We should warn you that being a financial expert has its occupational hazards. One is being cornered at cocktail parties by people who are eager to explain their system for making creamy profits by investing in common stocks. One of the few good things about a financial crisis is that these bores tend to disappear, at least temporarily.

We may exaggerate the perils of the trade. The point is that there is no easy way to ensure superior investment performance. Later in the book we show that in well-functioning capital markets it is impossible to predict changes in security prices. Therefore, in this chapter, when we use the concept of present value to price common stocks, we are not promising you a key to investment success; we simply believe that the idea can help you to understand why some investments are priced higher than others.

Why should you care? If you want to know the value of a firm’s stock, why can’t you look up the stock price in the newspaper? Unfortunately, that is not always possible. For example, you may be the founder of a successful business. You currently own all the shares but are thinking of going public by selling off shares to other investors. You and your advisers need to estimate the price at which those shares can be sold.

There is also another, deeper reason why managers need to understand how shares are valued. If a firm acts in its shareholders’ interest, it should accept those investments that increase the value of their stake in the firm. But in order to do this, it is necessary to understand what determines the shares’ value.

We begin with a look at how stocks are traded. Then we explain the basic principles of share valuation and the use of discounted-cash-flow (DCF) models to estimate expected rates of return.

These principles lead us to the fundamental difference between growth and income stocks. A growth stock doesn’t just grow; its future investments are also expected to earn rates of return that are higher than the cost of capital. It’s the combination of growth and superior returns that generates high price–earnings ratios for growth stocks. We explain why price–earnings ratios may differ for growth and income stocks. Finally we show how DCF models can be extended to value entire businesses rather than individual shares.

Still another warning: Everybody knows that common stocks are risky and that some are more risky than others. Therefore, investors will not commit funds to stocks unless the expected rates of return are commensurate with the risks. But we say next to nothing in this chapter about the linkages between risk and expected return. A more careful treatment of risk starts in Chapter 7.
General Electric (GE) has about 10.6 billion shares outstanding and at last count these shares were owned by about 600,000 shareholders. They included large pension funds and insurance companies that each own several million shares, as well as individuals who own a handful of shares. If you owned one GE share, you would own .00000001% of the company and have a claim on the same tiny fraction of GE’s profits. Of course, the more shares you own, the larger your “share” of the company.

If GE wishes to raise new capital, it can do so either by borrowing or by selling new shares to investors. Sales of shares to raise new capital are said to occur in the primary market. However, such sales occur relatively infrequently and most trades in GE take place on the stock exchange, where investors buy and sell existing GE shares. Stock exchanges are really markets for secondhand shares, but they prefer to describe themselves as secondary markets, which sounds more important.

The two principal stock exchanges in the United States are the New York Stock Exchange and Nasdaq. Both compete vigorously for business and just as vigorously tout the advantages of their trading systems. The volume of business that they handle is immense. For example, on an average day the NYSE trades around 2.8 billion shares in some 2,800 companies.

In addition to the NYSE and Nasdaq, there are a number of computer networks called electronic communication networks (ECNs) that connect traders with each other. Large U.S. companies may also arrange for their shares to be traded on foreign exchanges, such as the London exchange or the Euronext exchange in Paris. At the same time many foreign companies are listed on the U.S. exchanges. For example, the NYSE trades shares in Toyota, Royal Dutch Shell, Canadian Pacific, Tata Motors, Nokia, Brasil Telecom, China Eastern Airlines, and more than 400 other companies.

Suppose that Ms. Jones, a longtime GE shareholder, no longer wishes to hold her shares in the company. She can sell them via the NYSE to Mr. Brown, who wants to increase his stake in the firm. The transaction merely transfers partial ownership of the firm from one investor to another. No new shares are created, and GE will neither care nor know that the trade has taken place.

