
(p_u + \tau(A)) \left( R - \frac{B}{\Delta p} \right) = I(\tau(A)) - A.
\]
For \( A > A^* \), \( \tau(A) = \tau^* \).

(ii) Define \( \hat{\tau} \) by
\[
I'(\hat{\tau}) = 1 - (p_u + \hat{\tau}).R
\]
(\( \hat{\tau} \) maximizes a firm's NPV given that the other firm's choice is \( \tau \).) Borrower's incentive compatibility constraint is \((\Delta p)(1 - \hat{\tau})R0 \geq B\), where \( R0 \) is her reward in the case of income \( K \). So the pledgeable income is
\[
(p_u + \tau) \left( 1 - \hat{\tau} \right)R - \frac{B}{\Delta p}.
\]
(\( \tau, \hat{\tau} \) is a symmetric Nash equilibrium if and only if
\[
(p_u + \tau) \left( 1 - (p_u + \tau) \right)R - \frac{B}{\Delta p} \geq I(\tau) - A.
\]
This equation yields \( \hat{\tau} \).

(iii) \( (q, \hat{q}) \) is a symmetric Nash equilibrium for \( A = \hat{A} \).
• By choosing \( q_1 = \hat{q} + \epsilon \), borrower 1 deters entry by borrower 2.

Exercise 3.8 (equity multiplier and active monitoring). (i) See Section 3.4.
(ii) Suppose that monitoring at level \( c \) is to be induced. Two incentive compatibility constraints must be satisfied:
\[
(\Delta p)R_{\epsilon} \geq cl \quad \text{and} \quad (\Delta p)R_{\epsilon} \geq b(c)l.
\]
Because there is no scarcity of monitoring capital, the monitor contributes \( I_{\epsilon} \) to the project and breaks even:
\[
I_{\epsilon} = puR_{\epsilon} - cl = pu \frac{\Delta p}{\Delta p} (l - c).
\]
The equity multiplier, \( k \), is given by
\[
puR - R0 = I = A - I_{\epsilon}
\]
or
\[
pu \left( R - \frac{b(c) + c}{\Delta p} \right) I = I = A - pu \frac{\Delta p}{\Delta p} (l - c),
\]
that is,
\[
I = k(c)A
\]
where
\[
k(c) = \frac{1 + \epsilon}{1 - pu(R - b(c)/\Delta p)}
\]
\[
= \frac{1 + \epsilon}{1 - pu\Delta p/R0} + (pu/\Delta p) [b(c) + c - R0].
\]
The project's NPV (which includes the monitoring cost) is equal to
\[
pul - cl = (p_u - c)k(c)A.
\]
The borrower maximizes \( (p_u - c)k(c)A \) since the other parties receive zero utility and she therefore receives the project's NPV.

Exercise 3.9 (concave private benefit). (i) Suppose that the NPV per unit of investment is positive:
\[
puR > 1
\]
The entrepreneur's utility is equal to the NPV,
\[ U_e = (p_B R - I), \]
and so the entrepreneur chooses the highest investment that is consistent with the investors' breakeven constraint
\[ p_B \left( R I - \frac{B(I)}{\Delta P} \right) = I - A. \]
Because \( \lim_{\Delta P \to 0} B(I) = B \) and \( p_B (R - (B/\Delta P)) < 1 \), this upper limit indeed exists.
(ii) The shadow price is given by
\[ \nu = \frac{dU_e}{dA} = \frac{p_B R - 1}{p_B (R - (B/\Delta P)) - 1}. \]
Hence \( \nu \) increases with \( A \) (since \( B' < 0 \) and \( dI/dA > 0 \)).

Exercise 3.10 (congruence, pledgeable income, and power of incentive scheme).

(i) Either \( R_0 > B/\Delta P \) and the entrepreneur always behaves well. The NPV is
\[ \text{NPV}^I = p_B R - I + (1 - x)B \]
and the financing condition
\[ p_B \left( R - \frac{B}{\Delta P} \right) \geq I - A. \]
Or \( R_0 < B/\Delta P \). The NPV is then
\[ \text{NPV}^I = x(p_B R + B) + (1 - x)(p_B R + B) - I < \text{NPV}^I, \]
and the financing condition \( R_0 = 0 \) then maximizes the pledgeable income
\[ \max \{x(p_B + (1 - x)p_B) R \geq I - A. \]}
The pledgeable income is increased only if \( x \) is sufficiently low. The high-powered incentive scheme is always preferable if (1) is satisfied; otherwise, the parties may content themselves with a low-powered scheme (provided (2) is satisfied).
(iii) Suppose that the menu offers \( R_0^L, R_0^H \) when interests are divergent and \( R_0^H, R_0^L \) when interests are aligned. The state (divergent/congruent) is not observed by the investors and so this menu must be incentive compatible (the entrepreneur must indeed prefer the incentive scheme tailored to the state of nature she faces).

The interesting case is when the incentive scheme in the divergent state is incentive compatible \( (\Delta P) \times (R_0^L - R_0^H) \geq B \); otherwise, setting all rewards equal to 0 is obviously optimal.

In the congruent state, the entrepreneur must not pretend interests are divergent, and so
\[ p_B R_0^H + (1 - p_B) R_0^L \geq p_B R_0^H + (1 - p_B) R_0^L. \]
So one might as well take \( R_0^L - R_0^H \) and \( R_0^L - R_0^H \). This choice yields incentive compatibility in the congruent state and maximizes the pledgeable income.

Exercise 3.11 (retained-earnings benefit).

(i) Let us assume away any discounting for notational simplicity. The assumption on \( E' \) implies that retained earnings are always needed to finance the second project, as
\[ p_B \left( R^2 - \frac{B^2}{\Delta P} \right) < I^2 \]
for all \( B' \).

The borrower's utility is, as a function of date-1 earnings \( R_1^B \),
\[ U_B(R_1^B) = \begin{cases} R_1^B & \text{if the second project is not financed}, \\ R_1^B + \text{NPV}^I & \text{otherwise}, \end{cases} \]
where \( \text{NPV}^I = p_B R^2 - I^2 \) is independent of \( B' \).
Let \( R_1^B(B') \) denote the required level of retained earnings when the date-2 private benefit turns out to be \( B' \):
\[ p_B \left( R^2 - \frac{B^2}{\Delta P} \right) = I^2 = R_1^B(B'). \]
This equation also defines a threshold \( \hat{B}(R_0^I) \),

Thus, the expected utility is
\[ E[U_B(R_1^B)] = R_1^B + f(\hat{B}(R_0^I))[\text{NPV}^I]. \]
The shadow value of retained earnings is therefore
\[ \mu = \frac{dE[U_B(R_1^B)]}{dR_0^B} = 1 + f(\hat{B}(R_0^I)) \frac{dE[B^2]}{dR_0^B} \frac{\text{NPV}^I}{\Delta P}. \]
(ii) The date-1 incentive compatibility constraint is
\[ (\Delta P) \times (R_0^H + f(\hat{B}(R_0^I))[\text{NPV}^I]) \geq B'. \]
The pledgeable income,
\[ p_B \left( R^2 - \min_{x>1} R_0^I \right), \]
is therefore larger than in the absence of a second project. It is therefore more likely to exceed \( I^1 - A^1 \), where \( A^1 \) is the entrepreneur’s initial wealth.

**Exercise 3.12 (investor risk aversion and risk premium).** (ii) This condition says that the risk-free rate is normalized at 0. In other words, investors are willing to lend 1 unit at date 0 against a safe return of 1 unit at date 1.

(iii) With a competitive capital market, the financing condition becomes

\[
\max_{\{p_1 R_1^2, (1 - p_1) R_1^2 \}} \left( I - A \right),
\]

s.t.

\[
q_1 p_1 (R - R_0^2) + q_0 (1 - p_0) (- R_0^2) \geq I - A,
\]

\[
\frac{(\Delta p) R_1^2 - R_0^2}{\beta} \geq B,
\]

\[
R_0^2 \geq 0.
\]

Letting \( \mu_l, \mu_r, \) and \( \mu_1 \) denote the shadow prices of the constraints, the first-order conditions are

\[
p_1 [1 - \mu_1 q_1] + \mu_2 (\Delta p) = 0
\]

and

\[
(1 - p_1) [1 - \mu_1 q_0] - \mu_2 (\Delta p) + \mu_1 = 0.
\]

Conversely, (4) and (5) cannot be simultaneously binding, except when condition (1) is satisfied with exact equality.

- Suppose that constraint (4) is not binding (\( \mu_2 = 0 \)), which, from what has gone before, implies that \( R_0^2 = 0 \). Then \( \mu_1 = 1/q_0 \), and (7) can be satisfied only if

\[ q_0 > q_1. \]

- In contrast, suppose that constraint (5) is not binding (\( \mu_1 = 0 \)). Constraints (6) and (7) taken together imply that

\[ q_1 > q_0. \]

To sum up, the maximum punishment result \( (R_0^2 = 0) \) carries over to procyclical firms, because the incentive effect compounds with the “marginal rates of substitution” effect (the investors value income in the case of failure relatively more compared with the entrepreneur). But it does not in general hold for countercyclical firms. Then the investors care more about the payoff in the case of success, and the entrepreneur should keep marginal incentives equal to \( \beta/\Delta p \) and select \( R_0^2 > 0 \) (since the firm’s income is equal to 0 in the case of failure, this requires the firm to hoard some claim at date 0 so as to be able to pay the entrepreneur even in the case of failure).

Entrepreneurial risk aversion changes the incentive constraint (4) and the objective function (2). It may be the case that \( R_0^2 > 0 \) even for a procyclical firm.

**Exercise 3.13 (lender market power).** (i) If \( A \geq I \), then the “borrower” does not need the lender and just obtains the NPV \( (U_h = V) \). So let us assume that \( A < I \). The lender must respect two constraints. First, the standard incentive compatibility constraint:

\[
\frac{(\Delta p) R_0}{\beta} \geq B.
\]

Second, her net utility must be nonnegative:

\[
U_h - p_1 R_0 - A \geq 0.
\]

The lender maximizes

\[
U_h = p_1 R_0 - A/p_1
\]

subject to these two constraints.

Let us first ignore (ICs). The lender sets \( R_0 = A/p_1 \) and thus

\[
U_h = 0.
\]

The lender appropriates the entire surplus \( (U_l = V) \) as long as \( R_0 = A/p_1 \) satisfies the incentive
constraint, or
\[
\frac{\Delta P}{P_H} \geq B \iff A \geq \hat{A}.
\]
For \(A \in [\overline{A}, \hat{A})\), the lender cannot capture the borrower's surplus without violating the incentive constraint; then the borrower's net utility
\[
U_b = P_H \Delta P - A
\]
is decreasing in \(A\).
Lastly, the lender is willing to lend as long as
\[
U_l = V - U_b \geq 0 \quad \text{or} \quad A \geq \overline{A}.
\]
The borrower's net utility is as represented in Figure 1.

The borrower is "better off" (from the relationship) if she is either very rich (she does not need the lender) or poor (she cannot be expropriated by the lender)—although, of course, not too poor!

(ii) The lender solves
\[
\max U_l = P_H (R_b - R) - (I - A)
\]
s.t.
\[
\frac{\Delta P}{\Delta P} R_b \geq BI, \quad (I_b) \quad p_H R_b \geq A. \quad (I_b)
\]
If (\(I_b\)) were not binding, (\(I_b\)) would have to be binding (\(U_l\) is decreasing in \(R_b\)) and
\[
U_l = (p_H R - 1)I
\]
would yield \(I = \infty\), violating (\(I_b\)), a contradiction.
If (\(I_b\)) were not binding, (\(I_b\)) would have to be binding, and
\[
U_l = (p_H (R - \frac{B}{\Delta P}) - 1)I + A
\]
and so \(I = 0 = R_b\), contradicting (\(I_b\)).

Hence, the two constraints are binding, and so
\[
I = \frac{1}{p_H R (R - B/\Delta P)} A
\]
Recall that, in the presence of a competitive market,
\[
I^* = \frac{1}{1 - p_H (R - B/\Delta P)} A
\]
and so
\[
I < I^*.
\]
With variable-size investment, lender market power leads to a contraction of investment.

Exercise 3.14 (liquidation incentives). (i) Technically, the realization of \(\gamma\) is a "sufficient statistic" for inferring the effort chosen by the entrepreneur. Rewarding the entrepreneur as a function not only of \(\gamma\), but also of the realization of the final profit amounts to introducing into the incentive scheme noise over which the entrepreneur has no control. (We leave it to the reader to start with a general incentive scheme and then show that without loss of generality the reward can be made contingent on \(\gamma\) only.)

Second, it is optimal to liquidate if and only if \(\gamma = \gamma^*\). Hence, one can define expected profits:
\[
R^S \equiv \bar{\gamma} R \quad \text{and} \quad R^F \equiv L
\]
where "success" (\(S\)) now refers to a good signal, "failure" (\(F\)) to a bad signal, and \(R^S\) and \(R^F\) denote the associated continuation profits.

We are now in a position to apply the analysis of Section 3.2. Let \(R_b^*\) denote the entrepreneur's reward in the case of a good signal (\(\gamma = \gamma\)) and 0 that in the case of a bad signal. Incentive compatibility requires that
\[
\frac{\Delta P}{\Delta P} R_b \geq BI.
\]
The NPV is
\[
U_b = p_H R^* + (1 - p_H)L - I,
\]
and the pledgeable income is
\[
P^* = p_H R^* + (1 - p_H)L - p_H \frac{B}{\Delta P}.
\]
Financing is then feasible provided that \(A \geq \overline{A}\), where
\[
p_H \left(\frac{R^*}{\Delta P} + (1 - p_H)L - I - \overline{A}\right).
\]
(iii) Truth telling by the entrepreneur requires that
\[ y_R \geq I_h \geq y_R_0. \]

The entrepreneur’s other incentive compatibility constraint (that relative to effort) is then
\[ (\Delta p)(y_R - L_a) \geq B. \]

The investors’ payoff is then
\[ p_0 y(R - R_0) + (1 - p_0)(L - L_0). \]

As expected, it is highest when \( L_a = y_R_0 \) and \((\Delta p)(y_R - L_a) = B.\)

And so the pledgeable income is
\[ p_0 y(R - R_0) + (1 - p_0)(L - L_0) \]

for these values of \( L_a \) and \( R_0 \). Simple computations show that the financing condition amounts to
\[ p_0 y(R - R_0) + (1 - p_0)(L - L_0) \]

\[ - [p_0 y + (1 - p_0) \gamma p] \frac{B}{\Delta p} \geq I - A \]

\[ \text{or} \]
\[ A \geq x \frac{B}{\gamma p}. \]

Exercise 3.15 (project riskiness and credit rationing). The managerial minimum reward (consistent with incentive compatibility) is the same for both variants:
\[ \frac{B}{\Delta p} = \frac{B}{\Delta p}. \]

And so the investors’ breakeven condition can be written (with obvious notation) as
\[ I - A \leq p_0^A (R^A - \frac{B}{\Delta p^A}) \]

for variant A and
\[ I - A \leq p_0^B (R^B - \frac{B}{\Delta p^B}) \]

for variant B .

Because \( p_0^B > p_0^A \), the safer project (project A) is financed for a smaller range of cash on hand \( A \). That is, the safe project is more prone to credit rationing. Intuitively, the nonpledgeable income is higher for a safe project, since the entrepreneur has a higher chance to be successful and thus to receive the incentive payment \( B/\Delta p \).

This, however, assumes that good behavior is needed for funding either variant. Let us relax this assumption. Good behavior boosts the pledgeable income (as well as the NPV, for that matter) more when the payoff in the case of success is high, that is, for the risky project. Thus, suppose that the following conditions hold:
\[ I - A \geq p_0^A (R^A - \frac{B}{\Delta p^A}), \]
\[ I - A \geq p_0^B R^B, \]
\[ I - A \leq p_0^B R^B, \]
\[ I - p_0^B R^B + B. \]

The first two inequalities state that the risky variant cannot receive financing whether good behavior or misbehavior is induced by the managerial compensation scheme (note, for example, that the second inequality is automatically satisfied if \( p_0^B \) is close to its lowest feasible value \( \Delta p \)).

The third states that the risky project generates enough pledgeable income when the cash-flow rights are allocated exclusively to investors. Finally, the fourth inequality guarantees that the safe project’s NPV is positive.

To check that these inequalities are not inconsistent, assume, for example, that \( A = 0 \) and \( p_0^A R^A - I \) (or just above); then
\[ p_0^A R^A - I_{p_0^A} < I. \]

Lastly, for \( B \) large enough, the first inequality is satisfied. We conclude that the risky project may be more prone to credit rationing if high-powered incentives are not necessarily called for.

Exercise 4.15 investigates a different notion of project risk, in which a safe project yields a lower liquidation value and a lower long-term payoff and is less prone to credit rationing than a risky project.

Exercise 3.16 (scale versus riskiness tradeoff). The risky project’s NPV is
\[ I_{p_0^A} = (x p_1 - 1) I. \]

The investors’ breakeven condition can be written as
\[ x p_1 I_A = I - A. \]

And so
\[ I_{p_0^A} = x p_1 - 1 \frac{I_A}{x p_1} - 1 = 1 - x p_1 A \]

Note that this is the same formula as obtained in Section 3.4.2, except that the expected cost of bringing
1 unit of investment to completion is 1/x rather than 1.

Turn now to the safe project. The NPV is then $U^S = (R_\ell - X)/X$, and the investors’ break-even condition is

$$\rho_0 I = AX - A.$$ 

Hence,

$$U^S = \frac{R_\ell - X}{X} - \rho_0 I.$$ 

The expected cost of bringing 1 unit of investment to completion is now X.

Thus the safe project is strictly preferred to the risky one if and only if

$$X < \frac{1}{X} \quad \text{or} \quad XX < 1.$$ 

Exercise 3.17 (competitive product market interactions). The representative firm’s investment must satisfy

$$p_0 PR I \geq 1 - A,$$ 

(1)

since the manager’s reward in the case of success, $R_0$, must satisfy

$$\Delta p R_0 \geq B.I.$$

The representative entrepreneur wants to borrow up to her borrowing capacity as long as the NPV per unit of investment is positive:

$$p_0 PR I \geq 1.$$ 

(2)

In equilibrium $I = I$ and $P = P(p_0 R_I)$. Let $I^*$ (the optimal level from an individual firm’s viewpoint) be given by

$$p_0 R I = 1 \quad \text{and} \quad P = (p_0 R I)^{**}.$$

(4)

Two cases must therefore be considered, depending on whether $A$ is (a) large or (b) small:

(a) if

$$p_0 (p^* R - B/(\Delta p) I^*) \geq 1 + A,$$

then the borrowing constraint is not binding and $I = I^*$;

(b) if

$$p_0 (p^* R - B/(\Delta p) I^*) < 1 - A,$$

then (1) is binding, and so

$$I = \frac{A}{1 - p_0 (R R_0 R I) - B/(\Delta p)}.$$ 

Exercise 3.18 (maximal incentives principle in the fixed-investment model). Recall that, because the investors break even, the entrepreneur’s expected payoff when the project is financed is nothing but the project’s NPV. The entrepreneur’s expected payoff is therefore independent of the way the investment is financed. The financing structure just serves the purpose of guaranteeing good behavior by the entrepreneur. Let $R^S_\ell$ and $R^S_F$ denote the (nonnegative) rewards of the borrower in the cases of success ($R^S$) and failure ($R^F$), respectively. The incentive constraint can be written as

$$\Delta p (R^S_\ell - R^S_F) \geq B.$$ 

(1)

This constraint implies that setting $R^S_\ell$ at its minimum level (0) provides the entrepreneur with maximal incentives. So, the incentive constraint becomes

$$\Delta p R^S_\ell \geq B.$$ 

The pledgeable income is equal to total expected income minus the borrower’s minimum stake consistent with incentives to behave:

$$p_0 R^S_\ell + (1 - p_0) R^F - \frac{B}{\Delta p} = p_0 (R - \frac{B}{\Delta p}) + R^F.$$ 

Thus the project is financed if and only if

$$p_0 (R - \frac{B}{\Delta p}) \geq 1 - (A + R^F).$$ 

As one would expect, the minimum income $R^F$ plays the same role as cash or collateral. It is really part of the borrower’s net worth.

The optimum contract can be implemented through a debt contract: let $D$, $R^F < D < R^S$, be defined by

$$p_0 D + (1 - p_0) R^F = 1 - A.$$ 

(1)

That is, the borrower owes $D$ to the lenders. In the case of failure ($R^F$), the borrower defaults and the lenders receive the firm’s cash, $R^F$. Equation (1R) then guarantees that the lenders break even.

In this fixed-investment version of the model, the debt contract is, however, in general not uniquely optimal: a small reward $R^F > 0$ for the borrower in the case of failure would still be consistent with (ICa), and (IR) as long as condition (IR) is satisfied with strict inequality. By contrast, the standard debt contract is uniquely optimal in the variable-investment version of the model, as it maximizes the borrower’s borrowing capacity (see Section 3.4.3).
Exercise 3.19 (balanced-budget investment subsidy and profit tax). The total investment subsidy is \( I \) and the profit tax \( tR \). Budget balance then requires
\[
p0tR1 = I.
\]
The amount of income that is pledgeable to investors is
\[
p0\left[R - tR - \frac{B}{A}\right] I,
\]
and so the breakeven constraint is
\[
p0\left[(1 - t)R - \frac{B}{A}\right] I = (1 - s)I - A.
\]
Adding up the two equalities yields
\[
p0\left[R - \frac{B}{A}\right] I = I - A
\]
or
\[
I = \frac{A}{1 - p0}.
\]
Finally, the entrepreneur receives the NPV, \( (p0 - 1)I \), since both the investors and the government make no surplus.

Exercise 3.20 (variable effort, the marginal value of net worth, and the pooling of equity). (i) Let \( R_b \) denote the entrepreneur’s reward in the case of success. The entrepreneur is residual claimant when she does not need to borrow:
\[
R_b = R.
\]
And so she maximizes
\[
\max_p \{(pR - \frac{1}{2}p^2 - 1)\}
\]
yielding
\[
p = R
\]
and
\[
U_b = \frac{1}{2}R^2 - I > 0.
\]
(ii) More generally,
\[
p = R_b.
\]
The investors’ breakeven condition is
\[
p(R - R_b) \geq I - A
\]
or
\[
R_b (R - R_b) \geq I - A.
\]
Only the region \( R_b \geq \frac{1}{2}R \) is relevant: were \( R_b \) to be smaller than \( \frac{1}{2}R \), then \( R_b - R_b \) would yield the same pledgeable income, but a higher utility to the entrepreneur.

The highest pledgeable income is obtained when \( R_b = \frac{1}{2}R \). Thus a necessary condition for financing is that
\[
A \geq A_1,
\]
where
\[
\frac{1}{2}R^2 = I - A_1.
\]
It must further be the case that the project’s NPV be positive. That is, for the (maximum) value of \( R_b \) satisfying
\[
R_b (R - R_b) = I - A,
\]
then
\[
U_b = R_bR - I - \frac{1}{2}R_b^2 - A \geq 0.
\]
So, using the breakeven constraint to rewrite the NPV, let
\[
U_b = V(A) = \max\{R_bR - \frac{1}{2}R_b^2 - I\}
\]
s.t.
\[
R_b (R - R_b) \geq I - A.
\]
This yields the shadow price of equity, \( V'(A) \):
\[
V'(A) = \left[R - R_b(A)\right] \frac{dR_b(A)}{dA} > 0,
\]
where \( R_b(A) \) is given by the investors’ breakeven condition. For \( A > I \), we can define \( V'(A) = \frac{1}{2}R^2 - I \).

And so \( V'(A) = 0 \) (note that we discuss net utilities, so the no-agency-cost benchmark is a shadow price of cash on hand equal to 0; this benchmark is equal to 1 for gross utilities). When \( A < I \), the entrepreneur is residual claimant and exerts the socially optimal effort. For \( A < I \), \( V'(A) > 0 \), but \( V''(I) = 0 \): a local increase in the entrepreneur’s compensation just below \( R \) has only a second-order effect.