Ms. Jones and Mr. Brown do not trade the GE shares themselves. Instead, their orders must go through a brokerage firm. Ms. Jones, who is anxious to sell, might give her broker a market order to sell stock at the best available price. On the other hand, Mr. Brown might state a price limit at which he is willing to buy GE stock. If his limit order cannot be executed immediately, it is recorded in the exchange’s limit order book until it can be executed.

When they transact on the NYSE, Brown and Jones are participating in a huge auction market in which the exchange’s designated market makers match up the orders of thousands of investors. Most major exchanges around the world, such as the Tokyo Stock Exchange, the London Stock Exchange, and the Deutsche Börse, are also auction markets, but the auctioneer in these cases is a computer. This means that there is no stock exchange floor to show on the evening news and no one needs to ring a bell to start trading.

Nasdaq is not an auction market. All trades on Nasdaq take place between the investor and one of a group of professional dealers who are prepared to buy and sell stock. Dealer markets are relatively rare for trading equities but are common for many other financial instruments. For example, most bonds are traded in dealer markets.

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1 Trades are still made face to face on the floor of the NYSE, but computerized trading is expanding rapidly. In 2006 the NYSE merged with Archipelago, an electronic trading system, and transformed itself into a public corporation. The following year it merged with Euronext, an electronic trading system in Europe and changed its name to NYSE Euronext.
The prices at which stocks trade are summarized in the daily press. Here, for example, is how The Wall Street Journal’s Web site (www.wsj.com) recorded a day’s trading in GE in March 2009:

<table>
<thead>
<tr>
<th>Hi</th>
<th>Lo</th>
<th>Volume</th>
<th>Closing Price</th>
<th>Net Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.52</td>
<td>5.93</td>
<td>216,297,410</td>
<td>9.62</td>
<td>0.05</td>
</tr>
</tbody>
</table>

You can see that on this day investors traded a total of 216 million shares of GE stock. By the close of the day the stock traded at $9.62 a share, up $0.05 from the day before. Since there were 10.6 billion shares of GE outstanding, investors were placing a total value on the stock of $102 billion.

Buying stocks is a risky occupation. GE’s stock price had peaked at about $60 in 2001. By March 2009, an unfortunate investor who had bought in at $60 would have lost 84% of his or her investment. Of course, you don’t come across such people at cocktail parties; they either keep quiet or aren’t invited.

Most of the trading on the NYSE and Nasdaq is in ordinary common stocks, but other securities are traded also, including preferred shares, which we cover in Chapter 14, and warrants, which we cover in Chapter 21. Investors can also choose from hundreds of exchange-traded funds (ETFs), which are portfolios of stocks that can be bought or sold in a single trade. These include SPDRs (Standard & Poor’s Depository Receipts or “spiders”), which are portfolios tracking several Standard & Poor’s stock market indexes, including the benchmark S&P 500. You can buy DIAMONDS, which track the Dow Jones Industrial Average; QUBES or QQQQs, which track the Nasdaq 100 index, as well as ETFs that track specific industries or commodities. You can also buy shares in closed-end mutual funds that invest in portfolios of securities. These include country funds, for example, the Mexico and Chile funds, that invest in portfolios of stocks in specific countries.

Finding the value of GE stock may sound like a simple problem. Each quarter, the company publishes a balance sheet, which lists the value of the firm’s assets and liabilities. At the end of 2008 the book value of all GE’s assets—plant and machinery, inventories of materials, cash in the bank, and so on—was $798 billion. GE’s liabilities—money that it owes the banks, taxes that are due to be paid, and the like—amounted to $693 billion. The difference between the value of the assets and the liabilities was $105 billion. This was the book value of GE’s equity.

Book value is a reassuringly definite number. Each year KPMG, one of America’s largest accounting firms, gives its opinion that GE’s financial statements present fairly in all material respects the company’s financial position, in conformity with U.S. generally accepted accounting principles (commonly called GAAP). However, the book value of GE’s assets measures their original (or “historical”) cost less an allowance for depreciation. This may not be a good guide to what those assets are worth today. When GE raised money to invest in various projects, it judged that those projects were worth more than they cost. If it was right, its shares should sell for more than their book value.