Furthermore,
\[
V''(A) < 0.
\]
Let \( A_2 < I \) satisfy
\[
V(A_2) = 0.
\]
Then
\[
A = \max\{A_1, A_2\}.
\]
(iii) Let \( I \geq I_1 \). That is, we fix \( I_1 \) and the corresponding \( V(\cdot) \) function. In the absence of an ex ante arrangement between the two entrepreneurs, each receives a net utility:
\[
\frac{1}{2}V(A)
\]
(the gross utility is \( \frac{1}{2}(V(A) + A_1) \)). For, because \( R_bR - \frac{1}{2}R_b^2 \) is concave, it is optimal for both to have

\[
\frac{1}{2}V(A)
\]
the same reward if they both invest. Thus the strategy consisting in (a) pooling cash on hand, (b) investing, and (c) setting identical reward schemes and investment yields, for each entrepreneur,\[ V(A - \frac{1}{2}(I_0 - I_1)). \]

Alternatively, the two can pool resources but only the low-investment-cost project will be funded. The expected net utility of each is then\[ \frac{1}{2}V(\max(2A, I_1)), \]

so pooling is always optimal. The lucky entrepreneur cross-subsidizes the unlucky entrepreneur if and only if\[ V(A - \frac{1}{2}(I_0 - I_1)) > \frac{1}{2}V(A), \]

so pooling is always optimal. The lucky entrepreneur cross-subsidizes the unlucky entrepreneur if and only if\[ V(A - \frac{1}{2}(I_0 - I_1)) > \frac{1}{2}V(\max(2A, I_1)). \]

The unlucky entrepreneur cross-subsidizes the lucky one if this inequality is violated. Finally, because\[ V(A) > \frac{1}{2}V(\max(2A, I_1)), \]

the cross-subsidization is from the lucky to the unlucky for \(I_0\) below some threshold.

Exercise 3.21 (hedging or gambling on net worth?).

(i) Letting \(H\) denote the entrepreneur’s stake in success (and 0 in failure), the incentive compatibility constraint is\[ (\delta p)R_H \geq R, \]

Financing is feasible if and only if\[ p_0(R - \frac{B}{\delta A}) \geq I - A. \]

The entrepreneur’s date-1 gross utility is\[ [p_H R - I] + [A - \lambda] \text{ if } A \geq \lambda \]

and\[ A \text{ if } A < \lambda. \]

If \(A \geq \lambda\), the entrepreneur’s date-0 expected gross utility is\[ U_0^H = [p_H R - I] + A, \]

if she hedges.

By contrast, and letting \(F(\varepsilon)\) denote the cumulative distribution of \(\varepsilon\), her expected utility becomes\[ U_0^H = [1 - F(\lambda - A_H)]\{p_H R - I\} + m^*(\lambda) \]

and\[ F(\lambda - A_H)\{p_H R - I\} + F(\lambda - 0)\,. \]

where\[ m^*(\lambda) = E[A | A > \lambda], \]

\[ m^*(\lambda) = E[A | A < \lambda], \]

\[ [1 - F(\lambda - A_H)]\{p_H R - I\} + F(\lambda - A_H)\{p_H R - I\} = A_H. \]

(ii) Ex post the entrepreneur chooses \(p\) so as to solve\[ \max_{p_o}(\{p\}R_H - \frac{1}{2}p^2), \]

and so\[ p = R_H. \]

The pledgeable income is\[ V = R_H(R - R_H) \]

and the NPV, i.e., the entrepreneur’s expected net utility, in the case of financing is\[ U_0 = R_H(R - I). \]

Without loss of generality, assume that \(R_H > R\) if \((R_H - \frac{1}{2}R, R_H - R)\) yields the same \(V\) and a higher \(U_0\).

Assume that \(F(\lambda - A_H) < \frac{1}{2}(R_H - I). \) This condition means that the entrepreneur can receive funding if she hedges the highest pledgeable income is reached for \(R_H = \frac{1}{2}R\). She also receives funding even in the absence of hedging provided that the support of \(\varepsilon\) is small enough (the lower bound is smaller than \(\frac{1}{2}R - (I - A_H)\)).

Let\[ V(A) = R_H(A - I), \]

where \(R_H(A)\) is the largest root of\[ R_H(R - R_H) = I - A. \]

One has\[ \frac{dV}{dA} = R_H(R - R_H) - R > 0 \]

and\[ \frac{d^2V}{dA^2} = \frac{2R}{(2R_H(A) - R)^2} \frac{dR_H}{dA} < 0. \]
Hence, $V$ is concave and so

$$V(A_0) > E[V(A_0 + \varepsilon)].$$

The entrepreneur is better off hedging.

(iii) The investment is given by the investors’ break-even condition:

$$p_0 \left( R - \frac{B - I}{B} \right) = I - A.$$

This yields investment $I(A)$, with $I' > 0$ and $I'' < 0$ if $B' > 0$, $I'' > 0$ if $B' < 0$. The ex ante utility is

$$U^b_0 = (p_0 R - 1) E[I(A_0 + \varepsilon)]$$

in the absence of hedging. And so $U^b_0 > U^g_0$ if $B' > 0$ and $U^b_0 < U^g_0$ if $B' < 0$.

(iv) When the profit is unobservable by investors, there is no pledgeable income and so

$$I = A.$$

And so

$$U^b_0 = R(A_0) \quad \text{and} \quad U^g_0 = E[R(A_0 + \varepsilon)] < R(A_0)$$

since $R$ is concave.

(v) Quite generally, in the absence of hedging the realization of $\varepsilon$ generates a distribution $G(\cdot)$ over investment levels $I = I(\varepsilon)$ and over cash used in the project $A(\varepsilon) < A_0 + \varepsilon$ such that

$$P(I(\varepsilon)) > I(\varepsilon) - A(\varepsilon),$$

where $P$ is the pledgeable income. And so

$$E[P(I)] > E[I] - A_0.$$

Drawing $I$ from distribution $G(\cdot)$ regardless of the realization of $\varepsilon$ and keeping $A_0 - E[A(\varepsilon)]$ the entrepreneur as well off.

In general, the entrepreneur can do strictly better by insulating her investment from the realization of $\varepsilon$ (in the constant-returns-to-scale model of Section 3.4, though, she is indifferent between hedging and gambling).

Consider, for example, the case $A_0 < A$ in sub-question (i). Then we know that gambling is optimal. The probability that the project is financed is

$$1 - F(A - A_0) \quad \text{and} \quad [1 - P(A - A_0)] A < A_0.$$

This last inequality states that there is almost surely “unused cash”: either $A_0 + \varepsilon > A$ and then there is no investment, or $A_0 + \varepsilon > A$ and then there is “excess cash” $[A_0 + \varepsilon - A]$.

Consider therefore the date-0 contract in which the date-1 income $r = A_0 + \varepsilon$ is pledged to investors. The probability of funding is then $X$, which allows investors to break even:

$$A_0 = X \left( I - p_0 \left( R - \frac{B - I}{B} \right) \right) = X \lambda.$$

Clearly,

$$X > 1 - F(A - A_0),$$

and so the entrepreneur’s date-0 gross utility has increased from

$$[1 - F(A - A_0)] (p_0 R - I) + A_0$$

to

$$X (p_0 R - I) + A_0.$$

Of course, this is not quite a fair comparison, since we have allowed random funding under hedging and not under gambling. But, because there is excess cash in states of nature in which $A > A$, the same result would hold even if we allowed for random funding under gambling; when $A < A$, the project could be funded with probability $x(A) = A / A$. The total probability of funding under gambling would then be

$$\int_0^{A - A_0} \lambda dF(A - A_0) + [1 - F(A - A_0)]$$

$$\lambda \int_{A - A_0}^{A} A dF(A - A_0) + \frac{\int_{A - A_0}^{A} A dF(A - A_0) - A_0}{\lambda}.$$

For more on liquidity and risk management, see Chapter 5.

Exercise 4.1 (maintenance of collateral and asset depletion just before distress). (i) When $c = 0$ (no moral hazard on maintenance), the pledgeable income is equal to (A plus)

$$p_0 \left( R - \frac{B}{B} \right).$$

Consider $c > 0$. First, suppose that the entrepreneur receives $R_0$ in the case of success, and $\eta$ in the case of good maintenance. That is, the two incentives are not linked together. The IE constraints are

$$(\Delta p) R_0 \geq B \quad \text{and} \quad \eta_0 \geq c.$$

The pledgeable income is (A plus)

$$p_0 \left( R - \frac{B}{B} \right) - c.$$
However, and as in Diamond’s (1984) model (see Section 4.2), it is optimal to link the two incentives. Let us look for conditions that guarantee that the entrepreneur both exerts effort to raise the probability of success and maintains the collateral. We just saw that it is optimal to reward the entrepreneur only if the project is successful and the asset has been maintained. Let \( R_0 > 0 \) denote this reward.

There are three potential incentive compatibility constraints:

• (work, maintain) \( \geq \) (shirk, maintain)

\[ p_0 R_0 - c \geq p_1 R_0 - c + B \]

or

\[ (\Delta p) R_0 \geq B. \]

• (work, maintain) \( \geq \) (shirk, do not maintain)

\[ p_0 R_0 - c \geq B. \]

Note that this second constraint does not bind if the first constraint is satisfied, since by assumption \( p_1 R_0 (\Delta p) \geq c \).

• (work, maintain) \( \geq \) (work, do not maintain)

\[ p_0 R_0 - c \geq 0. \]

This third constraint is not binding either. The necessary and sufficient condition for financing is

\[ p_0 \left( R - \frac{R}{\Delta p} \right) \geq I - A, \]

and the NPV is

\[ \xi_c = [p_0 R - I] + (A - c). \]

(ii) The decision over whether to maintain the collateral now depends on the realization of the signal about the eventual outcome of the project. The entrepreneur stops maintaining the asset when learning that the project will fail. When no signal accrues, the conditional probability of success (assuming that the entrepreneur has chosen probability of success \( p \in [p_0, p_1] \)) is

\[ p = \frac{p}{p + (1 - p)(1 - \xi)}. \]

The borrower maintains the asset if and only if

\[ p + (1 - p)(1 - \xi) (R_0 + A) \geq c. \]

The ex ante incentive compatibility condition (relative to the choice of \( p \)) is then (for \( c \) not too large)

\[ p_0 (R_0 + A - c) + (1 - p_1) (1 - \xi) (-c) \geq p_1 (R_0 + A - c) + (1 - p_2) (1 - \xi) (-c) + B. \]

The interpretation of the term \((\Delta p) \xi_c\) in the inequality in the statement of question (ii) is that if the entrepreneur works, she reduces the probability of receiving a signal that enables her to avoid maintenance benefiting the lenders.

(iii) • Suppose, first, that the entrepreneur does not pledge the assets. Then the condition for financing is the familiar one (with the value of collateral, \( A \), being nonpledgeable to investors):

\[ p_0 \left( R - \frac{R}{\Delta p} \right) \geq I. \]

• If the entrepreneur pledges the assets in the case of failure, then the financing condition becomes

\[ p_0 \left( R - \left( \frac{R}{\Delta p} + \xi c - A \right) \right) + (1 - p_2) (1 - \xi) A \geq I. \]

Not pledging the asset in the case of failure facilitates financing if

\[ p_0 \xi c \geq [p_0 + (1 - p_0) (1 - \xi)] A, \]

which is never satisfied if \( A > c \). Note that (1) the NPVs differ (the NPV is higher in the absence of pledging since the asset is then always maintained) and (2) more generally one should consider pledging only part of the asset.

Exercise 4.2 (diversification across heterogeneous activities). (i) Under specialization, the entrepreneur’s net utility is (see Section 3.4)

\[ U^*_i = \frac{p_i - 1}{1 - p_i} A \]

for activity \( i \). So, the entrepreneur prefers the low-NPV, low-agency-cost activity \( i \) if and only if

\[ \frac{p_i^0 - 1}{1 - p_i^0} > \frac{p_i - 1}{1 - p_i}. \]

(ii) Let \( R_0 \) denote the entrepreneur’s reward if both activities succeed \((R_0 = 0)\). The entrepreneur must prefer behaving in both activities to misbehaving in both:

\[ \{p_i^0 + p_i^1\} R_0 \geq B_i p_i^0 + B_i^0 p_i^1. \]
Now if the ratios \( \rho^1/\rho^2 \) and \( \beta^1/\beta^2 \) are sufficiently close to 1, a case we will focus on in the rest of the question, then the entrepreneur does not want to misbehave in a single activity either (the proof is similar to that in Section 4.2).

The entrepreneur solves

\[
\max \{ \rho_2^1I^1 + \rho_2^2I^2 \} \quad \text{s.t.} \quad \rho_2^1I^1 + \rho_2^2I^2 - \frac{\rho_2^1}{\rho_0^1} - \frac{\rho_2^2}{\rho_0^2} \geq \frac{\rho_1^1}{\rho_0^1}B \]

In contrast, the specialization solution solves the same program but with \( \rho_2^1/p_2^1 - \rho_1^1/p_1^1 \) replaced by \( p_0/(p_0 - p_1^1) \), which is bigger. Let

\[
\rho_0^1 = p_0R^1 - \frac{\rho_2^1}{\rho_0^1} - \frac{\rho_2^2}{\rho_0^2} \quad \text{and} \quad \rho_0^2 > \rho_0^1.
\]

Diversification reduces the agency cost, if

\[
\frac{\rho_0^1}{\rho_0^2} < 1,
\]

then the optimum is to have

\[
l^0 > I^0.
\]

But \( I^0 = 0 \) is not optimal. We need to reintroduce the incentive constraint according to which the entrepreneur does not want to shirk in activity \( \beta \) only (the one that yields the highest total private benefit); condition (2) (satisfied with equality) so as to maximize borrowing capacity, and now labeled (2'),

\[
(p_0/p_1^1)R^2 = B^lI^0 + B^lI^2, \quad (2')
\]

does not imply

\[
p_0(p_1^1 + \Delta p)R^2 \geq B^lI^0 \quad (4)
\]

if the ratio \( I^0/I^2 \) is too small. Conditions (2') and (4) (satisfied with equality) together define the optimal ratios \( I^0/I^2 \).

Exercise 4.4 ("value at risk" and benefits from diversification). Let \( R_0, R_1, \) and \( R_2 \) denote the entrepreneur’s reward contingent on 0, 1, and 2 successes, respectively. The NPV given that the entrepreneur will never receive rewards strictly above \( R \), we can reason on the risk-neutral zone in \( u(\cdot) \) and use the NPV is

\[
2[p_0R - I].
\]

To see whether the two projects can be financed simultaneously, minimize the nonpledgeable part of this NPV,

\[
\frac{1}{2}[1 + \alpha|R_2| + \frac{1}{2}[1 - \alpha|R_1| + \frac{1}{2}[1 + \alpha|R_0|, \quad (1)
\]

while providing incentives. To compute the entrepreneur’s expected compensation above, note that the probability of two successes is

\[
Pr\text{project 1 succeeds | work on project 1)} \times Pr\text{project 2 succeeds | work on project 2 and success in project 1)}
\]

or \( \frac{1}{2}(1 + \alpha) \). And so forth.

(i) The two incentive constraints are

\[
\frac{1}{2}[1 + \alpha|R_2| + \frac{1}{2}[1 - \alpha|R_1| + \frac{1}{2}[1 + \alpha|R_0| \geq 2B - R_0 \quad (2)
\]

and

\[
\frac{1}{2}[1 + \alpha|R_2| + \frac{1}{2}[1 - \alpha|R_1| + \frac{1}{2}[1 + \alpha|R_0| \geq B + R_1 + \frac{1}{2}[1 + \alpha] \quad (3)
\]

(ii) If \( R \) is large, one can then reward the entrepreneur only in the upper tail:

\[
R_2 = \frac{-8B}{1 + \alpha}
\]

This value minimizes (1) subject to (2), and also satisfies (3).

(iii) When \( R < (8B)/(1 + \alpha) \), the entrepreneur can no longer be rewarded solely in the upper tail to satisfy (2). Note that \( R_0 = 0 \) is optimal from (2) and (3). (2) can be satisfied by \( (R_2 - R, R_1 = R, R_0 = 0) \) if and only if

\[
\frac{1}{2}[1 + \alpha]R \geq B \quad (4)
\]

The question is then whether (3) is also satisfied.

- For positive correlation \((\alpha > 0)\), increasing \( R_1 \) makes (3) harder to satisfy. Hence, minimizing the nonpledgeable income requires choosing the lowest \( R_1 \) that satisfies (2). This value satisfies (3) if and only if

\[
B \geq \frac{1}{2}[1 + \alpha]R \quad \text{or, after substitutions,}
\]

\[
B \leq \frac{1}{4}R,
\]

which is more constraining than (4).

- For negative correlation \((\alpha < 0)\), increasing \( R_1 \) makes it easier to satisfy (3). While it is still optimal to set \( R_2 = R \), the binding constraint may now be (3) (and thus the nonpledgeable income exceeds \( 2B - 2p_0R/\Delta p \) here). Financing may be feasible even
though it would not be so if project correlation were positive (but $\frac{\mu}{1-\alpha} R$ must exceed $B$).

**Exercise 4.5 (liquidity of entrepreneur’s claim).** The entrepreneur’s incentive constraint when the liquidity shock is observed by investors is

$$(1 - \lambda) (\Delta p) R_0 \geq B.$$ 

The NPV is

$$U_0 = \text{NPV} = \lambda (\mu - 1) \tau_0 + p_0 R - I,$$

while the breakeven constraint is

$$\lambda (\mu - 1) \tau_0 + p_0 R - (1 - \lambda) p_0 R_0 \geq I - A.$$ 

As in the text, it is optimal to compensate the entrepreneur by providing her with liquidity (since $\mu > 1$) once $R_0$ is equal to $B/(1 - \lambda) \Delta p$. The level of liquidity, $r^*_0$, given to the entrepreneur is set by the breakeven constraint

$$\lambda (1 - \mu_0) r^*_0 + (1 - A) = p_0 \left(R - \frac{B}{\Delta p}\right).$$

It increases when more of the proceeds of reinvestment become pledgeable.

(iii) If $\lambda$ is a choice variable, the entrepreneur faces multiple tasks. She solves

$$\max_{(\lambda \in (0, 1), \tau_0 \in (0, p_0])} U_0 (p, \lambda) = \lambda [\mu - \mu_0] \tau_0 + (1 - \lambda) p_0 R_0 - \lambda c + B [1 - p_1].$$

The NPV is

$$U_0 = \text{NPV} = \lambda (\mu - 1) \tau_0 + p_0 R - I - \lambda c.$$

For a given contract $(R_0, r_0)$ the entrepreneur chooses

$$\lambda = \bar{\lambda} = \bar{\lambda}$$

if $(\mu - \mu_0) \tau_0 - p_0 R_0 \geq c$.

Note that, for $p = p_0$, the entrepreneur does not “oversearch” for new investment opportunities as long as

$$(\mu - \mu_0) \tau_0 - p_0 R_0 \leq (\mu - 1) \tau_0 \Rightarrow (\mu - \mu_0) \tau_0 \leq p_0 R_0.$$ 

Suppose that one wants to implement $p = p_0$. Then

- either $\lambda = 0$, and then the outcome is the same as in the absence of a liquidity shock;
- or, more interestingly, $\lambda = \bar{\lambda}$ (which implies a *fortiori* that $\lambda = \bar{\lambda}$ if the entrepreneur deviates and chooses $p = p_0$):

$$U_0 (p_0, \bar{\lambda}) = U_0 (p_0, \bar{\lambda}) \iff (1 - \lambda) (\Delta p) R_0 \geq B.$$

Furthermore,

$$U_0 (p_0, \bar{\lambda}) > U_0 (p_0, 0) \iff (\mu - \mu_0) \tau_0 - p_0 R_0 \geq c.$$ 

Hence, $R_0 = B/(1 - \lambda) \Delta p$, and so an added constraint with respect to subquestion (i) is

$$\lambda (\mu - \mu_0) \tau_0 \geq c + p_0 \frac{B}{(1 - \lambda) \Delta p}.$$ 

**Exercise 4.6 (project size increase at an intermediate date).** Consider first the entrepreneur’s date-1 behavior when the size has been doubled. If the entrepreneur has worked on the initial project, and using the perfect correlation between the two projects, the incentive constraint is

$$p_0 R_0 \geq p_1 R_0 + B.$$ 

If she shirked on the first project, then it is optimal to shirk again.

The date-0 incentive constraint is then

$$\lambda (\mu - 1) \tau_0 + p_0 R - I - \lambda c.$$ 

A borrower’s not receiving a reward. The incentive constraint when the liquid-

ty shock is observed by investors is

$$\lambda (\mu - 1) \tau_0 + p_0 R - (1 - \lambda) p_0 R_0 \geq I - A.$$ 

To obtain the nonpledgeable income, minimize the left-hand side of the latter inequality subject to the incentive constraints, yielding

$$R_0 = B \frac{\lambda}{\Delta p} \text{ and } R_0 = B \frac{\lambda}{\Delta p}.$$ 

Thus the nonpledgeable income is

$$(1 + \lambda) p_0 B \frac{\lambda}{\Delta p}.$$ 

**Exercise 4.7 (group lending and reputational capital).** (i) By assumption,

$$p_0 \left(R - \frac{B}{\Delta p}\right) < p_0 \left(R - \frac{B}{1 + \alpha \Delta p}\right) < I - A.$$ 

Under individual borrowing, the pledgeable income is $p_0 \left[R - (B/\Delta p)\right]$, and so individual borrowing is not feasible. Under group lending, let $R_0$ denote the bor-
rower’s individual reward when both succeed. They get 0 when at least one of them fails. The idea is that a borrower is punished “twice” for her failure: she gets no reward and also suffers from the other borrower’s not receiving a reward. The incentive con-
straint is then

$$p_0 (\Delta p)(1 + \alpha) R_0 \geq B, \quad \text{(IC)}$$
yielding pledgeable income per borrower

\[ P = p_u R - p_u \left( \min R_b \right) = p_u R - \frac{B}{(1 + a)\Delta p} \]

Hence, group lending is not feasible either.

(iii) If both players are altruistic with \( a = \frac{1}{2} \), they both cooperate in the unique equilibrium of the "stage-2" game. They have payoff \( \frac{1}{2} \), since they enjoy the monetary gain of the other agent. More precisely, the utilities in the stage-2 game are as follows:

\[
\begin{array}{c|cc}
\text{Agent 2} & C & D \\
\hline
C & 2 & 2 \\
D & 0 & 0 \\
\end{array}
\]

Cooperating is a dominant strategy (\( \frac{2}{2} > 1 \) and \( 0 > \frac{1}{2} \)), and so both cooperate.

If both agents are selfish (\( a = 0 \)), the payoff given in the statement of the question are those of a standard prisoner's dilemma and both agents defect.

(iii) The structure of payoffs is such that the altruistic agent gets nothing in the second stage if she misbehaves in the first stage. Consider the incentive constraint facing altruistic agents:

\[ p_u (\Delta p) \left( 1 + a \right) R_b + \frac{1}{2} \delta > B \]

with \( a = \frac{1}{2} \). The pledgeable income per borrower is

\[ p_u \left( R - \frac{2B}{\Delta p} + \frac{\delta}{\Delta p} \right) \]

The financing is secured if

\[ p_u \left( R - \frac{2B}{\Delta p} + \frac{\delta}{\Delta p} \right) > I - A. \]

From this, the minimum discount factor to secure financing is

\[ \delta_{\text{min}} = \frac{\Delta p}{p_u} \left( I - A \right) - (\Delta p) R + \frac{\delta}{\Delta p} > 0, \]

by assumption. The intuition is that the altruistic agent behaves in order to separate herself from the selfish agent and to build a reputation for being altruistic. The term \( \delta/\Delta p \) reflects the gain from reputation and can be interpreted as the borrower's "social collateral."

Exercise 4.9 (borrower-friendly bankruptcy court).

(i) Monetary returns, such as \( I \) and \( r \), that are not subject to moral hazard (or adverse selection) are optimally pledged to investors if financing is a constraint. This increases the income that is returned to investors without creating bad incentives for the entrepreneur.

- The entrepreneur’s incentive constraint is (for a given realization of \( r \))

\[ \{ p_u (r) - p_v (r) \} R_b \geq B \text{ or } (\Delta p) R_b \geq B. \]

Condition (1) in the statement of the question says that continuation always maximizes social (total) value. However, systematic continuation (continuation for all \( r \)) generates too little pledgeable income to permit financing (right-hand side of condition (2) in the statement); on the other hand, systematic liquidation would generate enough pledgeable income (left-hand side of (2)).

Financing requires liquidating inefficiently. Intuitively, there is then no point giving \( R_b (r) > B/\Delta p \) for some \( r \) in the case of continuation. The difference serves no incentive purpose and can be used to boost pledgeable income, allowing for more frequent continuation (in other words, it is more efficient to compensate the management with continuation rather than with money as long as incentives are sufficient). (Note: to prove this, generalize the optimization program in subquestion (ii) to allow for a choice of \( R_b (r) \) for \( r \geq r^* \)).

(ii) The borrower solves

\[
\max \text{NPV} = \max \left\{ \mathbb{E}[r] + \int_{r^*}^{r} p_v(r)f(r)\,dr \right. \\
\left. + \int_{r^*}^{\bar{r}} Lf(r)\,dr \right\} \\
\text{s.t.} \\
\mathbb{E}[r] + \int_{r^*}^{r} p_v(r)f(r)\,dr + \int_{r^*}^{\bar{r}} Lf(r)\,dr \geq I - A.
\]

Clearly, \( r^* \) is the lowest value that satisfies the breakeven constraint. Condition (2) in the statement of the question implies that \( 0 < r^* < \bar{r} \). And, of course, \( \bar{r} \geq p_r (r^*) \).

(iii) With a short-term debt contract, \( d = r^* \), the firm will be able to repay its debt and continue if \( r \geq r^* \). If \( r < r^* \), the lenders are entitled to use default to liquidate. The investors do not want to renegotiate since \( R_b (r^*) < R_b (r) \).

- \( \text{dr}^* / \Delta t < 0 \). A lower amount of equity calls for more pledgeable income.
Exercise 4.10 (benefits from diversification with variable-investment projects).

(i) The analysis follows the lines of Section 3.4. The incentive constraint on project $i$ with size $I_i$ is

$$(\Delta p_i) R_i^u \geq B I_i,$$

where $R_i^u$ is the entrepreneur’s reward in the case of success in project $i$; and so the pledgeable income is $\rho_i I_i$.

The entrepreneur allocates $A_i$ to project $i$, where

$$A_1 + A_2 = A.$$

Her total utility is

$$U_b = \rho_1 - 1 - \rho_0 A.$$

It does not really matter how the entrepreneur allocates her wealth between the two projects. In particular, there is no benefit to having a second project.

(ii) As in the case of fixed-investment projects, it is optimal to reward the entrepreneur only if the two projects succeed ($R_2^u > 0$, $R_1^u = R_0^u = 0$). The two incentive constraints are

$$p_2 H R_2^u \geq p_2 L (R_2^u + R_1^u)$$

and

$$p_2 H R_2^u \geq p_2 L (R_1^u + R_3^u + R_4^u).$$

Let

$$I = I_1^u + I_2^u.$$

Then

$$U_b = NPV = \sum_i (p_i R_i^u - I_i) = (\rho_1 - 1) I$$

and the financing condition becomes

$$p_0 R_i - p_0 R_i^u \geq 1 - A.$$

Thus, everything depends only on total investment $I$, except for the first incentive constraint. For a given $I$, this constraint is relaxed by taking

$$I^u = I^u + \frac{1}{4} I.$$

The rest of the analysis proceeds as in Section 4.2. The first incentive constraint is satisfied if the second is. And so

$$U_b = \rho_1 - 1 - p_0 A.$$
Exercise 4.11 (optimal sale policy). (i) The entrepreneur maximizes NPV,

$$\int s \left( R - \frac{B}{\Delta p} \right) f(s) ds + F(s^*) L,$$

subject to the investors' break-even constraint:

$$\int s \left( R - \frac{B}{\Delta p} \right) f(s) ds + F(s^*) L \geq I - A,$$

where use is made of the fact that the proceeds from the sale should go to investors in order to maximize pledgeable income. One finds

$$s^* \left[ R + \mu (R - B/\Delta p) \right] = L.$$

Note that $s^* R = L$ if financing is not a constraint (A large), and

$$s^* \left[ R - \frac{B}{\Delta p} \right] < L.$$

The optimal $s^*$ trades off maximizing NPV (which would call for $s^* = L/R$) and pleasing investors (which would lead to $s^* = L/(R - (B/\Delta p))$).

(Showoffs: we have assumed that it is optimal to induce the entrepreneur to exert effort when the firm is not liquidated. A sufficient condition for this to be the case is

$$(s - \Delta p) R \leq \max \left\{ L \left( s - \frac{B}{\Delta p} \right) \right\};$$

that is, the pledgeable income is always lowest under continuation and shirking. To see this, consider state-contingent probabilities $x(s)$ of continuation and working, $y(s)$ of continuation and shirking, and $z(s)$ of liquidation.

Solve

$$\max_{x(s), y(s), z(s)} \left\{ \int s x(s) (s R) + y(s) [(s - \Delta p) R] + z(s) \left( s L \right) f(s) ds \right\}$$

s.t.

$$\int x(s) \left[ s - \frac{B}{\Delta p} \right] R + y(s) [(s - \Delta p) R] + z(s) L \left( f(s) ds \right) \geq I - A$$

and $x(s) + y(s) + z(s) = 1$ for all $s$.

(iii) Endogenizing $R_0(s) > B/\Delta p$ for $s \geq s^*$ (where the threshold may differ from the one obtained in (i)), the expression for the NPV is unchanged. The break-even constraint becomes

$$\int s \left( R - R_0(s) \right) f(s) ds + F(s^*) L \geq I - A.$$

The derivative with respect to $R_0(s)$ is negative and so

$$R_0(s) = \frac{B}{\Delta p} \text{ as long as } \mu > 0.$$

(iii) It is optimal to sell if $s = s_1$. Let $R_0^{\ast} (:= B/\Delta p$ from the assumption made) be defined by

$$s_1 \left( R - R_0^{\ast} \right) = I - A.$$

If $R_0 \leq R_0^{\ast}$, then the "career concerns" incentives are sufficient to prevent first-stage moral hazard. The only possible issue is then renegotiation. That is, if $s_1(R - B/\Delta p) > L$, the two parties are tempted to renegotiate.

If in contrast $R_0 > R_0^{\ast}$, then even in the absence of renegotiation, there is first-stage moral hazard. Financing becomes infeasible.

Exercise 4.12 (conflict of interest and division of labor). (i) The incentive constraints are

$$p_0 R_0 + (1 - p_0) \tilde{R}_0 - c \geq p_0 R_0 + (1 - p_0) \tilde{R}_0 - c + B \quad \text{(no shirking on project choice)}$$

$$p_0 \tilde{R}_0 \quad \text{(no shirking on maintenance)}$$

$$p_0 R_0 + B \quad \text{(no shirking on either dimension)}.$$

The first two constraints can be rewritten as

$$(\Delta p)(R_0 - \tilde{R}_0) \geq B \quad \text{and} \quad \tilde{R}_0 \geq \frac{c - p_0}{1 - p_0}.$$

The third,

$$\Delta p)(R_0 + (1 - p_0) \tilde{R}_0) \geq B + c,$$

is guaranteed by the other two.

(ii) The nonpledgeable income is

$$\min \left\{ p_0 R_0 + (1 - p_0) \tilde{R}_0 \right\} = p_0 \frac{B}{\Delta p} + \frac{c - p_0}{1 - p_0}.$$

The financing condition is

$$p_0 R + (1 - p_0)L - p_0 \frac{B}{\Delta p} - \frac{c - p_0}{1 - p_0} \geq I - A.$$

(iii) The agent in charge of maintenance is given
and 0 otherwise. Her incentive constraint is
\[(1 - p)\hat{R}_0 \geq c.\]
So when given \(\hat{R}_0 = c/(1 - p)\), this agent exerts
care in maintaining the asset and receives no rent.

The entrepreneur’s incentive constraint then be-
comes
\[(\Delta P)\hat{R}_0 \geq B.\]

The nonpledgeable income is now
\[\hat{p}_0 \frac{B}{\Delta P} + (1 - p)\hat{R}_0 = \hat{p}_0 \frac{B}{\Delta P} + c.\]

For more on the division of labor when multiple
tasks are in conflict, see Dewatripont and Tirole
(1999) as well as Review Problem 9.5

Exercise 4.14 (diversification and correlation). (i)
The two incentive constraints are
\[p'_1[R_2] \geq p'[R_2 + 2B] \text{ and } p'_2[R_2] \geq p_0p_1R_2 + B.\]
The first constraint can be rewritten as
\[p'_2[R_2] \geq \frac{2p'_2R}{(p_0 + p_1)\Delta P}.\quad (IC)\]
The second constraint is satisfied if the first is.
The pledgeable income is
\[2p_0R - \min_{[0]} [p'_2[R_2]],\]
hence the result.
(ii) The entrepreneur receives \(p_0R_2\) by behaving
on both projects. When misbehaving (either on one
or the two projects), the entrepreneur receives ex-
pected income \(p_0R_1\). And so she might as well mis-
behave in both. The incentive constraint is then
\[p_0R_1 \geq p_0R_2 + 2B.\quad (IC)\]
And so the pledgeable income is
\[2p_0R - \min_{[0]} [p_0R_2] = 2p_0R - 2p_0\frac{B}{\Delta P}.\]
This yields the financing condition.
(iii) The incentive constraints are
\[\{xp_0 + (1 - x)p'_1[R_2]\} \geq \{xp_0 + (1 - x)p'_2[R_2\} + 2B\]
and
\[\{xp_0 + (1 - x)p'_1[R_2]\} \geq \{xp_0 + (1 - x)p_0p_1[R_2\} + B.\]

The second turns out to be satisfied if the first is.
The financing condition becomes
\[p_0\left[ \frac{R}{\Delta P} + \frac{1 - (1 - x)(1 - p_0)}{1 - (1 - x)(1 - p_0) - B} \right] \geq I - A.\]

Ex ante (before financing), \(x = 0\) facilitates financ-
ing. Ex post (after the investors have committed their
funds), the entrepreneur’s payoff,
\[\{xp_0 + (1 - x)p'_1[R_2]\],\]
is increasing in \(x\) and so \(x = 1\). Note that the NPV is
independent of \(x\):
\[U_b = [NPV - 2|p_0R - I].\]

Exercise 4.15 (credit rationing and the bias to-
wards less risky projects). (i) Note, first, that the in-
centive compatibility constraint is the same regard-
less of the choice of project specification: letting \(R_b\)
denote the entrepreneur’s reward in the case of suc-
cess (as usual, there is no point rewarding the entre-
preneur in the case of failure), the incentive compat-
ibility constraints are
\[(p'_1 - p'_2)[R_b] \geq B\]
\[\iff (p'_2 - p'_1)[R_b] \geq B\]
\[\iff (\Delta P)\hat{R}_0 \geq B.\]

The pledgeable income is therefore
\[\gamma' = \{xp_0\left[ \frac{R}{\Delta P} + (1 - x)L^2\right] \quad \text{for the safe variant, and}\]
\[\gamma' = \{xp_0\left[ \frac{R}{\Delta P} + (1 - x)L^2\right] \quad \text{for the risky one.}\]

Because \(\gamma' > \gamma\), choosing the safe variant facili-
tates funding. Lastly, \(\hat{X}\) is defined by
\[\gamma' \geq I - \hat{X}.\]

The NPV is otherwise the same for both variants, hence, \(U_b\) is the same provided the project is funded.
(ii) The entrepreneur having discretion over the
choice of projects adds an extra dimension of moral
hazard. Providing her with “high-powered incen-
tives” (\(R_b\) in the case of success, 0 in the case of failure) is ideal for encouraging good behavior in
the case of continuation, but it also pushes the

---

entrepreneur to take risks, as\(^d\)
\[
x P_0 B_0 > x P_0 B_0.
\]
More generally, any incentive scheme that addresses the ex post moral-hazard problem \((\Delta P) (B_0 - B_0) > B_0\) encourages the choice of the risky variant unless the entrepreneur receives a reward (only) when the collateral value is high \((\hat{L}_i)\). But such a reward further reduces pledgeable income and may jeopardize financing altogether when \(A < \hat{A}, \) but \(B_0 > 1 - \hat{A}\).

Exercise 4.16 (fire sale externalities and total surplus-enhancing cartelizations). (i) The representative entrepreneur’s borrowing capacity \(i\) is determined by the investors’ break-even condition:
\[
|x P_0 + (1 - x) P| = i - A,
\]
where
\[
\rho_i \equiv \frac{P_0 (B - \hat{B})}{\Delta P}
\]
is the pledgeable income per unit of investment in the absence of distress.

Because it is individually optimal to resell all assets when in distress, \(J = (1 - x)/i\), and so
\[
P = P((1 - x)/i).
\]
Furthermore, in equilibrium \(i = \hat{i}\), and so
\[
\hat{I} = \frac{1 - |x P_0 + (1 - x) P| ((1 - x)/i)}{A}.
\]
The representative firm’s NPV (or utility) is
\[
U_0 = \left[x P_1 + (1 - x) P((1 - x)/i) - 1\right] I
\]
for the value of \(I\) just obtained.

(iii) In the case of cartelization, specifying that at most \(z < 1\) can be resold on the market, and so \(J = (1 - x)/i z\), these expressions become
\[
\hat{I} = \frac{1 - |x P_0 + (1 - x) z P((1 - x)/i) z|}{A}
\]
and
\[
U_0 = \left[x P_1 + (1 - x) z P((1 - x)/i) z - 1\right] I.
\]

6. Note that the choice of the risky project is perfectly detected in the case of liquidation, since liquidation then yields only \(\hat{L}_i\) instead of the higher level \(L_i\). The entrepreneur is, however, protected by limited liability and therefore cannot be punished for the wrong choice of project, for the reader interested in contract theory: if we endogenized limited liability through large risk aversion below 0, we would need to assume that the safe project yields the low-liquidation value \(\hat{L}_i\) at least with positive probability. Otherwise, the entrepreneur could be threatened with a negative income in the case of low liquidation value and there would be no moral hazard in the choice of project.

Let
\[
H(z, I) \equiv (1 - x) z P((1 - x)/z) L > 0.
\]
Then
\[
\frac{\partial H}{\partial z} = (1 - x) [P + J P'] > 0.
\]
Hence, \(H\) decreases with \(z\) if and only if the elasticity of demand is greater than 1.

Let us check that an elasticity of demand greater than 1 is consistent with the stability condition (incidentally, the same reasoning applies to the more general case in which only a fraction \(z\) of the assets are put up for sale). Simple computations show that
\[
\frac{d I}{d z} = \frac{1 - (1 - z)^2 P'}{A}
\]
and that the conditions
\[
\frac{d I}{d z} < 0, \quad P + J P' < 0
\]
are consistent if and only if
\[
1 > x P_0 + (1 - x) P.
\]
This latter condition is not guaranteed by the fact that investment is finite \((1 > x P_0 + (1 - x)/i)\), but it is satisfied when \(z > 1\) is large enough.

When the elasticity of demand exceeds 1,
\[
\hat{I} = \frac{1 - |x P_0 + H(z, I)|}{A}
\]
decreases with \(z\), and
\[
U_0 = \left[x P_1 + (1 - x) z P((1 - x)/i) z - 1\right] I
\]
decreases with \(z\) for two reasons: both the NPV per unit of investment and the investment decrease.

Simple computations show that
\[
[A - J P'] d I = (1 - x)^2 [P + J P'] dz,
\]
and so \(d I = 0\).

(iii) Let \(\rho_i \equiv x P_1 + (1 - x) z P\). The change in total surplus is given by
\[
d(U_0 + S^0) = [\{1 - x\}[P dz + z dP] + (\rho_i - 1) dI] - (1 - x) I dP,
\]
where the first term (in brackets) on the RHS measures the change in the entrepreneur’s utility and the second term the change in buyer surplus. And so
\[
d(U_0 + S^0) = (1 - x) P dz + (\rho_i - 1) dI.
\]
The term \((1 - x)P/dz\) corresponds to a better utilization of distressed assets (which are valued \(P\) by the marginal buyer) when \(dz > 0\), while the second term (the original one from the point of view of welfare analysis) stands for the social surplus created by an increase in borrowing capacity (associated with \(dz < 0\)).

The total surplus increases when \(z\) decreases as long as
\[
\hat{\rho}_1 - 1 > 1 - \hat{\rho}_0 - (1 - x)^2z^2(A/(1 - \hat{\rho}_0))P, \\
\eta - 1
\]
where \(\hat{\rho}_0 = x + (1 - x)zP\) and \(\eta = -P'/P\).

Note that \(\hat{\rho}_1\) can be increased without bound (by increasing \(p_1\) if \(p_0\) constant, i.e., by increasing \(B\) for a given \(p_0\) without altering any other variable. So for \(\hat{\rho}_1\) sufficiently large, total surplus increases.

Exercise 4.17 (loan size and collateral requirements). When collateral is pledged only in the case of success, the investors' break-even constraint can be written as
\[
The entrepreneur receives funding if and only if
\[
\hat{\rho}_1 = \frac{\hat{\rho}_0 - (1 - x)^2z^2(A/(1 - \hat{\rho}_0))P}{\eta - 1}.
\]

The entrepreneur's incentive compatibility constraint can be written as
\[
\hat{\rho}_1 - 1 > 1 - \hat{\rho}_0 - (1 - x)^2z^2(A/(1 - \hat{\rho}_0))P, \\
\eta - 1 \quad \text{if} \quad \hat{\rho}_1 > \hat{\rho}_0
\]
where \(\hat{\rho}_0 = x + (1 - x)zP\) and \(\eta = -P'/P\).

Note that \(\hat{\rho}_1\) can be increased without bound (by increasing \(p_1\) if \(p_0\) constant, i.e., by increasing \(B\) for a given \(p_0\) without altering any other variable. So for \(\hat{\rho}_1\) sufficiently large, total surplus increases.

Exercise 5.2 (credit rationing, predation, and liquidity shocks). (i) The entrepreneur wants to carry on both projects as often as possible, as this maximizes NPV. The pledgeable income in a contract that pays \(R_0 = B/(p_0 A)\) in the case of two successes and continues in the case of first success is
\[
R_0(\hat{\rho}_1 - I) + \left(\hat{\rho}_1 - I - \frac{\hat{\rho}_0 - B}{\hat{\rho}_0}\right).
\]

Hence, if it is weakly larger than 0, then the investors break even and the second project is financed if the first one was successful. If it is strictly larger than 0, then with investors breaking even, the entrepreneur has some additional income; it is optimal to take it in the form of a stochastic loan commitment in period 1.

(ii) Intuitively, \(\xi\) weakly increases in \(K, p_0\) and decreases in \(\hat{\rho}_1\). The optimal \(\xi\) is such that
\[
\begin{align*}
\hat{\rho}_1 + \xi (1 - \hat{\rho}_0) & \left(\hat{\rho}_1 - I - \frac{\hat{\rho}_0}{\hat{\rho}_0}\right) \\
+ \left(\hat{\rho}_1 - I - \frac{\hat{\rho}_0}{\hat{\rho}_0} - (1 - \xi)\Delta \rho\right) & = 0
\end{align*}
\]
or \(\xi = 1\) if the solution to the previous equation exceeds 1.

(iii) The contract is renegotiation proof. Indeed, either \(p_0 - I - \hat{\rho}_0\Delta\rho < 0\) and then the lenders will not invest in the second project unless obliged to \(\xi\) or \(\xi = 1\) and then the borrower wants to carry on the second project.

(iv) The described sequence of short-term contracts is behaviorally equivalent to the optimal long-term contract from (i).
• To prevent predation, the entrepreneur can (publicly) secure at date 0 a credit line equal to (I1 − p1)H, or else obtain a guarantee that the date-1 project will be funded.

• Such long-term contracts are not renegotiated because they are ex post efficient (social surplus is maximized if the date-1 project is undertaken, as I0R1 > I1).

• The condition implies that unconditional financing of the two projects and date-0 shirking cannot allow investors to break even.

• x∗ is given by

\[ (\Delta q)/(1 - x^*) \left( \frac{p_0R_1}{\Delta p} \right) \geq R_0. \]

Suppose that p1 > I1. In states of nature where the initial contract specifies that the date-1 project is not financed, investors can offer to finance the project. They and the entrepreneur then get an extra rent (for example, p1 − I1 and p0R1/\Delta p if the investors make a take-it-or-leave-it renegotiation offer).

(iv) Termination is no longer a threat under renegotiation. The only way to induce the entrepreneur to behave at date 0 and date 1 is to give her, in the case of success at date 1, H0 − R1/\Delta p if profit is equal to a, and H0 > R0 if it is equal to A, such that

\[ (\Delta q)/p_0(R_0 - R_0) \geq R_0. \]

This reduces the date-1 pledgable income from p1 to

\[ p_1 - q_0p_0(R_0 - R_0) = p_1 - q_0p_0. \]

The condition in the statement of the exercise then implies that funding cannot be secured at date 0.

Exercise 5.3 (asset maintenance and the soft budget constraint). (ii) Assume that the financiers can commit not to renegotiate the initial contract. The optimal contract for the entrepreneur maximizes the NPV,

\[ U_0 = \int_0^1 \left[ F(p^*(L))p_1 - \int_0^{p_1} \rho f(\rho) d\rho - 1 \right] + \left[ 1 - F(p^*(L)) \right] I \left( g(L) \right) dL \]

subject to the financing constraint,

\[ \left\{ \int_0^1 F(p^*(L))p_1 - \int_0^{p_1} \rho f(\rho) d\rho + \left[ 1 - F(p^*(L)) \right] I \left( g(L) \right) dL \right\} \geq I - A, \]

and the incentive compatibility constraint for maintenance,

\[ \left\{ \int_0^1 \left[ F(p^*(L))(p_1 - p_0) + \Delta(L) \right] I \left( g(L) \right) dL \right\} \geq R_0I, \]

where

\[ F(L) = \frac{b(L) - \bar{g}(L)}{\bar{g}(L)} \]

is the likelihood ratio, and p1 − p0 = B/\Delta p.

Letting µ and ν denote the shadow prices of these two constraints, one gets the formulae in the statement of the question by differentiating with respect to \( p^*(L) \) and \( \Delta(L) \).

(iii) The function \( p^*(\cdot) \) obtained under commitment has slope exceeding 1 (except for very large L, for which the slope is equal to 1). This slope can be positive or negative. The soft-budget-constraint problem arises when \( p \) is smaller than \( p_1 - L \) (allowing for negative values of \( p_1 \)), i.e., for L small.

Exercise 5.4 (long-term prospects and the soft budget constraint). Go through the same steps as in Exercise 5.3, replacing \( p_1^* \) by \( p_1 + R_1 \cdot "p_1^*" \) by \( "p_1 + R_1 \cdot \) eliminating the liquidation values, and making the functions \( p^*(\cdot) \) and \( \Delta^*(\cdot) \) functions of \( R_1 \) instead of L. One finds

\[ p^*(R_1) = R_1 \left( \frac{p_1 + \mu p_0}{1 + \nu} \right) \]

and

\[ \Delta^*(R_1) = 0 \quad \text{if } \nu R_1 < \nu \]

(and if \( \Delta^*(R_1) > 0 \), then \( p^*(R_1) = p_1 + R_1 \)).

Exercise 5.5 (liquidity needs and pricing of liquid assets). (i) The borrower’s utility, conditional on receiving funds, is equal to the project’s NPV. Letting \( \langle x_l, x_h \rangle \in [0,1]^2 \) denote the probabilities of continuation in low and high-liquidity shock states, we have

\[ U_0 = (1 - \lambda)(p_1 - p_2)x_l + \lambda(p_1 - p_0)x_h - (1 - \lambda - (q-1)(p_1 - p_0)x_h. \]
Funding is feasible if
\[(1 - \lambda)((\rho_1 - \rho_0)x_1 + \lambda(\rho_1 - \rho_0)x_0) \geq I - A + (q - 1)(\rho_1 - \rho_0)X_u.\]
For, the borrower needs no liquidity in order to cover the low shock because \(\rho_0 > \rho_1\), the investors are willing to let their claim be diluted in order to continue. In contrast, the borrower needs to hoard \((\rho_1 - \rho_0)\) Treasury bonds if \(x_0 = 1\), in order to make up the shortfall between the liquidity shock and what can be raised on the capital market by diluting existing claimholders.

Clearly, \(x_1 = 1\) as this both raises the borrower’s objective function and relaxes the financing constraint. In contrast, \(x_0 = 1\) raises the objective function as long as \((q - 1)(\rho_1 - \rho_0) \leq \lambda(\rho_1 - \rho_0)\) but reduces the pledgeable income. If condition (2) in the statement of the exercise is satisfied, then \(x_0 = 1\) is indeed optimal. Otherwise \(x_0 = 0\) is optimal given the financing constraint. (Note that, were we to allow \(0 \leq x_0 \leq 1\), that is, randomized liquidation, an \(x_0 = 0\) could be optimal when condition (2) is violated.)

(ii) Suppose neither (2) nor (3) is binding. Then each firm hoards \((\rho_1 - \rho_0)\) Treasury bonds. But then there is excess demand for Treasury bonds as \(T < \rho_1 - \rho_0\).