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Closed-end mutual funds issue shares that are traded on stock exchanges. Open-end funds are not traded on exchanges. Investors in open-end funds transact directly with the fund. The fund issues new shares to investors and redeems shares from investors who want to withdraw money from the fund.
Valuation by Comparables

When financial analysts need to value a business, they often start by identifying a sample of similar firms. They then examine how much investors in these companies are prepared to pay for each dollar of assets or earnings. This is often called valuation by comparables. Look, for example, at Table 4.1. The first column of numbers shows for some well-known companies the ratio of the market value of the equity to its book value. Notice that market value is generally higher than book value. There are two exceptions; GE’s stock was worth exactly book value while Dow Chemical stock was selling for much less than book.

The second column of numbers shows the market-to-book ratio for competing firms. For example, you can see from the first row of the table that the stock of the typical large pharmaceutical firm sells for three times its book value. Therefore, if you did not have a market price for the stock of Johnson & Johnson (J&J), you might estimate that it would also sell at three times book value. This would give you a stock price of $46, a bit lower than the actual market price of $52.

An alternative would be to look at how much investors in other pharmaceutical stocks are prepared to pay for each dollar of earnings. The first row of Table 4.1 shows that the typical price–earnings (P/E) ratio for these stocks is 10.9. If you assumed that Johnson & Johnson should sell at a similar multiple of earnings, you would get a value for the stock of just under $50, only a shade lower than the actual price in March 2009.

Valuation by comparables worked well for Johnson & Johnson, but that is not the case for all the companies shown in Table 4.1. For example, if you had naively assumed that Amazon stock would sell at similar ratios to comparable dot.com stocks, you would have been out by a wide margin. Both the market-to-book ratio and the price–earnings ratio can vary considerably from stock to stock even for firms that are in the same line of business. To understand why this is so, we need to dig deeper and look at what determines a stock’s market value.

<table>
<thead>
<tr>
<th>Company</th>
<th>Competitors</th>
<th>Company</th>
<th>Competitors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Johnson &amp; Johnson</td>
<td>3.4</td>
<td>11.3</td>
<td>10.9</td>
</tr>
<tr>
<td>PepsiCo</td>
<td>6.4</td>
<td>15.6</td>
<td>12.9</td>
</tr>
<tr>
<td>Campbell Soup</td>
<td>9.0</td>
<td>8.8</td>
<td>11.3</td>
</tr>
<tr>
<td>Wal-Mart</td>
<td>3.0</td>
<td>14.6</td>
<td>13.4</td>
</tr>
<tr>
<td>Exxon Mobil</td>
<td>2.9</td>
<td>7.6</td>
<td>5.3</td>
</tr>
<tr>
<td>Dow Chemical</td>
<td>0.5</td>
<td>12.5</td>
<td>10.6</td>
</tr>
<tr>
<td>Dell Computer</td>
<td>4.5</td>
<td>7.9</td>
<td>11.1</td>
</tr>
<tr>
<td>Amazon</td>
<td>11.2</td>
<td>46.9</td>
<td>22.2</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>4.4</td>
<td>14.1</td>
<td>13.6</td>
</tr>
<tr>
<td>American Electric Power</td>
<td>1.1</td>
<td>8.1</td>
<td>11.0</td>
</tr>
<tr>
<td>GE</td>
<td>1.0</td>
<td>4.6</td>
<td>8.8</td>
</tr>
</tbody>
</table>


* Figures are median ratios for competing companies.
The Determinants of Stock Prices

Think back to Chapter 2, where we described how to value future cash flows. The discounted-cash-flow (DCF) formula for the present value of a stock is just the same as it is for the present value of any other asset. We just discount the cash flows by the return that can be earned in the capital market on securities of comparable risk. Shareholders receive cash from the company in the form of a stream of dividends. So

\[ \text{PV(stock)} = \text{PV(expected future dividends)} \]

At first sight this statement may seem surprising. When investors buy stocks, they usually expect to receive a dividend, but they also hope to make a capital gain. Why does our formula for present value say nothing about capital gains? As we now explain, there is no inconsistency.