Next, note that, for \(\lambda\) small, condition (2) cannot bind. Hence, (iii) must bind:
\[q - 1 - \lambda(\rho_1 - \rho_0) - \frac{\rho_1 - \rho_0}{\rho_2} = 0.\]

(iii) The new asset yields no liquidity premium since it yields no income in the bad state, and so \(q^* = 1 - \lambda\).

Exercise 5.6 (continuous entrepreneurial effort; liquidity needs). (i) The entrepreneur chooses probability of success \(p\) such that
\[\max_p \mathcal{F}(pR_0 - \frac{1}{2}p^2).\]

Hence,
\[p = R_0.\]

The breakeven constraint is
\[p(R - R_0) = I - A \text{ or } R_0(R - R_0) = I - A.\]

Note that this equation is satisfied for \(R_0 = \frac{1}{4}R\).

(ii) The investors’ breakeven condition is
\[I - A + \int_0^{\tau^*} p f(p) \, dp = F(p^*)R_0(R - R_0).\]

The entrepreneur maximizes
\[F(p^*)R_0^2\]
subject to the breakeven condition.

Exercise 5.7 (decreasing returns to scale). (i) The optimal policy maximizes the entrepreneur’s expected utility, which is equal to the NPV,
\[U_0 = rI + F(p^*)p_0R(I) - \int_0^{\tau^*} p f(p) \, dp \, x - 1,\]
subject to the investors’ breakeven constraint,
\[rI + F(p^*)p_0R(I) - \frac{H}{\sigma^2} \geq I - A + \int_0^{\tau^*} p f(p) \, dp \, x .\]  

Let us assume that this constraint is binding. Taking the first-order conditions with respect to \(I\) and \(p^*\), we obtain, after some manipulations,
\[p_0 \left[ R(I) - R(l) \right] = 1 - \frac{1}{\sigma^2} \left( F(p^*) - \frac{\mu}{\sigma^2} \right) \int_0^{\tau^*} f(p) \, dp.\]  

(ii) The right-hand side of (i) is decreasing in the cutoff \(p^*\). The left-hand side of (i) is decreasing in \(I\). Thus \(p^*\) and \(I\) comove positively. From \(\mathbb{E}(R_0)\), when the balance sheet deteriorates \(A\) decreases, both \(I\) and \(p^*\) decrease. This implies, in particular, that the firm issues more short-term debt.

Exercise 5.8 (multistage investment with interim accrual of information about prospects). (ii) Start with variant (a) (uncertainty about \(\tau\)). The optimal contract specifies a cutoff \(\tau^*\) above which the firm should reinvest \(I_0\).

The NPV also equal to the entrepreneur’s utility under a competitive capital market is, for a given \(\tau^*\),
\[U_0(\tau^*) = \int_{\tau^*}^\infty \left( [p_0 + \tau]R - I_0 \right) f(\tau) \, d\tau - I_0.\]

As usual, the incentive constraint (in the case of continuation) requires a minimum stake \(R_0\) in the case of success for the entrepreneur; \(R_0\) must satisfy
\[(\Delta p)R_0 \geq B.\]
So the pledgeable income

\[
\mathcal{P}(\tau^*) = \int_{\tau_1}^{\tau_2} \left[ p\tau + \tau(R - \frac{E}{\Delta p}) - I_1 \right] f(\tau) \, d\tau.
\]

Financing requires that

\[
\mathcal{P}(\tau^*) \geq I_0 - A.
\]

\(U_b\) and \(\mathcal{P}\) are maximized at \(\tau_1^*\) and \(\tau_2^*\) such that

\[
(p\tau + \tau_1^* R - I_1) = \text{and}
\]

\[
(p\tau + \tau_2^* R - \frac{E}{\Delta p}) = I_1,
\]

respectively. The entrepreneur is more eager to continue with the investors' breakeven constraint and the first-best continuation threshold \(\tau^*_1\) is consistent with financing, so \(\mathcal{P}(\tau^*_1) \geq I_0 - A\). Otherwise, continuation must be less frequent as \(A\) declines:

\[
\mathcal{P}(\tau^*_1) = I_0 - A.
\]

But at the level \(A_0\) at which

\[
\mathcal{P}(\tau^*_2) = I_0 - A_0,
\]

there is no longer the possibility to increase pledgeable income at the expense of value. For \(A < A_0\), financing cannot be secured.

- The analysis of variant (b) proceeds similarly, with

\[
U_b(R^*) = \int_{\tau_1}^{\tau_2} \left[ p\tau R - I_1 \right] f(R) \, dR - I_0,
\]

\[
\mathcal{P}(R^*) = \int_{\tau_1}^{\tau_2} \left[ p\tau R - \frac{E}{\Delta p} - I_1 \right] f(R) \, dR - I_0,
\]

\[
p\tau_1 R^*_1 = I_1,
\]

\[
p\tau_2 R^*_2 = \frac{E}{\Delta p} = I_1.
\]

(iii) For \(A = A_0\), the entrepreneur must give the entire pledgeable income in order to secure funding. So, she only takes

\[
R_b = \frac{E}{\Delta p}
\]

in the case of continuation, and

\[
R = (p\tau + \tau \frac{E}{\Delta p} = \frac{E}{\Delta p} R^*),
\]

where \(E/(\Delta p) R < 1\) in variant (a), and \(R = p\tau E/(\Delta p) R^*\) in variant (b).

Exercise 5.9 (the priority game: uncoordinated lending leads to a short-term bias). (i) The first-best allocation maximizes the NPV:

\[
\max \left( r - I_1 + \left[ p + \tau (I_1) \right] R \right),
\]

yielding

\[
\tau^* (I_1^*) R = 1.
\]

Note that \(I_1^* < R\) by assumption, and so an amount \((r - I_1^*)\) can be distributed at date 1. The date-1 payoffs, \(v_b\) and \(v_l\) to borrower and lenders, and the date-2, success-contingent payoffs, \(R_b\) and \(R_l\), must satisfy

\[
v_b + v_l + I_1^* = r,
\]

\[
R_b + R_l = R,
\]

\[
R_b = \tau^* (I_1^*) R = 1 - \delta R_b.
\]

This yields one degree of freedom.

(ii) Suppose that the entrepreneur secretly proposes the following contract to a (representative) lender: the lender's short-term claim increases by \(\delta R_b\) in exchange for the transfer of his long-term claim to the entrepreneur (by assumption, the entrepreneur is not allowed to defraud other investors of their short- or long-term claims). The lender is willing to accept this deal as long as

\[
\delta R_b \geq \left[ p + \tau (I_1) \right] (\delta R_b),
\]

Deepening investment decreases:

\[
\delta I_1 = -\delta R_b.
\]

The entrepreneur’s interim utility increases by

\[
\delta U_b = \left[ \tau^* (I_1) (1 - \delta R_b) R_0 + \left[ p + \tau (I_1) \right] (\delta R_b) \right] = \left[ -\tau^* (I_1) R_0 + \left[ p + \tau (I_1) \right] (\delta R_b) \right] > 0
\]

when \(I_1 = I_1^*\), since \(\tau^* (I_1^*) R = 1\) and \(R_0 < R\).

Note that the incentive to sacrifice the long-term profitability by increasing short-term debt decreases as \(R_b\) increases. Thus, it is optimal for the borrower to hold the smallest possible short-term claim \((v_b = 0)\) and the largest long-term claim consistent with the investors' breakeven constraint and the collusion-proof constraint:

\[
I_1 = I_1^* \Rightarrow \int - I_1 + \left[ p + \tau (I_1) \right] (R - R_b)
\]

and

\[
\tau^* (I_1) R_b = 1,
\]

where \(I_1 < I_1^*\).
Exercise 5.10 (liquidity and deepening investment). (i) Let \( R_0 \) denote the entrepreneur’s reward in the case of success (she optimally receives 0 in the case of failure). The incentive constraint, as usual, is

\[(\Delta p) R_0 \geq B.\]

The necessary and sufficient condition for financing is that the pledgeable income exceeds the investors’ outlay:

\[p_B \left( R - \frac{R}{\Delta p} \right) \geq I - A.\]

(iii) The incentive compatibility condition is not affected by a deepening investment:

\[(p_B + \tau) - (p_B + \tau) R_0 \geq B \iff (\Delta p) R_0 \geq B.\]

The investors’ breakeven condition is

\[\{F(p_B + \tau) - F(p_B + \tau) p_B\} (R - R_0) \geq I - A + \int_0^{\tau_B} \rho f(\rho) \, d\rho.\]

(iii) The NPV (or borrower’s utility) is

\[\rho^* \Leftrightarrow \left[ F(p_B + \tau) - F(p_B + \tau) p_B \right] (R - R_0) \geq I - A + \int_0^{\tau_B} \rho f(\rho) \, d\rho.\]

This NPV is maximized at

\[\rho^* = \tau R = \rho_1.\]

Because

\[R_0 \geq \frac{R}{\Delta p},\]

the first best is implementable only in Case 1, which follows. 

Case 1:

\[\{F(p_B + \tau) - F(p_B + \tau) p_B\} (R - R_0) \geq I - A + \int_0^{\tau_B} \rho f(\rho) \, d\rho.\]

Case 2:

\[\{1 + \mu F(\rho)\} \rho_0 < I - A + \int_0^{\tau_B} \rho f(\rho) \, d\rho.\]

Case 3: in the intermediate case, \(\rho^*\) is given by

\[\{1 + \mu F(p^*)\} \rho_0 = I - A + \int_0^{\tau_B} \rho f(\rho) \, d\rho.\]

(ii) Whenever \(\rho^* > \rho_0\) (which is the generic case, conditional on financing), the firm must hold liquidity in order to avoid credit rationing at the intermediate stage. The investors’ maximal return on the deepening investment, \(\mu p_B\), is smaller than the total value, \(\mu \rho_1\), of this reinvestment.

Exercise 5.11 (should debt contracts be indexed to output prices?). (i) For a given policy \(\rho^*(P)\), the NPV is

\[U_B = \hat{P} R + E\left[F(\rho^*(P)) p_B PR\right] - I - E\left[\int_0^{\tau^*(P)} \rho f(\rho) \, d\rho\right].\]

where expectations are taken with respect to the random price \(P\). The investors’ breakeven constraint is

\[\hat{P} R + E\left[F(\rho^*(P)) p_B \left(PR - \frac{R}{\Delta p}\right)\right] \geq I - A + E\left[\int_0^{\tau^*(P)} \rho f(\rho) \, d\rho\right].\]

Let \(\mu\) denote the shadow price of the budget constraint (we assume that \(\mu > 0\)). Then, taking the derivative of the Lagrangian with respect to \(\rho^*(P)\) yields

\[\rho^*(P) = p_B PR - \left(\frac{\mu}{1 + \mu}\right) \frac{p_B R}{\Delta p}.\]

(ii) To implement the optimal policy through a state-contingent debt \(d(P)\), one must have

\[\rho^*(P) = [Pr - d(P)] + \left[p_B \left(PR - \frac{R}{\Delta p}\right)\right]

or

\[d(P) = Pr - \ell_0,\]

where

\[\ell_0 = \left(\frac{1}{1 + \mu}\right) \left[p_B \frac{R}{\Delta p}\right].\]

Exercise 6.1 (privately known private benefit and market breakdown). (i) If the borrower’s private benefit \(I\) were common knowledge, then, if financed, the borrower would receive \(R_0\) in the case of success, with

\[R_0 \geq \frac{B}{\Delta p},\]

so as to induce her to behave. The project would be funded if and only if the pledgeable income exceeded the investment cost:

\[p_B \left( R - \frac{R}{\Delta p} \right) \geq I.\]
Suppose that the borrower offers a contract specifying that she will receive $R_g$, in the case of success and 0 in the case of failure (offering to receive more than 0 in the case of failure would evidently raise suspicion, and can indeed be shown not to improve the borrower’s welfare). There are three possible cases:

(a) $R_b \geq B_h/\Delta p$ induces the borrower to work regardless of her type, and thus creates an information insensitive security for the lenders, who obtain

$$p_H (R - R_b) - I \leq p_H (R - B_h/\Delta p) - I < 0$$

using (1). So, such high rewards for the borrower cannot attract financing.

(b) $R_b < B_h/\Delta p$ induces the borrower to shirk regardless of her type. The lenders’ claim is again information insensitive, and from (2) fails to attract financing.

(c) $B_h/\Delta p \leq R_b < B_h/\Delta p$; suppose that, in equilibrium, the good borrower offers a contract with a reward in this range, and that this attracts financing. A bad borrower must then “pool” and offer the same contract: if she were to offer a different contract, her type would be revealed to the capital market and her project would not be funded. Furthermore, she receives utility from the project being funded at least equal to that of a good borrower (she receives the same payoff conditional on working and a higher payoff conditional on shirking). So, she is better off pooling with the good borrower than not being funded.

We conclude that equilibrium is necessarily a pooling equilibrium. It either involves no funding at all or funding of both types. From the study of cases (a) and (b), we also know that, in the case of funding, the good type behaves and the bad one misbehaves.

(iii) A necessary condition for funding is thus that

$$[\alpha p_H + (1 - \alpha) p_L](R - R_b) \geq I.$$ 

Since $R_b \geq B_h/\Delta p$, there cannot be any lending if $\alpha < \alpha^*$. Thus

$$[\alpha^* p_H + (1 - \alpha^*) p_L](R - B_h/\Delta p) = I.$$

Thus, if the proportion of good borrowers is smaller than $\alpha^*$, the borrower may now be able to receive financing. Suppose that the borrower, regardless of her type, offers to receive $R^*_g$ in the case of success and 0 in the case of failure, where

$$[\alpha p_H + (1 - \alpha) p_L](R - R^*_b) = I.$$ 

Because $\alpha > \alpha^*$, $R^*_b > B_h/\Delta p$ and so the good borrower behaves. The investors’ break-even condition is therefore satisfied. It is an equilibrium for both types to offer contract $(R^*_b, 0)$ and for the capital market to fund the project.8

(iii) The pooling equilibrium (which exists whenever $\alpha > \alpha^*$) exhibits no market breakdown. Indeed, there is more lending under adverse selection than under symmetric information.9

• It involves an externality between the two types of borrower. The good type obtains reward

$$R^*_b = R - I / [\alpha p_H + (1 - \alpha) p_L]$$

in the case of success below that, $R - I / p_B$, that she would obtain under symmetric information. The good type thus cross-subsidizes the bad type, who would not receive any funding under symmetric information.

• The project’s NPV conditional on being funded falls from $p_B R - I$ to $[\alpha p_H + (1 - \alpha) p_L] R - I$ due to asymmetric information. The quality of lending is thus affected by adverse selection.

Exercise 6.2 (more on pooling in credit markets). A loan agreement specifying reward $R_b$ in the case of success, and 0 in the case of failure, induces a proportion $H(B_b/\Delta p)$ of borrowers to behave. This proportion is endogenous and increases with $R_b$. Thus

$$\alpha^* = (1 - \alpha^*) p_L / \alpha.$$ 

8. A more formal analysis of equilibrium behavior and of the equilibrium set can be performed along the lines of Section 6.4. We prefer to stick to a rather informal presentation at this stage.

the lender’s expected profit is

\[ U_l = H(R_0 \Delta p) p_0 (R - R_0) + (1 - H(R_0 \Delta p)) p_1 (R - R_0). \]

Because \( p_0 > p_1 \) so with only high-quality types, the level of \( R_0 \) that satisfies the break-even constraint of lenders could be larger than \( R_0 \) when they face distribution \( H \) of borrowers. Thus, there is an externality among different types of borrowers.

Under a uniform distribution on \([0, R]\) and for \( p_1 = 0 \), the level of \( R_0 \) maximizing pledgeable income is given by

\[
0 = h(R_0 \Delta p) p_0 (R - R_0) \Delta p \\
- h(R_0 \Delta p) p_1 (R - R_0) \Delta p \\
- H(R_0 \Delta p) p_0 - (1 - H(R_0 \Delta p)) p_1 \\
= h(R_0 \Delta p) [(R - R_0) \Delta p]^2 - H(R_0 \Delta p) \Delta p - p_1 \\
= \frac{1}{\alpha} (R - R_0) \bar{p}_0 - \frac{R_0}{\alpha} \bar{p}_0.
\]

Thus the pledgeable income is

\[
P(R_0) = \frac{1}{\alpha} \bar{p}_0 \bar{p}_1 R^2,
\]

and is smaller than \( I \) for \( R \) large enough.

**Exercise 6.3 (reputational capital).** (i) In this one-period adverse-selection problem, the bad type is always more eager to go on with a project than the good type. Thus, we may only have a pooling equilibrium. The assumptions imply that if we induce the bad type to work, or if we do not induce the good type to work, then the pledgeable income will not cover investment expenses. So, the only chance to receive funding is to induce the good type to work and the bad type to shirk. Under this type of contract, the pledgeable income is

\[
\alpha \bar{p}_0 + (1 - \alpha) \bar{p}_1 \left( R - \frac{R_0}{\alpha} \right)
\]

\[
= (\bar{p}_0 - (1 - \alpha) \Delta p) \left( R - \frac{R_0}{\alpha} \right).
\]

(ii) First, note that the good type always works in the first period as \( \bar{p} < \Delta p \).

In a pooling equilibrium, the bad type would always work. But then, the updated belief on the probability of the good type would still be \( \alpha \) in period 2, and from the first inequality of the last displayed set of inequalities, and the result in (i), the project would not be financed in period 2. But this implies that the bad type would be better off shirking in period 1. So there is no pooling equilibrium.

In a separating equilibrium, the bad type would not work in period 1. Then, after a success in period 1, the updated belief on the probability of the good type would be \( \alpha_1 \), and conditional on success in period 1 the project would be financed in period 2 (by the last assumed inequality) and the payoff to the borrower in the case of success would be

\[
R - \frac{I - A}{\bar{p}_0 - (1 - \alpha_1) \Delta p}.\]

That, however, means that the bad type strictly prefers to work in period 1. Thus, there is no separating equilibrium.

The semiseparating equilibrium requires that the bad type is indifferent between working and shirking in period 1, that is,

\[
B = (\Delta p) \left[ \bar{p}_1 \left( R - \frac{I - A}{\bar{p}_0 - (1 - \alpha_1) \Delta p} \right) + B \right].
\]

This determines the updated belief \( \alpha_1 \) on the probability of the good type conditional on success in period 1, and thus determines the probability of the bad type working in period 1.

**Exercise 6.5 (asymmetric information about the value of assets in place and the negative stock price reaction to equity offerings with a continuum of types).** (i) The investors receive \( R_0 \) in the case of success and 0 in the case of failure. The entrepreneur therefore issues equity if and only if

\[
(p + \tau) (R - R_0) \geq p R \iff \tau R \geq (p + \tau) R_0
\]

and so there indeed exists a cutoff \( p^* \in [p, R] \) such that the entrepreneur issues equity if and only if \( p \leq p^* \).

(ii) The investors’ break-even condition is therefore

\[
\{ E[p \mid p \leq p^*] + \tau | R_0 = l \} \text{ or } R_0 = \frac{l}{m (p^* + \tau)}.
\]

If interior, the cutoff satisfies

\[
\tau R = (p^* + \tau) R_0 \text{ or } \tau R = \frac{p^* + \tau}{m (p^* + \tau)}.
\]

Note also that \( p^* > p \); if \( p^* \) were equal to \( p \), then \( m (p^* - p) = 0 \) and so types \( p \) and just above would be strictly better off issuing equity. The condition
(m⁻¹) ≤ 1 does not suffice to guarantee uniqueness, though. Uniqueness, however, prevails if (m⁻¹) is bounded away from 1 (for example, (m⁻¹) = 2 in the case of a uniform distribution) and if R/I is close to 1.

For p* = p, m⁻¹(p*) = E[p] (the prior expectation). And so the condition stated in (ii) ensures that the cutoff is interior.

Finally, if there are multiple equilibria, the one with the highest p* yields the lowest stigma for equity issues since

\[ R_0 = \frac{f}{m\cdot (p^*) + \tau} \]

is then smallest among equilibria.

For a uniform density, the equilibrium is, as we noted, unique, and, if interior, is given by

\[ \left[ \frac{1}{2}(p^* + p) + \tau \right] R = (p^* + \tau)I. \]

(iii) Let us now look at the stock price reaction. The market value prior to the announcement of the equity issue is equal to total value (given that investors will break even on average): \[ V_0 = E[p]R + F(p^*)[R - I] \]

\[ = [F(p^*)m^{-1}(p^*) + (1 - F(p^*))m^{-1}(p^*)]R \]

\[ + F(p^*)[R - I]. \]

The ex post value of shares upon an announcement is

\[ V_1 = [m^{-1}(p^*) + \tau] R - I. \]

And so

\[ V_0 - V_1 = [1 - F(p^*)] R \]

\[ \times [m^{-1}(p^*)] R = \left[ \left[ m^{-1}(p^*) + \tau \right] R - I \right]. \]

In the case of an interior equilibrium,

\[ V_0 - V_1 = [1 - F(p^*)] R \]

\[ \times \left[ m^{-1}(p^*) - \frac{p^*}{p^* + \tau} \right] R \]

\[ = m^{-1}(p^*) - \frac{p^*}{p^* + \tau}. \]

But

\[ m^{-1}(p^*) > 1 > \frac{m^{-1}(p^*) + \tau}{p^* + \tau}. \]

Hence,

\[ V_0 - V_1 > 0. \]

(iv) Let

\[ H(p^*, \tau) = \frac{R}{T} \left[ m^{-1}(p^*) + \tau \right] - [p^* + \tau]. \]

At the Pareto-dominant, interior equilibrium,

\[ H_{p^*} < 0 \]

(where the subscript denotes a partial derivative).

Furthermore, and using the fact that H = 0 at an equilibrium,

\[ H_r = [m^{-1}(p^*) + \tau] \frac{R}{T} - \frac{p^* - m^{-1}(p^*)}{m^{-1}(p^*) + \tau}. \]

Hence, p* increase with \( \tau \). So does the volume \( 1 - F(p^*)I \).

**Exercise 6.6 (adverse selection and rating).** (i) Condition (1) means that the pledgeable income of a good (bad) borrower exceeds (is lower than) the investors’ investment \( I - A \). The pledgeable income is equal to the expected income, \( p_BR \), minus the entrepreneur’s incompressible share, \( p_BI/\Delta p \) (or \( p_BI/\Delta p \)).

- To see that no lending occurs in equilibrium, note that the bad type (type B) always derives a (weakly) higher surplus from being financed than a good type (type A). Hence, contracts that provide financing to a good type will also provide financing to a bad one (pooling behavior).

Condition (1) implies that one cannot offer a breakeven contract that induces the bad type to work. So any breakeven contract must induce misbehavior by the bad type. But condition (2) in turn implies that pooling contracts with stakes for the borrower in the interval \( [b/\Delta p, B/\Delta p] \) generate a loss for the investors.

(ii) In a separating equilibrium the good type chooses \( x \) and then offers \( R_0 \) and the bad type, which is recognized, chooses \( x = 0 \) and, from condition (1), receives no funding. Were the bad type to mimic the good type, she would get funding with probability \( 1 - x \); for, either the signal reveals the type and then she gets no funding, or the signal reveals nothing and the investors still believe they face a good type (we here use the fact that the equilibrium is separating).

Letting \( R_0 \) denote the good type’s “full information” (with net capital \( A - rx \)) contract (given by \( p_B(R - R_0) = I - A + rx \)), it must be the case that the bad type does not want to mimic the good type and prefers to keep her capital \( A \) instead. That is,

\[ A \geq (1 - x)[p_BR_0 + B] + x(A - rx) \]
R remained a monopoly at date 2 and hence could offer even if it faced no competition at date 1 and it re-
implies that a foreign bank would not lend at date 1
the local bank lends only to the good type. It offers
thermore, the condition

\[ \alpha q _{1} + \alpha p _{1} - \alpha p _{1} R = 0 \]

implies that it would not lend to a bad type even
if it faced no competition in either period. Hence,

\[ \alpha q _{1} + \alpha p _{1} - \alpha p _{1} R = 0 \]

Thus, only the local bank will lend at date 1. Fur-
thermore, the condition

\[ q R - \delta q (R - I) < 0 \]

implies that it would not lend to a bad type even
if it faced no competition in either period. Hence, the
local bank lends only to the good type. It offers

\[ R _{1} = 0. \]

In the absence of information sharing, foreign
banks do not know whether the borrower succeeded
at date 1, and therefore at date 2 (they put probabil-
ability \( p \) on the borrower’s being successful at date 2).

Note that the foreign banks do not want to make
offers to the local borrower at date 2, suppose that
they offer \( B _{0} < R \). Either the borrower will succeed
and then the local, incumbent bank will offer a bit
more \( (R _{0} + \varepsilon) \), or it will fail and then the incum-
bent will not bid. Hence, a foreign bank can win the
contest for the local firm only if the latter will fail.
Hence, they do not bid, and the incumbent bank bids
\( R _{1} = 0 \) if the borrower is successful (and does not fi-
nance otherwise). The local bank’s profit (and thus
each bank’s profit since banks do not make profits in
foreign markets) is

\[ \pi^{n s} = \alpha (p R - I + \delta p (R - I)) \]

where “ns” means “no sharing.”

The borrower’s ex ante utility is

\[ U _{b}^{n s} = 0. \]

Suppose now that banks share their information.
They are then Bertrand competitors at date 2 and
make no profit at that date. But the local bank still
lends at date 1 if the borrower’s type is \( p \); the profits
and utilities are

\[ \pi^{s} = \alpha (p R - I) \quad \text{and} \quad U _{b}^{s} = \delta \alpha p (R - I). \]

Hence, banks do not want to share their information.

(ii) Suppose now that \( \alpha \) is endogenous. Then \( C (\alpha) \)
needs to be subtracted from the borrower’s previous
utility (which is now a gross utility) in order to obtain
the net utility.

In the absence of information sharing, the bor-
rrower is held up by the local bank, and so

\[ \alpha^{ns} = \pi^{ns} - U _{b}^{ns} = 0. \]

Under information sharing, the borrower’s investment
is given by

\[ \max _{\alpha ^{s}} (\delta \alpha p (R - I) - C (\alpha ^{s})), \]

and so, for an interior solution,

\[ C (\alpha ^{s}) = \delta \alpha p (R - I). \]

Then

\[ \pi^{s} = \alpha ^{s} (p R - I) > \pi^{ns} \]

and

\[ U _{b}^{s} = \delta \alpha ^{s} p (R - I) - C (\alpha ^{s}). \]

Exercise 6.8 (pecking order with variable invest-
ment). (i) The separating program is

\[ \max \quad \left[ p _{u} R _{u}^{2} + (1 - p _{u}) R _{u}^{2} \right] \]

s.t.

\[ \left\{ p _{u} (R _{u}^{2} - R _{u}^{2}) + (1 - p _{u}) (R _{u}^{2} - R _{u}^{2}) \right\} \geq I - A, \quad (B _{u}) \]

\[ q _{u} R _{u}^{2} + (1 - q _{u}) R _{u}^{2} \leq U _{b}^{u}, \quad \text{(M)} \]

\[ (\Delta p) (R _{u}^{2} - R _{u}^{2}) \geq B. \quad \text{(C)u} \]

Note that \((B _{u})\) implies that the bad borrower
works if she mimics the good one.

(ii) The key observation is that the solution to the
separating program satisfies

\[ R _{u}^{2} = 0. \]

That is, the good borrower receives nothing in the
case of failure. In particular, if \( R _{u}^{2} \) stands for the sal-
vage value of the leftover assets, this salvage value
is entirely transferred to the investors in the case of
failure.
The proof of this observation is instructive. Suppose that \( R^*_2 > 0 \). Consider a small increase \( \delta R^*_1 > 0 \) in the borrower’s reward in the case of success and a small decrease \( \delta R^*_1 < 0 \) in her reward in the case of failure such that:

\[
p_1(\delta R^*_1) + (1 - p_1)(\delta R^*_1) = 0.
\]

This change alters neither the objective function nor the investors’ profit from the good borrower (see \( R^*_1 \)), but it relaxes the moral-hazard constraint \( (\text{CH}_1) \), and interestingly the mimicking constraint\(^\dagger\) as well since \( q_0 < p_0 \). In words, a good borrower, who has a higher probability of success, cares relatively more about her income in the case of success and relatively less about her income in the case of failure than a bad borrower.

iii) Because the weak monotonic-profit assumption is satisfied, Proposition 6.2 in the supplementary section implies that the separating allocation is the unique perfect Bayesian equilibrium allocation if and only if prior beliefs lie below some threshold \( \alpha^* \).

**Exercise 6.9 (herd behavior).** Entrepreneur 1, who moves first, chooses his best project, regardless of the state of nature. The investors then attach probability of success:

\[
m = \alpha p + (1 - \alpha) q
\]

to the project. They are willing to go along with compensation \( R^*_1 \) such that:

\[
m(R - R^*_1) = 1.
\]

Now consider entrepreneur 2. In the unfavorable environment, she has no choice but choosing the strategy that gives a probability of success. Suppose now that she herds with entrepreneur 1 in the favorable environment. Her overall probability of success when she selects the same strategy as entrepreneur 1 is:

\[
q(R - R^*_1) = 1
\]

So let \( R^*_2 \) and \( R^*_2 \) denote the second entrepreneur’s compensation in the case of success depending on whether the environment is unfavorable or favorable, respectively:

\[
q(R - R^*_1) = 1 \text{ and } (\delta R + (1 - \theta)\delta R - R^*_1) = 1.
\]

Herd behavior requires that:

\[
\frac{\partial R^*_2}{\partial \theta} \geq \frac{\partial R^*_2}{\partial \theta}
\]

This condition requires in particular that, despite herding, the choice of the same strategy by both entrepreneurs is sufficiently good news about the environment \((\delta R + (1 - \theta)\delta R - q)\) and therefore brings about much better financing terms for entrepreneur 2. It is satisfied, for example, if the project is hardly creditworthy in the unfavorable environment \((qR = l)\) and \(q\) is not too small.

**Exercise 6.10 (maturity structure).** In this simple example the good borrower can costlessly separate from the bad one by not hoarding any liquidity (i.e., setting short-term debt \( d = r \)). Because \( R^*_2 > 0 \), the good borrower knows that she will be able to find sufficient funds by going to the capital market at date 1 and diluting existing external claims. By contrast, the project will be stopped at date 1 for the bad borrower in the absence of liquidity hoarding. This would not be the case if the borrower resorted to hoarded liquidity rather than to the capital market to meet the liquidity shock.

This example is very special but it conveys the basic intuition: going back to the capital market is less costly for a good borrower than for a bad one if information about the firm’s quality accrues in between.

\(\dagger\) The mimicking constraint can be shown to be binding. If it were not binding, the solution to the separating program would be the good borrower’s full information contract. The borrower would then obtain reward \( R^*_1 \) in the case of success, and 0 in the case of failure, where \( R^*_1 \) is determined by the good borrower’s symmetric-information debt capacity. But then:

\[
a_R R^*_1 (1 - a_R R^*_1) + q_R a_R R^*_1 / 2 = q_R R^*_1 / 2.
\]

where \( a_R \) is determined by the bad borrower’s symmetric-information debt capacity. Because under symmetric information a good borrower can borrow more than a bad one, \( F^* = H^* > 0 \), and so \( IR^*_2 \) must be binding after all.
Exercise 7.1 (competition and vertical integration).
(i) The project can be financed because there is
enough pledgeable income from condition (1).
• Feasible contracts:
$$R^2 + \delta M \geq I \quad \text{and} \quad (\Delta \rho)(1 - \delta)M \geq B.$$  
For example, the debt contract,
$$R^2 = \delta I \quad \text{and} \quad \delta_0 = (I - R^2)/M$$
which amounts to a debt $D = I$, is an optimal con-
tact. To obtain it as the unique optimal contract,
one could, for example, add variable investment.
(ii) The entrepreneur obtains
$$\delta_0 = R^2 + M - I$$
under an exclusive contract with the supplier.
By contrast, the industry profit when the rival ob-
tains the enabling technology is
$$2(R^2 + D - I) + K < R^2 + M - I$$
from condition (2) and the profit-destruction effect.
Because neither the supplier’s nor the rival’s rent
(which is 0 under exclusivity) can decrease, the entre-
preneur cannot gain from nonexclusivity.
• The supplier will not find it profitable to supply
the enabling technology to the rival if and only if
$$R^2 + \delta M \geq R^2 + \delta D + \left[ R^2 + \left( D - \frac{R}{\Delta \rho} \right) - (1 - K) \right]$$  
(3)
or
$$\delta (M - D) \geq R^2 + \left( D - \frac{R}{\Delta \rho} \right) - (1 - K).$$

The term in square brackets in (3) is the differ-
ence between the rival’s pledgeable income and the
extra investment cost $I - K$. The solution is thus
to offer enough equity to the supplier. Note that
the borrower can always achieve this while main-
taining borrower incentives: $(\Delta \rho)(1 - \delta)D \geq B$. If
the borrower chose effort after observing the sup-
plier’s action, the incentive constraint would become
$(\Delta \rho)(1 - \delta_0)(M \geq B)$.

Remark. For some parameter values an optimal
debt/equity mix might involve a larger expected pay-
ment for the supplier than the investment $I$, but that
is not a problem as the entrepreneur may demand a
lump-sum payment equal to the difference up front,
thus leaving the supplier with no rent.

Exercise 7.2 (benefits from financial muscle in a
competitive environment).
(i) If $p > \rho_i(R)$, then the entrepreneur will not be able to withstand
the liquidity shock if it occurs. Hence, it needs a liquidity
cushion, perhaps in the form of a credit line.
• The NPV is
$$\left(1 - \lambda \right) [p_1(R) + \lambda (1 - \lambda) \delta p] - I, \quad \text{where} \quad z - 1 \quad \text{if the firm withstands the liquidity shock, and} \quad z = 0 \quad \text{otherwise. Hence,}
\begin{align*}
(a) & \quad z = 0 \quad \text{if} \quad p < \rho_i(R); \\
(b) & \quad z = 1 \quad \text{if} \quad p > \rho_i(R) \quad \text{and there is enough pledge-
able income to “secure a credit line,”}
\end{align*}
\begin{align*}
\rho_i(R) & \geq I - A + \lambda p \\
\text{or} \quad (1 - \lambda) p_1(R) + (1 - A) & > \lambda (p - \rho_i(R)); \quad \text{(5)}
\end{align*}
(c) $z = 0$ if (5) is not satisfied and
$$\left(1 - \lambda \right) [p_1(R) - (1 - A)] \geq \lambda (p - \rho_i(C));$$
(d) no investment takes place if
$$\left(1 - \lambda \right) [p_1(R) - (1 - A) < 1 - \lambda.$$  
(ii) Simultaneous choices: under simultaneous
choices, there is no commitment effect. Condition
(1) and question (i) imply that the incumbent does
not want to withstand her liquidity shock regard-
less of the existence of the entrant. The left inequal-
ity in (2) then implies that the entrant has enough
pledgeable income to “secure a credit line,”
$$\rho_i(R) > I - A + \lambda p$$
and there is enough pledge-
able income to “secure a credit line,”
where $z = 1$ if the firm withstands the liquidity shock, and $z = 0$ otherwise. Hence,
\begin{align*}
(a) & \quad z = 0 \quad \text{if} \quad p < \rho_i(R); \\
(b) & \quad z = 1 \quad \text{if} \quad p > \rho_i(R) \quad \text{and there is enough pledge-
able income to “secure a credit line,”}
\end{align*}
\begin{align*}
\rho_i(R) & \geq I - A + \lambda p \\
\text{or} \quad (1 - \lambda) p_1(R) + (1 - A) & > \lambda (p - \rho_i(R)); \quad \text{(5)}
\end{align*}
(c) $z = 0$ if (5) is not satisfied and
$$\left(1 - \lambda \right) [p_1(R) - (1 - A)] \geq \lambda (p - \rho_i(R));$$
(d) no investment takes place if
$$\left(1 - \lambda \right) [p_1(R) - (1 - A) < 1 - \lambda.$$  
(iii) Simultaneous choices: under simultaneous
choices, there is no commitment effect. Condition
(1) and question (i) imply that the incumbent does
not want to withstand her liquidity shock regard-
less of the existence of the entrant. The left inequal-
ity in (2) then implies that the entrant has enough
pledgeable income to “secure a credit line,”
$$\rho_i(R) > I - A + \lambda p$$
and there is enough pledge-
able income to “secure a credit line,”
where $z = 1$ if the firm withstands the liquidity shock, and $z = 0$ otherwise. Hence,
\begin{align*}
(a) & \quad z = 0 \quad \text{if} \quad p < \rho_i(R); \\
(b) & \quad z = 1 \quad \text{if} \quad p > \rho_i(R) \quad \text{and there is enough pledge-
able income to “secure a credit line,”}
\end{align*}
\begin{align*}
\rho_i(R) & \geq I - A + \lambda p \\
\text{or} \quad (1 - \lambda) p_1(R) + (1 - A) & > \lambda (p - \rho_i(R)); \quad \text{(5)}
\end{align*}
(c) $z = 0$ if (5) is not satisfied and
$$\left(1 - \lambda \right) [p_1(R) - (1 - A)] \geq \lambda (p - \rho_i(R));$$
(d) no investment takes place if
$$\left(1 - \lambda \right) [p_1(R) - (1 - A) < 1 - \lambda.$$  
(iii) Simultaneous choices: under simultaneous
choices, there is no commitment effect. Condition
(1) and question (i) imply that the incumbent does
not want to withstand her liquidity shock regard-
less of the existence of the entrant. The left inequal-
ity in (2) then implies that the entrant has enough
pledgeable income to “secure a credit line,”
$$\rho_i(R) > I - A + \lambda p$$
and there is enough pledge-
able income to “secure a credit line,”
where $z = 1$ if the firm withstands the liquidity shock, and $z = 0$ otherwise. Hence,
\begin{align*}
(a) & \quad z = 0 \quad \text{if} \quad p < \rho_i(R); \\
(b) & \quad z = 1 \quad \text{if} \quad p > \rho_i(R) \quad \text{and there is enough pledge-
able income to “secure a credit line,”}
\end{align*}
\begin{align*}
\rho_i(R) & \geq I - A + \lambda p \\
\text{or} \quad (1 - \lambda) p_1(R) + (1 - A) & > \lambda (p - \rho_i(R)); \quad \text{(5)}
\end{align*}
(c) $z = 0$ if (5) is not satisfied and
$$\left(1 - \lambda \right) [p_1(R) - (1 - A)] \geq \lambda (p - \rho_i(C));$$
(d) no investment takes place if
$$\left(1 - \lambda \right) [p_1(R) - (1 - A) < 1 - \lambda.$$  
(iii) Simultaneous choices: under simultaneous
choices, there is no commitment effect. Condition
(1) and question (i) imply that the incumbent does
not want to withstand her liquidity shock regard-
less of the existence of the entrant. The left inequal-
ity in (2) then implies that the entrant has enough
pledgeable income to “secure a credit line,”
$$\rho_i(R) > I - A + \lambda p$$
and there is enough pledge-
able income to “secure a credit line,”
where $z = 1$ if the firm withstands the liquidity shock, and $z = 0$ otherwise. Hence,
\begin{align*}
(a) & \quad z = 0 \quad \text{if} \quad p < \rho_i(R); \\
(b) & \quad z = 1 \quad \text{if} \quad p > \rho_i(R) \quad \text{and there is enough pledge-
able income to “secure a credit line,”}
\end{align*}
\begin{align*}
\rho_i(R) & \geq I - A + \lambda p \\
\text{or} \quad (1 - \lambda) p_1(R) + (1 - A) & > \lambda (p - \rho_i(R)); \quad \text{(5)}
\end{align*}
(c) $z = 0$ if (5) is not satisfied and
$$\left(1 - \lambda \right) [p_1(R) - (1 - A)] \geq \lambda (p - \rho_i(C));$$
(d) no investment takes place if
$$\left(1 - \lambda \right) [p_1(R) - (1 - A) < 1 - \lambda.$$  
(iii) Simultaneous choices: under simultaneous
choices, there is no commitment effect. Condition
(1) and question (i) imply that the incumbent does
not want to withstand her liquidity shock regard-
less of the existence of the entrant. The left inequal-
ity in (2) then implies that the entrant has enough
pledgeable income to “secure a credit line,”
$$\rho_i(R) > I - A + \lambda p$$
and there is enough pledge-
able income to “secure a credit line,”
where $z = 1$ if the firm withstands the liquidity shock, and $z = 0$ otherwise. Hence,
Exercise 7.3 (dealing with asset substitution). (i) The liquidation value $L_0$ is fully pledgeable. By contrast, only $R - R_0$ is pledged in the case of success, where

\[ p_0R_0 \geq p_0R_0 + B. \]

Hence, the left-hand side of (1) is the pledgeable income.

• With a competitive capital market, the entrepreneur’s utility is the NPV:

\[ U \equiv (1 - x)L_0 + xP_0R - I. \]

• Optimal contracts must satisfy

\[ (1 - x)(L_0 - \tau_h) + xP_0(R - R_0) = I - A, \]

with

\[ R_0 \geq B/\Delta p. \]

For $A \equiv R$ the optimal contract is necessarily a debt contract ($\tau_h = 0$).

(ii) Interpretation of equation (2). The NPV is

\[ (1 - x)L + x(P_0 + \tau)(L)R - I. \]

Hence, $L = L_0$ maximizes the NPV, which is then equal to $U^*_0$.

• Consider a “step-function” contract: in the case of liquidation, the entrepreneur receives

\[ 0 \quad \text{if } L < L_0, \]

\[ \tau_h \quad \text{if } L \geq L_0. \]

Furthermore, the entrepreneur receives $R_0 = B/\Delta p$ in the case of continuation and success (this value minimizes both the nongleddable income and the incentive to cut down on maintenance to raise future profit). With this incentive scheme, the entrepreneur’s utility

\[ (1 - x)\tau_h(L) + x\{P_0 + \tau(L)\}R_0 \]

is maximized either at $L = L_0$ or at $L = 0$. One therefore needs

\[ (1 - x)\tau_h + xP_0B/\Delta p \geq x\{P_0 + \tau(0)\}B/\Delta p. \]

The threshold for financing that does not encourage asset substitution is given by

\[ I - A^* = (1 - x)(L_0 - \tau_h) + xP_0\{R - B/\Delta p\}, \]

where $\tau_h$ is given by the first inequality satisfied with equality.

Exercise 7.4 (competition and preemption). Let us first compute the first date $t_b < t_l$ at which lenders are willing to finance an entrepreneur who will later on be a monopolist:

\[ I - e^{-\rho(t - t_0)}A = e^{-\rho(t - t_l)}(N - B/\Delta p). \]

Thus no financing is feasible before date $t_l$.

Next, compute the earliest date $t_b < t_l$ at which the entrepreneur prefers to invest (as a monopolist) rather than just consuming her endowment:

\[ NPV = e^{-\rho(t - t_b)}p_0M - I = 0, \]

where the NPV is computed from date $t_b$ on.

The condition in the statement of the question,

\[ p_0M \geq p_0\{M - B/\Delta p\} + A, \]

is equivalent to

\[ t_b \geq t_l. \]

Note that $t_b < t_l$ if $A = 0$.

(a) If $t_b > t_l$, then the equilibrium involves rent equalization, as in Fudenberg and Tirole (1985). Only one entrepreneur invests, and this at date $t_l$.

(b) If $t_b < t_l$, then we are back to a situation similar to the static game. Entrepreneurs are unable to invest before $t_l$, even though, starting from $t_b$, they would like to preempt their rival. (Again, we refer to Fudenberg and Tirole (1985) for more details about this type of situation.)

Exercise 7.5 (benchmarking). (ii) Let us write the NPV, the break-even constraint, and the incentive constraint. First, the NPV accounts for deadweight losses due to negative incomes:

\[ \tilde{\psi}_b = NPV = p[D_p(1 - \rho_1)D_b] + (1 - \rho)[p_0D + p_0(1 - \rho_1)(M + \bar{\psi}_b)] - (1 - p_0)\tilde{\psi}_b - I. \]

The breakeven constraint is $\rho(p_a(D - a_2) + (1 - \rho)p_l(b_1))$, 
+ \rho \geq 1 - p_l(a_1 + b_1) + (1 - p_l)^2b_2 \geq I - A$. 

Lastly, the incentive constraint is $\rho(a_2 + (1 + \theta)b_1)$ 
+ \rho \geq 1 - p_l(a_2 + (1 + \theta)b_1)
+ (1 - p_l)(a_2 + (1 + \theta)b_1) \geq \frac{B}{\Delta\rho}$

To show that one can set $a_2 = b_1 = 0$ without loss of generality, write the Lagrangian and the first-order condition. Equivalently, if $a_2 > 0$, we can decrease $a_2$ and increase $a_1$ so as to keep both (IR) and (IC) unchanged, and note that these two variables do not enter into the expression of the NPV; while, if $b_2 > 0$, we can decrease it and increase $b_1$ so that (IR) and the NPV are kept intact, but (IC) is then not binding.

The diagrammatic representation of the problem in the $(a_1, b_1)$-space is as in Figure 4.

(iii) When $\rho$ tends to 1, $b_1$ going to infinity has almost no cost in terms of NPV. Thus (IC) becomes costless to satisfy, as in Section 7.1.1 in the case of perfect correlation.

- When $\theta$ goes to 0, then punishments are almost costless, and so again (IC), can be satisfied without jeopardizing (IR). Again there is basically no agency cost (as in the case in which firms have a large amount of collateral that the lenders value almost as much as the borrower).

**Exercise 7.7 (optimal contracts in the Bolton–Scharfstein model).** Consider a more general long-term contract in which the entrepreneur's reward contingent on different events is $r_i^g$ if date-0 profit is $D$ but there is no refinancing at date 1 (with probability $\rho$). And, if refinanced, $g_i^1$ ($g_i^2$) when the entrepreneur succeeds in both periods (when she fails at date 0, but succeeds at date 1, respectively). When reinvesting at date 1, to "commit to" high effort, the entrepreneur should keep a high enough stake, i.e., $r_i^1$ and $r_i^2$ need to be high enough to satisfy the incentive constraint.

Fixing the continuation policy $z^3$ and $z^4$ as long as the high effort is guaranteed the predation deterrence constraint is not affected by this enrichment of the contract space: $D \geq (z^4 - z^3)(M - D)$. (PD)

The date-0 incentive compatibility constraint and investor's breakeven constraint, however, need to be modified:

$z^n \geq B \geq 0 \geq p_a(z^n \geq 1 - z^n r_i^g) + (1 - p_a)z^n r_i^g$

$\geq (1 - p_a)z^n r_i^g \geq B \geq 0$ (IC)

and

$I - A \leq z^n(D + D - R_i^2 - 1) + (1 - z^n)(D - R_i^2)$

$\geq (1 - A) \leq D + z^n(D - 1 - R_i^2) - (1 - z^n)r_i^g$ (IR)

The entrepreneur's expected utility is $
U_j = z^n R_j^1 + (1 - z^n)A - APV - D - I + z^n(D - 1)$, as usual, when (IC) is binding.

As in Section 7.1.2, suppose (PD) is binding, (IC) is binding; for, if it were not, $z^3$ could be increased to relax (PD) without violating (IC).

Then one can show that

- $R_j^1 \geq R_j^2$ ($\geq B/Ap$); if $R_j^1 < R_j^2$, then $R_j^2$ could be reduced so as to relax (IC), which would
contradict the fact that \((IC')\) is binding. And so 
\[ R_1^b = B/\Delta p. \]
• \(r_0^b = 0\): suppose \(z^b \in (0,1)\) and \(r_0^b > 0\) (if \(z^b = 1\), we could simply set \(r_0^b = 0\)). From \((FD)\) being binding, the incentive constraint can be written as 
\[ z^b(R_1^b - R_1^B) + (1 - z^b)r_0^b + \frac{D}{M-D}R_1^b - \frac{B}{\Delta p}. \]
Keeping \(z^b\) unchanged, we can decrease \(r_0^b\) and increase \(R_1^b\) so that \(z^b(R_1^b - R_1^B) + (1 - z^b)r_0^b\) remains the same, i.e., in the case of date-0 success, one rewards the entrepreneur only in the case of continuation. There is no loss of generality in doing so since no constraint is affected, nor is the entrepreneur’s objective function.

Exercise 7.8 (playing the soft-budget-constraint game vis-à-vis a customer). (i) At date 2, given success and in the absence of a date-1 contract, the customer would offer a purchasing price equal to 0 (or any arbitrarily small but positive amount) and the entrepreneur would accept. In this event, the entrepreneur and the investors get zero profit. Therefore, by playing wait-and-see, the customer would enjoy expected payoff \(p_1 v\) since the entrepreneur would shirk under this strategy. The same outcome prevails if the customer offers \(R = 0\) at date 1.

Given that the entrepreneur has obtained funding at date 0, to induce a high probability of success at date 1 the customer needs to offer a price \(R = R_1 + B/\Delta p\). This is more profitable for the customer than offering a contract that is not incentive-compatible:
\[ p_1\left(v - R_1 - \frac{B}{\Delta p}\right) > p_1 v. \]
When this inequality holds, the NPV is
\[ p_1(R_1 + B/\Delta p) - 1, \]
which is smaller than \((\Delta p)v - 1\). On the other hand, if the condition above is violated, it is optimal for the customer to offer \(R = 0\). But in this case the entrepreneur shirks and the project is not financed at date 0.

(ii) Suppose now that the entrepreneur issues short-term debt \(r_1\) at date 0. At time 1 the customer has to cover \(r_1\) in order for the firm to continue. It is as if date 1 were an initial financing stage at which the customer finances an investment with size \(r_1\).

The short-term debt can be chosen such that the customer finances the project only if the entrepreneur works, i.e.,
\[ p_2 v < r_1. \]

Then, to induce the high effort, the customer offers a transfer price \(R = B/\Delta p\), on top of \(r_1\). The customer gets
\[ p_1\left(v - \frac{B}{\Delta p}\right) - r_1. \]

By assumption \(p_1(v - R/\Delta p) > p_1 v\). It is possible to extract the full surplus from the customer by setting \(r_1 = p_2(v - R/\Delta p)\). This amount is greater than \(I - A\) by assumption and so investors are willing to finance the project at date 0. The entrepreneur then gets
\[ p_1\left(v - \frac{B}{\Delta p}\right) - A + (r_1 - (I - A)), \]
which is equal to the NPV, \(p_2 v - I\). This is intuitive since both the initial investors and the customer get zero profit.

Exercise 7.9 (optimality of golden parachutes).

Consider the following class of contract: when the entrepreneur reports a signal \(s \in \{r,q\}\), the probability of continuation is \(z^s\). She is paid \(R_1^s\) in the case of continuation and success, and \(y^s\) in the case of termination. In the latter event, the investors get
\[ U_s = I - T^s \leq L. \]

In the case of continuation, in order to overcome the moral-hazard problem, both \(R_1^s\) and \(R_1^b\) must exceed \(B/\Delta p\). For the \(q\)-type entrepreneur, the (NM) constraint is now
\[ z^q(q_s - v)R_1^q + (1 - z^q)T^q \leq z^q q_0 R_0^q + (1 - z^q)T^q, \]
\((NM)\)

The investors’ breakeven condition is
\[ I - A \leq \alpha(z^q q_0 R_0^q + (1 - z^q)T^q) \]
\[ + (1 - \alpha)(z^q q_0 R_1^q + (1 - z^q)T^q) - A \]
and the entrepreneur gets expected payoff
\[ U_h = \alpha(z^q q_0 R_0^q + (1 - z^q)T^q) \]
\[ + (1 - \alpha)(z^q q_0 R_1^q + (1 - z^q)T^q) \]
\[ - \text{NPV} \]
\[ = \alpha(z^q q_0 R + (1 - z^q)L) \]
\[ + (1 - \alpha)(z^q q_0 R + (1 - z^q)L) - I, \]
under the investors’ breakeven condition.
We claim that the following properties hold.

• (NM') is binding. Otherwise, we could decrease either \( R_t' \) or \( T^a \) and increase the pledgeable income unless \( R_t' = B/\Delta p \) and \( T^a = 0 \). But, in the latter case, from (NM) being slack, we must have \( z^d > 0 \), then from \( L > q_0(R - B/\Delta p) \) the pledgeable income can be increased by reducing \( z^d \).

• \( R_t' = B/\Delta p \); if \( R_t' > B/\Delta p \), decreasing it boosts pledgeable income and relaxes (NM).

• \( T^a = 0 \); suppose \( T^a > 0 \) and \( z^d < 1 \) (when \( z^d = 1 \), we can simply set \( T^a = 0 \)). Following the logic of Section 7.2.1, a simultaneous change of \( T^a \) and \( z^d \) that keeps the pledgeable income constant must satisfy

\[
\left[ r_1 \left( R - \frac{B}{\Delta p} - L + T^a \right) \right] d z^d = (1 - z^d) d T^a.
\]

By doing so, the LHS of (NM') changes by an amount equal to

\[
\left[ r_1 \left( R - \frac{B}{\Delta p} - L + (q_0 - \tau) \frac{B}{\Delta p} \right) \right] d z^d;
\]

(NM') is relaxed by a simultaneous decrease of \( T^a \) and \( z^d \). (If \( z^d = 0 \), we could instead decrease \( T^a \) to relax (NM') and increase the pledgeable income.)

Incorporating these findings, the program becomes

\[
\text{max} \text{NPV} = \alpha[L + z^d (r_1 (R - L))]
\]

s.t. \( z^d (q_0) = \alpha \left[ \frac{L + z^d (q_0 (R - L)) - L}{L} \right] \)

\( 1 - A = \alpha \left[ \frac{L + z^d (q_0 (R - L)) - L}{L} \right] \)

\( \frac{1 - \alpha}{L} = \alpha \left[ \frac{L + z^d (q_0 (R - L)) - L}{L} \right] \)

\( \frac{(1 - \alpha)(1 - \rho)}{L} \)

• When \( q_0 R > L \), it is optimal not to adopt the golden parachute policy, \( T^a = 0 \): suppose \( T^a > 0 \). First, note that to satisfy (NM') as an equality, \( T^a < q_0 B/\Delta p \) in \( q_0 B/\Delta p \) as long as \( t > 0 \). Therefore, an increase in \( z^d \) relaxes (NM') and increases the NPV. Consider a simultaneous change in \( z^d \) and \( T^a \) that leaves (NM') unchanged:

\[
(q_0 B/\Delta p - T^a) d z^d = -(1 - z^d) d T^a.
\]

Since \( T^a < q_0 B/\Delta p \), a decrease in \( T^a \) comes with an increase in \( z^d \), which increases the NPV. This change is feasible since the pledgeable income is increased:

\[
\text{dP} = \left[ q_0 (R - B/\Delta p) - 1 + T^a \right] d z^d - (1 - z^d) d T^a
\]

\( = \left[ q_0 R - L \right] d z^d > 0. \)

• When \( q_0 R < L \), a golden parachute is optimal, \( T^a = 0 \) and \( z^d = 0 \). From \( T^a < q_0 B/\Delta p \), the relevant part in the pledgeable income can be written as

\[
L + z^d (q_0 R - L - (q_0 B/\Delta p) - T^a) - T^a,
\]

therefore decreasing \( z^d \) raises both the pledgeable income and the NPV. At the optimum \( z^d = 0 \), and the optimal \( T^a \) is determined by (NM)

\[
T^a = z^d (q_0 - \tau) \frac{B}{\Delta p}.
\]

It is also easy to check for both cases that the (NM) constraint of the \( r \)-type entrepreneur is not binding.

Exercise 7.10 (delaying income recognition). We look for a “pooling equilibrium” in which the entrepreneur keeps a low profile \((\gamma_1 = 0)\) when successful \((\gamma_1 = R_1)\). To this end, let us compute the posterior probability \( \alpha_2 \) that the entrepreneur has high ability at date 2 (H2) following (reported) profit 1 at date 1 and (actual and reported) profit 2 at date 2:

\[
\alpha_2 = \text{Pr}(H_2 | (0, R_1)) = \frac{A + B}{\tau + D}
\]

where \( A = \alpha p(r + \tau + q), B = (1 - \alpha)(1 - p)(r + q + \tau), C = \alpha(\rho - (1 - \rho)q + \tau), D = (1 - \alpha)(1 - \rho)(r + \rho q + \tau) \). The numerator represents the probability that the entrepreneur has ability \( H_2 \) and succeeds at date 2 with probability \( \alpha_2 \); she has high ability at date 1 and still has high ability and so has average probability of success \( r + \tau \) (due to the date-1 hidden savings made when she is successful at date 1, which has probability \( \alpha_2 \)) with probability \( (1 - \alpha)(1 - \rho) \) she had low ability at date 1 (and therefore had hidden savings with probability \( q \) and became expert in the task (and so has probability \( r + q + \tau \). The denominator represents the total probability of date-2 success in this pooling equilibrium, and is computed in a similar way.

By contrast, the probability that the entrepreneur has type \( H_2 \) when she fails at date 2 is

\[
\alpha_3 = \frac{E + F}{G + H} < \alpha_2,
\]
where \( E = \alpha\rho[r - q\tau], F = (1 - \alpha)(1 - \rho)[1 - r - q\tau], G = (1 - \alpha)(1 - \rho)[1 - r + q\tau] \), and \( H = (1 - \alpha)[1 - (1 - \rho)r + \rho q + q\tau] \).

Suppose now that the entrepreneur reports \( \hat{y}_1 = R_i \). Let

\[
\begin{align*}
\sigma_{\text{BM}} &= \Pr(H_2 | (R_1, R_2)) = \frac{I}{J + K} \\
(\text{where } I = [\alpha\rho r + (1 - \alpha)(1 - \rho)q]\tau, J = [\alpha\rho r + (1 - \alpha)(1 - \rho)q]\tau, \text{ and } K = [\alpha(1 - \rho)r + (1 - \alpha)p]\tau) \text{ and} \\
\hat{\sigma}_{\text{BM}} &= \Pr(H_2 | (R_1, 0)) = \frac{N}{M + O} \\
(\text{where } M = [\alpha\rho r + (1 - \alpha)(1 - \rho)q](1 - \tau), N = [\alpha\rho r + (1 - \alpha)(1 - \rho)q](1 - \tau), \text{ and } O = [\alpha(1 - \rho)r + (1 - \alpha)p](1 - \tau))
\end{align*}
\]

the case of success

\( \hat{\sigma}_{\text{BM}} \) denote the posterior beliefs when such an "early bloomer" (EB) succeeds and fails at date 1 when

\[\alpha > \alpha_{\text{EB}} \text{ and } \hat{\sigma}_{\text{BM}} > \sigma_{\text{EB}}.\]

Intuitively, a late success is more telling than an early one if either the type has a reasonable probability to evolve or if an early success confirms what one already knows, namely, that the entrepreneur has high ability.

Now assume that

\[\alpha_{\text{EB}} > \alpha > \hat{\sigma}_{\text{BM}} > \sigma_{\text{EB}}.\]

Then, the entrepreneur keeps his job at date 3 if and only if she succeeds at date 2. Keeping a low profile at date 1 when \( y_1 = R_i \), is then the optimal strategy because it increases the probability of date-2 success by \( \tau \).

Exercise 8.1 (early performance measurement boosts borrowing capacity in the variable-investment model). In the variable-investment model, the private benefit of shirking is \( B_I \), and the income in the case of success \( R_I \). Using the notation of Section 8.2.2, the incentive compatibility constraint is

\[\sigma_{\text{MB}} - \sigma_{\text{UB}} R_0 \geq B_I,\]

where \( R_0 \) is the entrepreneur's reward in the case of success. The borrowing capacity is then given by the investors' break-even constraint:

\[p_3 R_I - \sigma_{\text{MB}} \frac{B_I}{\sigma_{\text{UB}}} - \sigma_{\text{MB}} = I - A.\]

And so

\[U_3 = \sigma_{\text{MB}} R_0 - A - (p_3 R_I - 1)I = \frac{1 - (p_3 - 1)A}{\sigma_{\text{MB}} - \sigma_{\text{UB}}} - A.\]

In the absence of an intermediate signal, the expression is the same except that \( \sigma_{\text{MB}}(\sigma_{\text{MB}} - \sigma_{\text{UB}}) \) is replaced by \( p_3 / (p_3 - p_1) \).

Exercise 8.2 (collusion between the designated monitor and the entrepreneur). When the signal is high, there is no collusion. In the absence of collusion, the entrepreneur obtains \( \hat{b}_0 \), since it is in the interest of the monitor to exercise his options. Furthermore, the entrepreneur cannot receive more than \( \hat{b}_0 \), from the assumption that the entrepreneur cannot receive income without being detected.

Suppose therefore that the signal is low. In the absence of collusion, the entrepreneur and the monitor both receive 0. Suppose that the entrepreneur instead offers to funnel resources to the monitor. For a given choice of \( \tau \), the monitor agrees to collude if and only if his loss from exercising the options is compensated by the diverted resources:

\[s[p_3 - (\nu_1 - \tau)]R < T(\tau).\]

There is no collusion provided that

\[H(s) = \max_{\nu_1}(T(\tau) - s[p_3 - (\nu_1 - \tau)]R) < 0.\]

Because \( \partial H / \partial s < 0 \), there is no collusion provided that \( s \) exceeds some threshold.

Exercise 9.1 (low-quality public debt versus bank debt). Consider the three possible financing options. High-quality public debt. Such debt has probability \( p_3 \) of being reimbursed. As usual, the incentive constraint is

\[p_3 R_I - \sigma_{\text{MB}} \frac{B_I}{\sigma_{\text{UB}}} - \sigma_{\text{MB}} = I - A,\]

and so such financing is doable only if

\[A_3 = I - p_3 \left( \frac{R_I - B_I}{\sigma_{\text{UB}}} \right).\]
The entrepreneur’s utility is then the NPV:

\[ U_0^I = p_0 R - I > 0. \]

Low-quality public debt. Such debt corresponds to the case in which the entrepreneur has too low a stake to behave; and this debt is repaid with probability \( p_1 \):

\[ (\Delta p) R_0 < B \quad \text{and} \quad p_1(R - R_0) = I - A. \]

Hence,

\[ A_1 = I - p_1 R. \]

The entrepreneur’s utility is then

\[ U_0^I = p_1 R + B - I > 0. \]

Monitoring. Follow the treatment in Chapter 9. To secure such financing with stake \( R_0 \) for the monitor:

\[ (\Delta p) R_0 = c \quad \text{and} \quad p_0 R_0 - c = I_0. \]

And so a necessary and sufficient condition is

\[ p_0 \left( R - \frac{b}{\Delta p} \right) - c \geq I - A. \]

yielding threshold

\[ A_2 = I + c - p_0 \left( R - \frac{b}{\Delta p} \right), \]

and NPV

\[ U_0^I = p_0 R - I - c. \]

Summing up, under the assumptions made in the statement of the exercise:

\[ U_0^I > U_0^I > U_0^I > 0 \quad \text{and} \quad A_1 > A_2 > A_1. \]

So, financing is arranged as described in the statement of the question.

(A similar framework is used by Morrison\(^{12}\), except that the monitor is risk averse (which makes it more costly to hire). Morrison allows the monitor to contract with a “protection seller” in the credit derivative market in order to pass the default risk on to this third party and to thereby obtain insurance. This reduces the monitor’s incentive to monitor.)

Exercise 9.2 (start-up and venture capitalist exit strategy). (i) When the date-2 payoff can be verified at date 1, and there is no active monitor, the entrepreneur’s reward, \( R_0 \), in the case of success must ensure incentive compatibility and allow investors to recoup their date-0 outlay:

\[ (\Delta p) R_0 > B \quad \text{and} \quad p_0(R - R_0) \geq I - A. \]

Because

\[ I - p_0 \left( R - \frac{b}{\Delta p} \right) > A, \]

these two conditions are mutually inconsistent.

Suppose, in contrast, that an active monitor receives \( R_A \) in the case of success. We now have two incentive compatibility conditions and one breakeven condition:

\[ (\Delta p) R_A > B, \]

\[ (\Delta p) R_A > c_A, \]

and

\[ p_0(R - R_0 - R_A) \geq I - A. \]

Because

\[ A > I - p_0 \left( R - \frac{b + c_A}{\Delta p} \right), \]

these inequalities are consistent. The second and the third inequalities then bind, and so the NPV for the entrepreneur (which is equal to the total value created by the project minus the rent received by the monitor) is

\[ p_0 R_0 - A = p_0 \left( R - \frac{b + c_A}{\Delta p} \right) - I. \]

(ii) The conditions are

\[ p_0 s[R - P] \geq c_P \]

(the speculator makes money when he acquires information and exercises his call option in the case of good news),

\[ (\Delta p)sP \geq c_A \]

(this is the previous IC constraint with \( R_A = sP \)), and

\[ P \geq p_0 R \]

(the speculator cannot make money by refusing to monitor and purchasing the shares at price \( P \)).

Ignoring the last constraint yields the condition in the statement of the exercise. The third constraint requires that

\[ \frac{c_A}{c_P} \geq \frac{1 - p_0}{p_0(\Delta p)s}. \]

If this condition is not satisfied, the speculator does not have enough incentives to acquire the information when only the shares of the active monitor are

brought to the market at date 1. This means that the active monitor should be granted the right to “drag along” the shares (or some of the shares) of the limited partners in order to ensure the stock receives enough attention.

Exercise 9.3 (diversification of intermediaries).
(i) Straightforward. Follows the lines of Chapters 3 and 4.
(ii) Similar to Chapter 4’s treatment of diversification.
The presence of a monitor facilitates funding if and only if
\[ p_m R_m \geq p_u p_h R_u + c \]
(no shirking on monitoring one firm)
\[ \geq p_m R_m + 2c \]
(no shirking on monitoring both firms).
As usual, it can be checked that only the latter constraint is binding. So
\[ R_m \geq \frac{2c}{(\Delta p)/(p_u + p_h)}. \]
The nonpledgeable income (aggregated over the two firms) is
\[ \Delta R = \left[ \frac{p_m b}{\Delta p} + p_u \left( \frac{p_u}{p_u + p_h} \right) c \right] I. \]

Exercise 9.4 (the advising monitor model with cap-
tion).
The entrepreneur’s utility when enlisting a monitor is now equal to the NPV minus the rent derived by the monitor:
\[ U^e = (p_u + q_0) \left( R - \frac{c}{\Delta p} \right) - I. \]
Note that \( U^e \) may no longer exceed
\[ u_{\text{non}} = p_u R - I, \]
even when \( (\Delta p)R > c. \)
Funding with a monitor on board is feasible if and only if
\[ (p_u + q_0) \left( R - \frac{c}{\Delta p} \right) \geq I - A. \]
The presence of a monitor facilitates funding if and only if
\[ (p_u + q_0) \left( R - \frac{c}{\Delta p} \right) > p_u \left( R - \frac{B}{\Delta p} \right) \]
or
\[ q_0 R > c + p_u \frac{c}{\Delta p} + q_0 \frac{B}{\Delta p}. \]
The left-hand side is the increase in expected revenue; the right-hand side is the sum of the monitoring cost and the extra rents for the two agents.

Exercise 9.5 (random inspections).
(i) Suppose first that the entrepreneur behaves with probability \( 1; \) then there is no gain from monitoring and so \( y = 0. \)
But, in the absence of monitoring, the entrepreneur prefers to misbehave:
\[ (\Delta p)R_0 < E, \]
a contradiction. Conversely, suppose that the entre-
preneur misbehaves with probability \( 1; \) because
\[ vR_m > c, \]
the monitor monitors for certain \( (y = 0). \)
But then the entrepreneur prefers to behave as
\[ p_m R_m > 0. \]
Hence, the entrepreneur must randomize. For her to be indifferent between behaving and misbehaving, it must be the case that
\[ p_m R_m = y(p_u R_u + B) + (1 - y) \cdot 0 \]
or
\[ y = \frac{p_m R_m}{p_u R_u + B}. \]
Similarly, the monitor must randomize. Indifference between monitoring and not monitoring implies that
\[ (1 - x)p_u R_u + x(p_u + v) R_m - c \]
\[ = (1 - x)p_m R_m + x p_h R_m \]
or
\[ x vR_m = c \iff x = \frac{c}{\nu R_m}. \]
(ii) Assume that \( p_u (R - B/\Delta p) < I - A, \) so that financing is not feasible in the absence of a monitor. As usual, one should be careful here: because the monitor has no cash and thus cannot be asked to contribute to the investment and gets a rent, the borrower’s utility differs from the NPV,
\[ U_b = (1 - x)p_u R_u + xy(R + p_h R_m) - A \]
\[ = p_m R_m - A, \]
using the indifference condition for the entre-
preneur. The uninformed investors’ break even condi-
tion is
\[ P = (1 - x)p_H(R - R_0) + x(y)p_0(R - R_0) + (1 - y)(p_0 + v)(R - R_0) \]
\[ > I - A. \]

Note that \( y = 0 \) maximizes \( P \). First, if \( x > 0 \), a smaller \( y \) increases the amount of money returned to uninformed investors when correcting misbehavior. Second, it raises managerial discipline (reduces the level of \( R_0 \) necessary to obtain incentive compatibility); indeed \( R_0 \) can be taken equal to 0! (Note this would no longer hold if the entrepreneur could capture private benefit to uninformed investors when correcting misbehavior.)

The pledgeable income is then
\[ P = [(1 - x)p_H + x(p_0 + v)](R - c - \frac{c}{x}). \]

Noting that \( \frac{dP}{dx} > 0 \) at \( x = 0 \) and \( \frac{dP}{dx} < 0 \) at \( x = 1 \), the pledgeable income is maximized for \( x \) between 0 and 1. (The optimum does not, of course, involve \( R_0 = 0 \). We are just computing what it takes to obtain financing.)

(iii) We know from Chapter 8 that the entrepreneur works best rewarded on the basis of a sufficient statistic for her performance. Here, the monitor’s information is not garbled by exogenous noise, unlike the final outcome. Hence, it would in principle be better to reward the management on the basis of information disclosed in an incentive-compatible way by the monitor. We leave it to the reader to derive the optimal contract when one allows the monitor to report on his observation of the entrepreneur’s choice of effort.

Exercise 9.6 (monitor’s junior claim). Let \( R_0^1 \) and \( R_0^2 \) denote the entrepreneur’s rewards in the cases of success and failure. We are interested in situations in which the entrepreneur would choose the bad project if left unmonitored:
\[ (\Delta p)\left( R_0^1 - R_0^2 \right) < b. \]

Under monitoring, incentive compatibility requires that
\[ (\Delta p)\left( R_0^1 - R_0^2 \right) \geq b, \]
where \( \Delta p = p_H - p_0 \).

Similarly, the monitor’s compensation scheme must satisfy
\[ (\Delta p)\left( R_0^2 - R_0^1 \right) \geq c. \]

The uninformed investors are willing to lend if and only if
\[ p_H(R^2 - R_0^2) + (1 - p_H)(R^1 - R_0^1) \geq I - A. \]

Finally, the borrower’s utility is
\[ p_H R_0^2 + (1 - p_H) R_0^1 - c, \]
subject to his incentive constraint,
\[ (\Delta p)\left( R_0^2 - R_0^1 \right) \geq c. \]

This yields
\[ R_0^1 = 0 \text{ and } R_0^2 = \frac{c}{\Delta p}. \]

A necessary and sufficient condition for the borrower to have access to financing is
\[ p_H \left( R^2 - \frac{b + c}{\Delta p} \right) + (1 - p_H) R^1 \geq I - A. \]

Exercise 9.7 (intertemporal recoupment). (i) Long-term contracts. The potential NPV is
\[ V = 2 p_H R - (I_1 + I_2) - 2c. \]

Under competition among monitors, the borrower can obtain \( V \), for example, by proposing a contract specifying that the selected monitor at date \( t \), \( t = 1, 2 \), contributes \( E_t^1 \) and receives \( R_0^1 \), in the case of success (and 0 in the case of failure) such that
\[ p_H (R_0^1 + E_0^1) = I_1 + E_0^1 + 2c, \]
\[ (\Delta p) E_0^1 = c. \]

(The reader familiar with Sections 4.2 and 4.7 will notice that considering two incentive constraints, one per period, is in general not optimal. More on this later. However, we here show that the upper bound on the borrower’s utility can be reached, and so we do not need to enter the finer analysis of “cross-pledging.”)

Similarly, giving a stake \( R_0^2 \) in the case of success (and 0 in the case of failure) such that
\[ (\Delta p) E_0^2 \geq b \]
The three conditions say that if the entrepreneur is rewarded $R^*_2$ the firm's payoff in the case of continuation that is consistent with the investors' breaking even. The entire short-term income ($r$ in the case of success and $L$ in the case of failure) is pledged to investors, and the project continues only in the case of date-1 success. The three conditions say that if the entrepreneur is rewarded $R^*_2$ in the case of date-2 success, then

- $R^*_2 \geq B/\Delta p$: her date-2 incentive compatibility constraint is satisfied;
- $p_0(R - R^*_2) > L$: interference reduces the investors' income; and
- $(p_1 - p)[p_0(R^*_2)] \geq L_0; the entrepreneur's date-1 incentive compatibility constraint is also satisfied.

(ii) From the definition of $R^*_2$, the project is financed, and from the three conditions, high efforts in both periods are guaranteed. Although there is an efficiency loss in terminating the project in the case of date-1 failure, this relaxes the date-1 incentive constraint and is optimal if $p_0^1$ is large enough, that is, if the probability of interference is low enough.

The incentive scheme offered to the entrepreneur is that she is rewarded $R^*_2$ if and only if she is successful in both periods; and the project is terminated if the date-1 income is equal to 0.

To implement this incentive scheme, the entrepreneur can issue two kinds of securities with different cash flow and control rights:

- short-term debt $d \in (0, \min(L/p_0, r))$: debt-holders receive control if $d$ is not repaid at date 1; and
- long-term equities associated with control at time 1 if $d$ is paid, and the following cash-flow rights: at date 1 equity-holders receive the residual revenue ($r - d$ in the case of a date-1 failure, and max($0, L - d$) in the case of a date-1 success); at date 2 they receive $R - R^*_2$ in the case of success.

Debtholders interfere and terminate the project if there is no date-1 income, since

\[ pd - \min(L, d). \]

Equityholders, when in control, do not interfere and so the project continues.

(iii) Suppose $R^*_2 = B/\Delta p$, and all three conditions still hold. Now if the entrepreneur is also paid
\( r_0 \in (0, r) \) in the case of date-1 success, the date-1 incentive constraint is relaxed:

\[
(p_1^0 - p_1^1)[r_0 + p_0^2] \geq R_b.
\]

But given that it is satisfied for \( r_0 = 0 \), there is no benefit to boosting incentives in this way. Indeed, a positive \( r_0 \) reduces the pledgeable income. The breakeven constraint of investors becomes more stringent:

\[
I - A \leq p_1^0(r - r_0) + (1 - p_1^0)L + p_0^1(R - R_b).
\]

A positive \( r_0 \) is not optimal as it makes the financing more difficult to arrange but has no incentive effect.

In general, a short-term bonus reduces the pledgeable income, while incentives are best provided by vesting the manager’s compensation.

Exercise 10.2 (allocation of control and liquidation policy). (i) As usual, if financing is a binding constraint it is optimal to give 0 to the entrepreneur in the case of failure and to allocate the entire liquidation value \( L \) to investors in the case of liquidation. This increases the pledgeable income without perverse incentive effects or destruction of value. The entrepreneur maximizes her expected utility,

\[
U_0 = E_u(x(L, U_0^0)p_0^0 - I) + E_u(x(L, U_0^1)p_0^0 - I - A),
\]

subject to the incentive constraint,

\[
(\Delta p)b_0 \geq B,
\]

and the investors’ breakeven constraint,

\[
E_u[x(L, U_0^0)p_0^0(R - R_b) + (1 - x(L, U_0^0))L] \geq I - A.
\]

The interesting case is when both the incentive and the participation constraints are binding. Let us rewrite the program as

\[
\max E_u[x(L, U_0^0)(p_1 - p_0) + (1 - x(L, U_0^0))L - I - A]
\]

subject to

\[
E_u[x(L, U_0^0)p_0^0 + (1 - x(L, U_0^0))L] = I - A.
\]

Let \( \mu \geq 1 \) denote the multiplier of the participation constraint. We obtain

\[
x^{18}(u) = 1 \text{ if and only if } p_1 - U_0^0 \geq -\mu - 1)p_0 + \mu L,
\]

where "SB" stands for "second best."

As one would expect, continuation is less desirable when the liquidation value and the entrepreneur’s alternative employment become more attractive (and, because of the difficulty of attracting financing, the liquidation value receives a higher weight than the entrepreneur’s fallback option).

(ii) The first-best continuation rule is given by

\[
x^{FB}(u) = 1 \text{ if and only if } p_1 - U_0^0 \geq L
\]

(that is, \( u = 1 \)). \( \Omega^{FB} \) is included in \( \Omega^{18} \), as described in Figure 5. More generally, \( \Omega^{FB} \) shrinks as \( \mu \) decreases.

To show this, note that for \( L < p_0 \), everyone prefers to continue. So the interesting region is \( L > p_0 \).

(iii) When the entrepreneur has control, the entrepreneur can guarantee himself \( p_1 - p_0 \) by choosing to continue. Second, renegotiation always leads to the first-best efficient outcome.

(a) Continuation is first-best efficient. If the initial contract makes the entrepreneur want to continue in the absence of renegotiation, there is nothing to renegotiate about (a necessary condition for renegotiation is the existence of gains from trade). If the entrepreneur prefers to liquidate (because of the existence of a golden parachute), the investors will want to compensate the entrepreneur to induce him to continue (the split of the gains from renegotiation depend on the relative bargaining powers).

(b) Liquidation is first-best efficient. Again, if the entrepreneur prefers to liquidate in the absence of renegotiation there is nothing to renegotiate about. Otherwise, the investors will "bribe" the entrepreneur to liquidate.

So

\[
\Omega^{FB} = \Omega^{18}
\]

Compare the investors’ return with the pledgeable income derived in question (i). In \( \Omega^{18} \) and outside
Ω^P, the decision rule is unchanged, and the investors cannot get more than \( p_0 \) and \( \ell \), respectively. In \( \Omega^M - \Omega^P \), the investors get at most \( p_0 \), while they were getting \( L > p_0 \). Thus, the project cannot be financed.

(iv) Under investor control, and in the absence of a golden parachute,

\[ x^{10}(\omega) = 1 \quad \text{if and only if} \quad p_0 \geq L. \]

If \( p_0 < L \), then investors cannot get more than under liquidation (there is no way the entrepreneur can compensate them). If \( p_0 > L \), but \( p_0 R_L < L \), then the entrepreneur can offer a reduction of her stake in the case of success (while keeping \( R_0 > B/\Delta p \)).

The project is financed since the investors get the same amount as in (i), except when \( L > p_0 \) and \( \omega \in \Omega^M \) for which they get more (\( L \) instead of \( p_0 \)).

(v) In the absence of renegotiation, the investors liquidate if and only if

\[ L - r_b \geq p_0. \]

The policy is renegotiated (toward liquidation) if

\[ (p_1 - p_0) - I_{\Omega}^0 \leq L - p_0 < r_b. \]

In contrast, if

\[ (p_1 - p_0) - I_{\Omega}^0 > L - p_0 > r_b, \]

then there is no renegotiation and there is (inefficient relative to the first best) liquidation.

A small golden parachute increases the NPV while continuing to satisfy the financing constraint (an alternative would be to ask the investors to finance more than \( L - 4 \) and let the entrepreneur save as to be able to "bribe" the investors to induce continuation).

Exercise 10.3 (large minority blockholding). If \( \xi < (\tau + \mu) x R \), then the large shareholder and the uninformed (majority) investors have aligned interests. The majority shareholders therefore always follow the large shareholder’s recommendation.

Let us therefore assume that \( \xi > (\tau + \mu) x R \). Let us look for an equilibrium in which the entrepreneur makes her suggestion "truthfully" (just announces her preferred modification). In state 2, the large shareholder secures the entrepreneur’s proposal. He makes a counterproposal in states 1 and 3.

The majority shareholders then go along with the joint proposal (in state 2). In the case of disagreement, the majority shareholders select the entrepreneur’s proposal, that of the large shareholder, or the status quo so as to solve

\[ \max \{-\beta \mu + \tau (1 - \xi), \beta \tau - \mu (1 - \beta) (1 - \xi), 0\}. \]

Note that in the equilibrium under consideration both the entrepreneur and the minority blockholder have incentives to report their preferences truthfully (and that there are other equilibria where this is not the case).

Exercise 10.4 (monitoring by a large investor). Let \( U_b(x) = p_u R + [\xi + (1 - \xi) x] [\tau R - y] - c_u(x) - I \) denote the NPV (the NPV is equal to the borrower’s utility because there is no scarcity of monitoring capital, and therefore no rent to be left to the monitor). Let

\[ p(x) = [p_u + \xi + (1 - \xi) x] \left( R - \frac{B}{\Delta p} \right) - c_u(x). \]

denote the income that can be pledged to investors given that (a) the entrepreneur’s stake must exceed \( B/\Delta p \) in order to elicit good behavior, and (b) the monitor’s expected income must compensate him for his monitoring cost. Concerning the last point, the monitor’s reward \( R_m \) in the case of success and investment contribution \( I_m \) must satisfy the following break-even and incentive conditions:

\[ p_u R_m = I_m = c_m(x) \quad \text{and} \quad (1 - \xi) \tau R_m = c^*_m(x). \]

Note that

\[ U_b(x) - p(x) = [\xi + (1 - \xi) x] \left( R - \frac{B}{\Delta p} \right) + \gamma, \]

constant, and so is decreasing in \( x \).

If there is a shortage of pledgeable income, the optimal monitoring level given by (10.11) and maximizing the NPV,

\[ c^*_m(x^*) = (1 - \xi) (\tau R - y), \]

is no longer adequate. Indeed

\[ U_b(x^*) - p(x^*) \Rightarrow p(x^*) > 0. \]

Thus, the monitoring intensity must increase beyond \( x^* \):

\[ c_m(x) > (1 - \xi) (\tau R - y). \]
If funding is feasible, then \( x \) is given by (the smallest value satisfying)
\[
P(x) = I - A.
\]
Let \( \hat{x} (\geq x^*) \) be defined by
\[
\hat{c}'_m(\hat{x}) \equiv (1 - \xi)\tau \left( R - B \frac{\Delta p}{s} \right).
\]
Because the pledgeable income no longer increases above \( \hat{x} \), funding is feasible only if
\[
P(\hat{x}) \geq I - A.
\]

Exercise 10.5 (when investor control makes financing more difficult to secure). (i) The incentive constraint is as usual
\[
p_H R_b \geq p_L R_b + B,
\]
yielding pledgeable income
\[
P_1 \equiv p_H \left( R - B \frac{\Delta p}{s} \right).
\]
The entrepreneur can receive funding if and only if
\[
P_1 \geq I - A.
\]
(ii) Assume entrepreneur control. Either
\[
\nu R_b \leq \gamma,
\]
and then the entrepreneur does not engage in damage control when shirking. The relevant incentive constraint remains (1), or
\[
\nu R_b > \gamma,
\]
and the incentive constraint becomes
\[
p_H R_b \geq (p_L + \nu)R_b + B - \gamma,
\]
if \( \nu \left( B \frac{\Delta p}{s} \right) < \gamma \),
then the incentive constraint is unchanged when \( R_b = R / \Delta p \), and so the pledgeable income (the maximal income that can be pledged to investors while preserving incentive compatibility) is still \( P_1 \).
(iii) Under investor control, the damage-control action is selected, and so the incentive constraint becomes
\[
p_H R_b - \gamma \geq (p_L + \nu)R_b + B - \gamma
\]
or
\[
(\Delta p - \nu)R_b \geq B.
\]
The new pledgeable income is
\[
P_2 = p_H \left( R - B \frac{\Delta p}{s} - \gamma \right),
\]
and is smaller than under entrepreneur control.

Exercise 10.6 (complementarity or substitutability between control and incentives). (i) As usual, this condition is
\[
p_H \left( R - B \frac{\Delta p}{s} \right) \geq I - A.
\]
(ii) Under entrepreneur control, the profit-enhancing action is not chosen in combination with the high effort since
\[
(p_H + \tau_H)R_b - \gamma < p_H R_b
\]
(since \( \tau_H R_b < \gamma \)).
Thus, to induce the high effort, \( R_b \) must satisfy
\[
(\Delta p)R_b \geq B.
\]
But then it is also optimal for the entrepreneur not to misbehave and choose the profit-enhancing action simultaneously:
\[
(p_L + \tau_L)R_b + B - \gamma \leq p_L R_b + \tau_L R_b - \gamma < p_L R_b,
\]
since \( R_b < R \). The analysis is therefore the same as in (i).
Under investor control, it is a dominant strategy for the investors to select the profit-enhancing action. Hence, the manager’s incentive constraint becomes
\[
(p_H + \tau_L)R_b \geq (p_L + \tau_L)R_b + B
\]
or
\[
(\Delta p + \Delta \tau)R_b \geq B.
\]
The pledgeable income increases with investor control if and only if
\[
(p_H + \tau_L) \left( R - B \frac{\Delta p}{s} + \gamma \right) \geq p_H \left( R - B \frac{\Delta p}{s} \right).
\]
This condition is necessarily satisfied if \( \Delta \tau \geq 0 \) (complementarity or separability), but it may fail if \( \Delta \tau \) is sufficiently negative.

Exercise 10.7 (extent of control). The NPV is larger under limited investor control:
\[
(p_H + \tau_L)R - \gamma_\lambda > (p_H + \tau_L)R - \gamma_\lambda
\]
We will assume that these NPVs are positive.
So the entrepreneur will grant limited control as long as this suffices to raise funds, i.e.,
\[
(p_u + \tau x) \left( R - \frac{E B}{\Delta p} \right) \geq I - A.
\]
If this condition is not satisfied, the entrepreneur must grant extended control in order to obtain financing. Financing is then feasible provided that
\[
(p_u + \tau o) \left( R - \frac{E B}{\Delta p} \right) \geq I - A.
\]
Lastly, note that
\[
\tau x R - y_\lambda \geq 0
\]
is a sufficient condition for ruling out entrepreneurial control (but entrepreneurial control may be suboptimal even if this condition is not satisfied; for, it may conflict with the investors’ breakeven condition).

Exercise 10.8 (uncertain managerial horizon and control rights). (i) The assumption
\[
(p_u + \tau x) \left( R - \frac{E B}{\Delta p} \right) \geq y
\]
means that the new manager is willing to take on the job even if control is allocated to investors. Because his reward \( p_u \) must satisfy
\[
(\Delta p)\mathbb{R}_0 \geq B,
\]
regardless of who has control, the new manager receives rent
\[
(p_u + \tau y) \left( R - \frac{E B}{\Delta p} \right) - y y
\]
(smaller than the rent, \( p_u B/\Delta p \), that he would receive if he were given control rights).

The entrepreneur’s utility is (if the project is undertaken)
\[
U_0 = (1 - \lambda)(p_u + \tau x) R - y x
\]
\[
+ \lambda (p_u + \tau y) \left( R - \frac{E B}{\Delta p} \right) - I.
\]
The financing condition is
\[
(1 - \lambda)(p_u + \tau x) \left( R - \frac{E B}{\Delta p} \right) + \lambda (p_u + \tau y) \left( R - \frac{E B}{\Delta p} \right) \geq I - A.
\]
(iii) Clearly, \( y = 1 \) both maximizes \( U_0 \) and facilitates financing.

Also, a necessary condition for \( U_0 \) to be positive is that \( \lambda \) not be too big.

Letting \( p_0 \leq p_u \frac{R - B}{\Delta p} \), if financing is feasible for \( x = 0 \), \( (1 - \lambda)p_0 + \lambda p_0^* \geq I - A \), then \( x = 0 \) is optimal. The entrepreneur invests if and only if \( U_0 \geq 0 \), or
\[
(1 - \lambda)p_0 + \lambda p_0^* \geq I.
\]
If \( (1 - \lambda)p_0 + \lambda p_0^* < I - A \), then, in order to obtain financing, the entrepreneur must set \( x \) in the following way:
\[
(1 - \lambda)p_0 + \lambda p_0^* + \tau x \left( R - \frac{E B}{\Delta p} \right) = I - A.
\]
Financing then occurs if and only if \( U_0 \geq 0 \) for this value of \( x \).

Exercise 10.9 (continuum of control rights). (i) Let \( \mathbb{R}_0 \) denote the entrepreneur’s reward in the case of success. The entrepreneur maximizes her utility, which is equal to the NPV,
\[
\max_{(I_x, y)} \{ \mathbb{R}_0[(x(t), g)] - I - E_t[g x(t, g)] \}
\]
subject to the constraint that investors break even,
\[
\{ p_u + E_t[(x(t), g)] \} [R - I - E_t[g x(t, g)]],
\]
and to the incentive compatibility constraint,
\[
(\Delta p)\mathbb{R}_0 \geq B.
\]
Clearly, \( \mathbb{R}_0 = B/\Delta p \) if the investors’ breakeven constraint is binding. Let \( \mu \) denote the shadow price of this constraint. Writing the Lagrangian and taking the derivative with respect to \( x(t, g) \) for all \( t \) and \( g \) yields
\[
x(t, g) = 1 \iff t R - g + \mu \left( R - \frac{E B}{\Delta p} \right) \geq 0.
\]
This defines a straight line through the origin in the \((t, g)\)-space under which \( x = 1 \) and over which \( x = 0 \).

(ii) When \( A \) decreases, more pledgeable income must be harnessed. So the straight line must rotate counterclockwise (add \( t > 0 \) realizations and subtract \( t < 0 \) ones). In the process, both \( t \) and \( y \) increase.

(iii) If \( x(t, g) = 1 \) and \( t > 0 \), the control right can be given to investors, if \( x(t, g) = 1 \) and \( t < 0 \) (which implies \( g < 0 \)) the decision yields a private benefit to
the entrepreneur), then the control can be allocated to the entrepreneur. Because

\[ |g| > |R| > |R_0|, \]

the entrepreneur chooses \( x(t, g) = 1 \). Furthermore, \( x(t, g) = 1 \) is not renegotiated since it is first-best efficient.

One proceeds similarly for \( x(t, g) = 0 \). (iv) Assume that \( g \) is the same for all rights and is positive. The optimal rule becomes

\[ t \geq t^* = \frac{\theta}{R + \mu(R - \beta^2 \Delta g)}. \]

Let \( H(t) \) denote the cumulative distribution function over \( t \):

\[ y = g[1 - H(t^*)], \]

\[ \tau = \int_{t^*}^{\infty} dH(t). \]

Hence,

\[ \frac{dy}{\tau} = \frac{\theta}{R} \]

and \( \frac{d^2y}{\tau^2} > 0 \).

One can, as earlier, envision that \( \tau \) increases as \( A \) decreases, for example.

Exercise 12.1 (Diamond–Dybvig model in continuous time). To provide consumption \( c(t) \) to consumers whose liquidity need arises between \( t \) and \( t + dt \) (in number \( f(t) dt \), one must cut \( x(t) dt \), where

\[ x(t)R(t) dt = c(t)f(t) dt. \]

Together with the fact that the total number of trees per representative depositor is 1, this implies that the first-best contract solves

\[ \max \left\{ \int_{t}^{\infty} u(c(t)) f(t) dt \right\} \]

s.t.

\[ \int_{t}^{\infty} f(t) dt \leq 1. \]

The first-order condition is then, for each \( t \),

\[ \frac{\nu'(c(t))}{R(t)} = \frac{\mu}{R(t)}, \]

where \( \mu \) is the shadow price of the constraint.

(iii) Take the (log-) derivative of the first-order condition:

\[ u'(c(t))R(t) = \mu \Rightarrow \frac{c}{u'} \frac{\dot{c}}{c} = \frac{R}{R}. \]

Because the coefficient of relative risk aversion exceeds 1,

\[ \frac{c}{u'} > \frac{R}{R}. \]

Note that, from the constraint, the average \( c/R \) is equal to 1. The existence of \( t^* \) follows (drawing a diagram may help build intuition).

(iii) Suppose that a depositor who has not yet suffered a liquidity shock withdraws at date \( \tau \). Reinvesting in the technology, she will obtain \( c(\tau)R(t - \tau) \) if the actual date of the liquidity shock is \( t > \tau \). Withdrawing is a “dominant strategy” (that is, yields more regardless of the future events) if

\[ c(\tau)R(t - \tau) > c(t) \] for all \( t > \tau \).

The log-derivative of \( (c(\tau)R(t - \tau)/c(t)) \) with respect to \( t \) is, for \( t \) close to 0,

\[ \frac{R(t - \tau)}{R(t - \tau)} \frac{c(t)}{c(t)} - \frac{c(\tau)}{c(t)} > 0, \]

We thus conclude that the first-best outcome is not incentive compatible.

Exercise 12.2 (Allen and Gale (1998) on fundamentals-based panic). (i) Let \( i_1 \) and \( i_2 \) denote the investments in the short- and long-term technologies. The social optimum solves

\[ \max \left\{ E[\lambda u(c_1(R)) + (1 - \lambda) u(c_2(R))] \right\} \]

s.t.

\[ \lambda c_1(R) \leq i_1, \]

\[ (1 - \lambda) c_2(R) \leq (i_1 - \lambda c_1(R)) + Ri_2, \]

\[ i_1 + i_2 = 1. \]

This yields

(a) \( c_1(R) = c_2(R) = i_1 + Ri_2 \) for \( R \leq \frac{1 - \lambda i_1}{\lambda i_2} = R^* \),

(b) \( c_1(R) = c_1(R^*), \)

and

\[ c_2(R) = \frac{Ri_2}{\lambda} \geq c_1(R) \] for \( R > R^* \).

For low long-term payoffs, \( \lambda c_1(R) < i_1 \) and the impatient types share risk with the patient types, as their short-term investment can be rolled over to


\[ \text{Journal of Finance 53:}1245–1283. \]
give some income to the latter. In contrast, high long-term payoffs (for which $\lambda c_i(R) = i_i$) are enjoyed solely by the patient types, who have no means of sharing the manna from heaven with the impatient types.

The optimal allocation is depicted in the Figure 6.

(iii) Let

$$\tilde{c}_1 = \frac{i_1}{\lambda}$$

Suppose that the deposit contract promises

$$\min(c_1, i_1(\lambda + (1 - \lambda)x))$$

and that a fraction $x(R)$ of the patient depositors withdraw at date 1. For $R \leq R^*$, we claim that the following equations characterize the equilibrium:

$$\frac{\lambda + (1 - \lambda)x(R) c_1 - i_1}{(1 - \lambda)(1 - x(R))} = c_1 \leq \tilde{c}_1.$$ 

First, note that, for $R > 0$, $x(R) = 1$ is not an equilibrium behavior, as patient consumers could consume an infinite amount by not withdrawing. Similarly, for $R < R^*$, $x(R) = 0$ is not part of an equilibrium because $B_1(R)/(1 - \lambda)$ is smaller than $i_1/\lambda$. Hence, a fraction in $(0, 1)$ of patient consumers must withdraw at date 1. This implies that patient consumers are indifferent between withdrawing and consuming, or $c_1 = \tilde{c}_1$.

Exercise 12.4 (random withdrawal rate). (i) This follows along standard lines. Asset maturities should match those of consumptions, $\lambda c_1 = i_2$ and $\lambda c_2 = i_1 R$.

$$(1 - \lambda)c_2 = i_1 R:$$

$$\max_{c_1} \left\{ u(c_1) + (1 - \lambda)u\left(1 - \frac{\lambda c_1}{1 - A}\right) \right\}$$

implies

$$u'(c_1) = R u'(c_2).$$

For CRRA utility, $c_1 c_2 = R^{-1/\gamma}$. So $i_2$ grows and $i_1$ decreases as risk aversion ($\gamma$) increases.

(ii) The optimal program solves

$$\max_{(i_1, i_2, \eta, \lambda, R)} \left\{ \frac{\lambda}{\lambda + 1} u \left( \frac{1}{\lambda} (i_1 + i_2 R) \right) \right\}$$

$$+ \left( 1 - \frac{\lambda}{\lambda + 1} \right) u \left( \frac{1}{\lambda} (i_1 + i_2 R) \right)$$

$$+ \left( 1 - \frac{\lambda}{\lambda + 1} \right) u \left( \frac{1}{\lambda} (i_2 (1 - \eta) + i_1 R (1 - \lambda)) \right)$$

Clearly, $z_\omega > 0 = y_\omega = 1$ and $y_\omega < 1 = z_\omega = 0$. Also, $y_\omega = 1$ implies $\gamma_\omega = 1$, and $z_\omega = 0$ implies $\lambda_1 = 0$.

For $\ell = 0$, the optimum has $z_\omega = 0$. It may be optimal to roll over some of $i_1$ in state $L$. For $\ell$ close to 1, $i_2$ serves to finance date-1 consumption in state $H$.

Exercise 13.1 (improved governance). (i) The pledgeable income is $p_\| (R - B/\Delta p)$. The financing constraint is

$$(1 + \gamma) (I - A^*) \leq p_\| (R - B/\Delta p).$$

(ii) The cutoff $A^*$ is given by

$$(1 + \gamma) (I - A^*) = p_\| (R - B/\Delta p).$$

Market equilibrium:

$$S(r) + \int_{r}^{\delta} \gamma(A) dA = \int_{r}^{\delta} (I - A^*) (I - A^*) \gamma(A) dA$$

or, equivalently,

$$S(r) + \int_{r}^{\delta} \gamma(A) dA = [1 - G(A^*)] \|.$$
Exercise 13.2 (dynamics of income inequality).

(i) See Section 13.3:
\[ U_1(y_1) = y_1. \]

(ii) The incentive constraint is
\[ (\Delta y) R_\alpha \geq B, \]
and so the pledgeable income is
\[ p_0 \left( \frac{R - B}{\Delta y} \right) I_t = \rho_1 I_t, \]
yielding an investment level given by
\[ I_t = (1 + r)(I_1 - A_\alpha) \text{ or } I_t = \frac{A_\alpha}{1 - \rho_1/(1 + r)}. \]

A project’s NPV is
\[ \{p_0 R - (1 + r)] I_1 = \{p_1 - (1 + r)\} I_t, \]
by assumption, \( \rho \) converges to \( A_\alpha \), yielding an investment level given by
\[ \rho_d = (1 + r)(I_1 - A_\alpha). \]

By assumption, \( p_1 \geq 1 + r \), and so entrepreneurs prefer to invest in a project rather than lending their assets. Income is
\[ y_t = (p_1 - \rho_1) I_t, \]
and so
\[ A_{t+1} = \frac{\rho_1 - \rho_2}{1 - \rho_1/(1 + r)} A_t + \hat{A}, \]
which converges to \( A_\alpha \) as \( t \) tends to \( \infty \).

(iii) The threshold is given by
\[ \frac{A_\alpha^\prime}{1 - \rho_1/(1 + r)} = I_t. \]
The limit wealth of poor dynasties is the limit point of the following first-order difference equation:
\[ A_{t+1} = \alpha (1 + r) A_t + \hat{A}, \]
or
\[ A_\alpha^\prime = \frac{\hat{A}}{1 - \alpha (1 + r)}. \]

(iv) If \( p_1 = 1 + r \), individuals are indifferent between being investors and becoming entrepreneurs. Note that wealths are equalized at
\[ A_\alpha = \frac{\hat{A}}{1 - \alpha (1 + r)}, \]
corresponding to investment
\[ I_\alpha = \frac{A_\alpha}{1 - p_0/(1 + r)} = \frac{\rho_1 \hat{A}}{1 - \alpha p_1/(1 + r)}. \]
Equilibrium in the loan market requires that
\[ \kappa(A_\alpha) = (1 - \kappa)(I_\alpha - A_\alpha) \]
or
\[ \kappa(p_1 - \rho_0) = (1 - \kappa) p_0. \]

\[ \bullet \text{ If } p_1 > (1 + r), \text{ then lenders must be unable to become entrepreneurs and so have wealth } A_\alpha^\prime. \text{ Thus } \kappa A_\alpha^\prime = (1 - \kappa)(I_\alpha - A_\alpha), \]
where \( I_\alpha \) was derived in question (ii).

Exercise 13.3 (impact of market conditions with and without credit rationing).

(i) The representative entrepreneur’s project has NPV (equal to the entrepreneur’s utility)
\[ U_3 = p_0 PR(I) - I - K, \]
and the scale of investment \( I \) can be financed as long as the pledgeable income exceeds the investors’ initial outlay:
\[ P(I) = p_0 PR(I) - B \geq I + K - A_\alpha. \]

(ii) Thus, there is at least some region (to the left of \( P_6 \) in the figure) in which the expansionary impact of the product price (the contractionary impact of past investment) is stronger in the presence of credit rationing, i.e., when the presence of \( R \) makes the financing condition binding.
Exercise 14.2 (alternative distributions of bargain-
ing power in the Shleifer–Vishny model).

Recalling that
\[ \hat{\alpha} = \alpha \quad \text{and} \quad \hat{\kappa} = \kappa + \kappa, \]
and simplifying
\[ \hat{\alpha} = \alpha - (1 - \mu)(\rho_0 - P). \]

Recalling that \( (1 - x)(1 - \nu) = x(1 - \mu) \), note that
\[ \hat{\alpha} + \hat{\kappa} = \alpha + \kappa, \]
as it should be from the fact that a change in bargaining power induces a mere redistribution of wealth for given investments.

Firm i’s borrowing capacity is now given by
\[
[\lambda x_0 + (1 - x)(1 - \nu)]I_i
+ x(1 - \mu)(\rho_0 - P)I_f - J_i - A_i
\]
or
\[
I_i = \frac{A_i + x(1 - \mu)(\rho_0 - P)I_f}{1 - (1 - x)(1 - \nu)(\rho_0 - P)},
\]
\[ - p_0[\nu(1 - \mu)(1 - \nu) - \nu(x(1 - x)(1 - \nu))]. \]

In symmetric equilibrium \( (\Delta_1 = \Delta_2 = \kappa, I_1 = I_2 = I) \),
\[
I \equiv \frac{\rho_0[\nu(1 - \mu)(1 - \nu) - \nu(x(1 - x)(1 - \nu))]}{1 - p_0[\nu(1 - \mu)(1 - \nu) - \nu(x(1 - x)(1 - \nu))]},
\]
is independent of \( \rho \).

Exercise 14.3 (liquidity management and acquisi-
tions). (i) Suppose that the acquirer expects price
demand \( P \) for the assets when the risky firm is in
distress (which has probability \( 1 - x \)). The NPV for a
given cutoff \( \rho^* \) is given by
\[
\begin{align*}
U_0^* &= (\rho_0 - 1) + (1 - x)\int_0^{\rho^*} [p_0 - (P + \rho^*)] d\bar{F}(\rho).
\end{align*}
\]
The borrowing capacity in turn is given by
\[
\begin{align*}
p_0I &+ (1 - x)\int_0^{\rho^*} [p_0 - (P + \rho^*)] d\bar{F}(\rho) = I - A.
\end{align*}
\]
And so
\[
\begin{align*}
U_0^* &= (\rho_0 - 1) + (1 - x)\int_0^{\rho^*} [p_0 - (P + \rho^*)] d\bar{F}(\rho).
\end{align*}
\]
Maximizing with respect to \( \rho^* \) and simplifying yields
\[
\rho^* = 1 - P.
\]
And so
\[
\rho_0 + L^* = P + \rho^* = 1.
\]
(ii) Anticipating that the safe firm has extra liquid-
ity \( L^* \), the seller chooses price \( P \) so as to solve
\[
\max F(p_0 + L^* - P)p,
\]
since the acquirer can raise funds only when \( P + \rho \leq \rho_0 + L^* \).

The derivative of this objective function is
\[
- f(p^*)P + f(1 - P)F(1 - P),
\]
where this derivative is positive at \( P = 0 \) and
negative at \( P = 1 \). Furthermore, \( -P + F(1 - P)/f(1 - P) \)
is a decreasing function of \( P \) from the monotone
hazard rate condition and so the equilibrium price
is unique and belongs to \((0, 1)\).
Suppose next that $I_0$ increases for some reason (and that this is observed by the seller). The first-order condition then becomes

$$-p + F(p_0 + l - P) = 0$$

and so

$$\left(1 + \left(\frac{F}{f}\right)\right) \frac{dp}{dp} - \left(\frac{F}{f}\right) = 0.$$ 

Because $(F/f) > 0$, $0 < \frac{dp}{dp} < 1$.

This implies that the cutoff, and thus the probability of a sale, increases despite the price adjustment.

(iii) Suppose that the distribution $F$ converges to a spike at $\hat{\rho}$. Consider thus a sequence $F_n(\rho)$ with

$$\lim_{n \to \infty} F_n(\rho) = 0 \text{ for } \rho < \hat{\rho}$$

and

$$\lim_{n \to \infty} F_n(\rho) = 1 \text{ for } \rho > \hat{\rho}.$$ 

Let us give an informal proof of the result stated in (iii) of the question. Choosing a price $P$ that triggers a cutoff that is smaller than $\hat{\rho}$ and does not converge with $n$ to $\hat{\rho}$ would yield (almost) zero profit, and so choosing an alternative price that leads to a cutoff a bit above $\hat{\rho}$ would yield a higher profit. Conversely, if the cutoff is above $\hat{\rho}$ and does not converge to $\hat{\rho}$, then $P \rho_n = 0$ and $F_n = 1$, and so the first-order condition is not satisfied. (This proof is loose. A proper proof must consider a sequence having the former or latter property.)

Exercise 14.4 (inefficiently low volume of asset reallocations). At the optimum, firm 1’s assets are resold in the secondary market if and only if $p_0 < \rho_1^*$. Furthermore, it is optimal for the contract to specify that the proceeds from the sale to firm 2 go to the investors in firm 1 (so as to maximize the pledgeable income). And so the investment $I_0$ is given by the investors’ breakeven constraint:

$$\int_{\rho_0}^{\rho_1^*} p_0 \, dp_0 \, dF(p_0) = I - A,$$

which yields

$$I = I(p_1^*).$$ 

The entrepreneur’s utility is

$$U_\mu = \text{NPV} - \int_{\rho_0}^{\rho_1^*} p_0 \, dp_0 \, dF(p_0) \, F(p_0).$$

The optimal cutoff maximizes $U_\mu$ and satisfies

$$\hat{\rho}_0 - \Delta p < \rho_1^* < \hat{\rho}_0.$$ 

Exercise 15.1 (downsizing and aggregate liquidity). (i) The incentive constraint is

$$(\Delta p) R_2^0 \geq B I$$

in the case of no shock, and

$$(\Delta p) R_2^0 \geq B J$$

in the presence of a liquidity shock.

So the pledgeable incomes are $p_0 R(J) - B(I - \Delta p)$ and $p_0 R(I) - B(J - \Delta p)$, respectively.

The investors’ breakeven constraint is

$$(1 - \lambda)p_0 R(J) - \lambda B(I - \Delta p) \geq I - A.$$ (1)

The entrepreneur’s utility is equal to the NPV:

$$U_\mu = \text{NPV} - \int_{\rho_0}^{\rho_1^*} p_0 \, dp_0 \, dF(p_0) \, F(p_0).$$

Let $\mu$ denote the shadow price of constraint (1). Maximizing $U_\mu$ subject to (1) (and ignoring the constraint $J \leq I$) yields first-order conditions with respect to $I$ and $J$:

$$[1 - \lambda]p_0 R(J) - 1[1 + \mu] = \mu \lambda B(I - \Delta p).$$

or

$$p_0 R(J) = \frac{1}{1 - \lambda} \mu \lambda B(I - \Delta p),$$

and

$$\lambda \mu p_0 R(J) - \mu[1 + \mu] = \mu \lambda B(I - \Delta p).$$

Comparing (3) and (4), one observes that ignoring the constraint $J \leq I$ is justified if and only if

$$\rho > \frac{1}{1 + \lambda}.$$ 

That is, when the cost of continuation in the state of nature with a liquidity shock exceeds the cost of
one more unit of investment in the state without. This simple comparison comes from the fact that the per-unit agency cost is the same in both states of nature. Let \( (I', J') \) denote the solution (obtained from (1), (5), and (4)).

(iii) Under perfect correlation, no inside liquidity is available. So, in order to continue in the case of a liquidity shock, each firm requires

\[ L = \rho J' \]

Hence, \( L^* = \rho J' \).

- If \( L < L^* \), then

\[ J = \frac{L}{\rho} < J' \].

The solution is obtained by solving the modified program in which the extra cost associated with the liquidity premium, \( (q - 1)\rho J \), is subtracted in \( \lambda_0 \) (in (2)), and added to the right-hand side of (1), yielding a modified investor breakeven constraint—let us call it \( (1') \). Equation (5) is unchanged, while (4) becomes

\[ p_\sigma R'(J) = p_\rho \left( 1 + \frac{d - 1}{\lambda} \right) + \frac{\mu}{\lambda} p_\sigma R(J) \frac{B(J)}{\lambda}. \]

So \( J < 1 \) a fortiori.

The liquidity premium is obtained by solving \( (1') \), (3), (4'), and (5).

(iii) Under independent shocks, exactly a fraction \( \lambda \) of firms incur no shock. Assuming \( q = 1 \) for the moment, (1) yields (provided \( I > A \))

\[ V = (1 - \lambda)p_\sigma R(I) \frac{B(I)}{\lambda p} + \lambda p_\sigma R(J) \frac{B(J)}{\lambda p} \]

\[ \lambda \rho J \]

\( V \) is the value of the stock index after the shocks have been met. And so the corporate sector, as a whole, can by issuing new claims raise enough cash to meet average shock \( \lambda \rho J \). So there is, in principle, no need for outside liquidity.

- This, however, assumes that liquidity is not wasted, if each entrepreneur holds the stock index, then, when facing a liquidity shock, the entrepreneur can raise \( p_\sigma R(J) \frac{B(J)}{\lambda p} \) by issuing new claims on the firm.

Meeting the liquidity shock then requires that

\[ p_\sigma R(J) \frac{B(J)}{\lambda p} + [V - \lambda \rho J] \geq \rho J \]

or

\[ (1 - \lambda)p_\sigma R(J) \frac{B(J)}{\lambda p} \geq (1 + \lambda)\left[ \rho J - p_\sigma R(J) \frac{B(J)}{\lambda p} \right] \]

which is not guaranteed. It is then optimal to pool the liquidity, for example, through a credit line mechanism.

Exercise 15.2 (news about prospects and aggregate liquidity).

(i) \( \text{NPV} = \int_{y} y \, dG(y) \) \[-\{1 - G(y^*)\}] = I - I.

Investors’ net income

\[ = \int_{y} y \, dG(y) \) \[-\{1 - G(y^*)\}] = I - I. \]

(ii) The NPV is maximized for \( y^* = y^*_0 = \frac{J}{J} \). So, if

\[ \int_{y} y \, dG(y) \) \[-\{1 - G(J)\}] = I - A \iff A > A^*_0, \]

then \( y^* = J \).

Otherwise, by concavity of the NPV, the contract raises \( y^* \) so as to attract investment:

\[ \int_{y} y \, dG(y) \) \[-\{1 - G(y^*)\}] = I - A. \]

The pledgeable income can no longer be increased when \( y^* = y^*_0 = J + R \).

So, for \( A < A^*_0 \), no financing is feasible.

- If \( A > A^*_0 \), then \( y^* < J + R \). Hence, for \( y^* < J + R \), investors have negative profit from continuation, and the firm cannot obtain financing just by going back to the capital market.

(iii) If productivities are drawn independently, the financing constraint,

\[ \int_{y} y \, dG(y) \) \[-\{1 - G(y^*)\}] = I - A, \]

implies

\[ \int_{y} y \, dG(y) \) \[-\{1 - G(y^*)\}] = I + R > 0, \]

and so, collectively, firms have enough income to pledge when going back to the capital market.

(iv) Suppose, in a first step, that there exists a large enough quantity of stores of value, and so
q = 1 (there is no liquidity premium). Then the break-even condition can be written as

$$E_0 \left[ \int_{y = 0}^{1} (y - f) dG(y | \theta) \right] \geq l - A.$$

- Maximize

$$E_0 \left[ \int_{y = 0}^{1} (y - f) dG(y | \theta) \right] - I$$

subject to the financing constraint (let $\mu$ denote the multiplier of the latter). Then

$$y^*(\theta) - f + \mu [y^*(\theta) - f - R] = 0 \quad \Rightarrow \quad y^*(\theta) = f + \frac{\mu}{1 + \mu} R.$$

- The lowest amount of pledgeable income,

$$\min_{\theta} \int_{y = 0}^{1} (y - f - R) dG(y | \theta),$$

may be negative. It must then be complemented by an equal number of stores of value delivering one for certain, say,

- If there are not enough stores of value, they trade at a premium ($q > 1$).

Exercise 15.3 (imperfectly correlated shocks). A shortage of liquidity may occur only if the fraction $\theta$ of correlated firms faces the high shock (the reader can follow the steps of Section 15.2.1 to show that in the other aggregate state there is no liquidity shortage.

The liquidity need is then, in aggregate,

$$[\theta + (1 - \theta) \lambda](\rho_1 - \rho_0) \mu.$$

The net value of shares in the healthy firms is

$$(1 - \theta)(1 - \lambda)(\rho_1 - \rho_0) \mu.$$

Using the investors’ break-even condition and the assumption that liquidity bears no premium:

$$[(1 - \lambda)(\rho_1 - \rho_0) - \lambda(\rho_2 - \rho_0)]I - I - A.$$

And so the corporate sector is self-sufficient if

$$(1 - \theta)(1 - \lambda)(\rho_0 - \rho_1) I \geq [\theta + (1 - \lambda) \lambda](\rho_1 - \rho_0) I$$
or

$$(1 - \theta)(I - A) \geq \theta(\rho_0 - \rho_1) I.$$

Exercise 15.4 (complementarity between liquid and illiquid assets). The NPV per unit of investment is equal to

$$(1 - \lambda + \lambda x) p_1 - [1 + (1 - \lambda) p_1 + (\lambda p_2 + (q - 1)(\rho_1 - \rho_0)] x.$$

We know that this NPV is negative for $x = 0$. Thus, either its derivative with respect to $x$ is nonpositive,

$$\lambda p_1 < \lambda p_2 + (q - 1)(\rho_1 - \rho_0),$$

and then there is no investment ($I = 0$). The absence of corporate demand for liquidity, and so $q = 1$, which contradicts the fact that $p_1 > p_0$. Hence, the derivative with respect to $x$ must be strictly positive:

$$\lambda p_1 > \lambda p_2 + (q - 1)(\rho_1 - \rho_0),$$

implying that $x = 1$.

For a low supply of liquid assets, this in turn implies that

(a) investment is limited by the amount of liquid assets,

$$L^A = (\rho_1 - \rho_0) I,$$

(b) the entrepreneurs compete away the benefits associated with owning liquid assets, and so they are indifferent between investing in illiquid and liquid assets and not investing at all,

$$p_1 = 1 + \rho + (q - 1)(\rho_1 - \rho_0).$$

Furthermore, for a low supply of liquid assets, entrepreneurs do not borrow as much as their borrowing capacity would allow them to. This borrowing capacity, denoted $I$, is given by

$$p_1 I = [1 + \rho + (q - 1)(\rho_1 - \rho_0)] I - A = p_1 I - A.$$

When $L^5$ reaches $L^A$, given by

$$L^A \equiv \frac{\rho_0 - \rho_1}{p_1 - \rho_0},$$

then $I = I$. For $L^5 > L^A$, $q$ decreases with $L^5$ and investment,

$$I = \frac{A}{1 + \rho + (q - 1)(\rho_1 - \rho_0) - p_1} = \frac{L^A}{p_0 - p_1},$$

increases until $L^5 = L^A$ (i.e., $q = 1$), after which it is no longer affected by the supply of liquid assets.
Exercise 16.1 (borrowing abroad). (i) Investing abroad is inefficient since \( \mu < 1 \). So it is optimal to prevent investment abroad. Letting \( R_l \) denote the return to investors in the case of success, the incentive compatibility constraint is
\[
p(R_l - R_l) \geq \mu I.
\]
The breakeven constraint is
\[
pR_l = I - A.
\]
The NPV, \( U_b = (pR_l - \mu)I \), is maximized when \( I \) is maximized subject to the incentive compatibility and breakeven constraints, and so
\[
I = A \frac{\mu}{(pR_l - \mu)}.
\]
and so \( U_b = \frac{pR_l - 1}{1 - (pR_l - \mu)}A \).
This is a reinterpretation of the basic model with \( p_H = p, p_L = 0, B = \mu \).
Investing abroad brings the probability of success of the domestic investment down to 0. And because investors are unable to grab any of the diverted funds, their proceeds are but a private benefit for the entrepreneur.
(ii) One has
\[
p[(1 - \tau)R_l - R_l] \geq \mu I
\]
and
\[
pR_l + (1 - p)\sigma R_l = I - A.
\]
The government’s breakeven condition is
\[
pR_l(1 - \tau)\sigma R_l = I - A.
\]
This yields \( I(\tau) \).
The government maximizes
\[
(p_H + \tau)R_l - \gamma = (p_L + \tau)R_l
\]
and
\[
I = \frac{A - D}{1 - (pR_l - \mu)}.
\]
Hence, \( \gamma = \gamma(\tau) \).
(iii) In the case of government commitment, \( \mu = \mu_0 \), maximizes \( U_b \). In the absence of commitment, suppose that investors expect \( \mu = \mu_0 \). Then the entrepreneurs receive
\[
p(R_l - R_l) = \mu I \quad \text{if} \quad \mu = \mu_0
\]
and
\[
\max(p(R_l - R_l), \mu_0 I) = \mu_0 I \quad \text{if} \quad \mu = \mu_0.
\]
Hence, \( \mu = \mu_0 \). And \( U_b \) is decreased.
(iv) The exchange rate is given at date 2 by
\[
e = pR_l.
\]
(Assuming that there is no excess supply of tradables \( R_l \); otherwise \( e = 1 \).) One has
\[
p(R_l - R_l) = \mu I
\]
and
\[
\frac{pR_l}{e} = I - A.
\]
Then
\[
I = R_l + A - \frac{A}{1 - (pR_l - \mu)/e}.
\]
e \geq 1 is equivalent to \((1 + A/R_l)(pR_l - \mu) \geq 1 \).
Exercise 16.2 (time-consistent government policy).
(i) The incentive constraint is
\[
(p_H + \tau)(R_l - R_l) \geq BI.
\]
And so the investors’ breakeven condition is
\[
(p_H + \tau)\left[R_l - \frac{B}{\sigma}\right] = I - A.
\]
This yields \( I(\tau) \).
The government maximizes
\[
[(p_H + \tau)R_l - y(\tau)]I.
\]
Hence,
\[
y' = 0 = R.
\]
(ii) \( \max_{\tau} [(p_H + \tau)R_l - 1 - y(\tau)]I \)
\[
\Longrightarrow [y'(\tau^*) - R]I = [(p_H + \tau)R_l - 1 - y(\tau^*)]I.
\]
(iii) \( \tau < \tau^* \) then.
Exercise 16.3 (political economy of exchange rate policies).
(i) \( d^* = p_H R^*_I \) and \( d = p_H R^*_I + (1 - p_H) R^*_e \).

(ii) The entrepreneur’s incentive constraint (expressed in tradables) is

\[
\Delta p \left[ R^*_I + \frac{R^*_I - R^*_e}{e} \right] \geq BI.
\]

The foreign investors’ breakeven constraint can be written as

\[
d^* + d_e = p_H R^*_I + p_H R^*_S + (1 - p_H) R^*_F \geq I - A.
\]

And so, adding up these two inequalities,

\[
p_H (R^*_I - B \Delta p) I + p_H S I + (1 - p_H) R^*_F - p_H R^*_F b \geq I - A.
\]

Thus, if the NPV per unit of investment is positive (which we will assume), it is optimal to set \( R^*_F b = 0 \) and \( R^*_F l = SI \).

The investment is therefore

\[
I(e) = A \frac{1}{1 - \left( S/e + p_H \right)}.
\]

It decreases as the exchange rate depreciates because part of the firm’s production is in nontradables.

(iii) Commitment. Suppose, first, that the government chooses \( g^* \) before entrepreneurs borrow abroad.

The representative entrepreneur has expected utility

\[
S I + \max \left\{ u(c^*_1 - ec^*_1) + v(g^*) \right\}.
\]

In the end, the entrepreneur’s average consumption of nontradables is \( SI \) and the (average and individual) consumption of tradables is

\[
R^* - g^* + \left[ p_H R - 1 \right] I + A
\]

since the NPV, \( (p_H R - 1) I + SI \), must accrue to them from the investors’ breakeven condition.

Hence, the government chooses \( g^* \) so as to solve

\[
\max \left\{ S I + u(R^* - g^* + \left[ p_H R - 1 \right] I) + v(g^*) \right\}
\]

subject to (1) and the market-clearing equation,

\[
p_H R(e) + R^* - g^* = c^*_1(e) + I(e) - A.
\]

The first-order condition is (using \( u' = e \))

\[
v'(g^*) = e \left[ 1 - \left( S \frac{1}{p_H} + (p_H R - 1) \right) \frac{d}{de} \frac{de}{dg^*} \right] > 0.
\]

Noncommitment. Under noncommitment, investment is fixed at some level \( I \) at the date at which \( g^* \) is chosen. So the government solves

\[
\max \left\{ S I + u(R^* - g^* + p_H I - d^* + d) + v(g^*) \right\}
\]

and so

\[
v'(g^*) = e \left[ 1 - \frac{d}{de} \frac{de}{dg^*} \right] < 0.
\]

(iv) Note that under noncommitment \( g^* \) increases as the debt expressed in nontradables, \( d \), increases. Overspending imposes a negative externality on foreigners when their claims are in nontradables, but each borrower also has an individual incentive to use nontradables as collateral so as to maximize borrowing capacity.