Today's Price

The cash payoff to owners of common stocks comes in two forms: (1) cash dividends and (2) capital gains or losses. Suppose that the current price of a share is \( P_0 \), that the expected price at the end of a year is \( P_1 \), and that the expected dividend per share is \( \text{DIV}_1 \). The rate of return that investors expect from this share over the next year is defined as the expected dividend per share \( \text{DIV}_1 \) plus the expected price appreciation per share \( P_1 - P_0 \), all divided by the price at the start of the year \( P_0 \):

\[ \text{Expected return} = r = \frac{\text{DIV}_1 + P_1 - P_0}{P_0} \]

Suppose Fledgling Electronics stock is selling for $100 a share \( (P_0 = 100) \). Investors expect a $5 cash dividend over the next year \( (\text{DIV}_1 = 5) \). They also expect the stock to sell for $110 a year hence \( (P_1 = 110) \). Then the expected return to the stockholders is 15%:

\[ r = \frac{5 + 110 - 100}{100} = .15, \text{ or } 15\% \]

On the other hand, if you are given investors’ forecasts of dividend and price and the expected return offered by other equally risky stocks, you can predict today’s price:

\[ \text{Price} = P_0 = \frac{\text{DIV}_1 + P_1}{1 + r} \]

For Fledgling Electronics \( \text{DIV}_1 = 5 \) and \( P_1 = 110 \). If \( r \), the expected return for Fledgling is 15%, then today’s price should be $100:

\[ P_0 = \frac{5 + 110}{1.15} = 100 \]

What exactly is the discount rate, \( r \), in this calculation? It’s called the market capitalization rate or cost of equity capital, which are just alternative names for the opportunity cost of capital, defined as the expected return on other securities with the same risks as Fledgling shares.

Many stocks will be safer than Fledgling, and many riskier. But among the thousands of traded stocks there will be a group with essentially the same risks. Call this group Fledgling’s risk class. Then all stocks in this risk class have to be priced to offer the same expected rate of return.

Let’s suppose that the other securities in Fledgling’s risk class all offer the same 15% expected return. Then $100 per share has to be the right price for Fledgling stock. In fact it is the only possible price. What if Fledgling’s price were above \( P_0 = 100 \)? In this case investors would shift their capital to the other securities and in the process would force down the price of Fledgling stock. If \( P_0 \) were less than $100, the process would reverse.
Investors would rush to buy, forcing the price up to $100. Therefore at each point in time all securities in an equivalent risk class are priced to offer the same expected return. This is a condition for equilibrium in well-functioning capital markets. It is also common sense.

**But What Determines Next Year’s Price?**

We have managed to explain today’s stock price $P_0$ in terms of the dividend $D_1$ and the expected price next year $P_1$. Future stock prices are not easy things to forecast directly. But think about what determines next year’s price. If our price formula holds now, it ought to hold then as well:

\[ \frac{P_1}{1 + r} = \frac{D_1 + P_2}{1 + r} \]

That is, a year from now investors will be looking out at dividends in year 2 and price at the end of year 2. Thus we can forecast $P_1$ by forecasting $D_2$ and $P_2$, and we can express $P_0$ in terms of $D_1$, $D_2$, and $P_2$:

\[ P_0 = \frac{1}{1 + r} (D_1 + P_1) = \frac{1}{1 + r} \left( \frac{D_1 + D_2 + P_2}{1 + r} \right) = \frac{D_1}{1 + r} + \frac{D_2 + P_2}{(1 + r)^2} \]

Take Fledgling Electronics. A plausible explanation for why investors expect its stock price to rise by the end of the first year is that they expect higher dividends and still more capital gains in the second. For example, suppose that they are looking today for dividends of $5.50 in year 2 and a subsequent price of $121. That implies a price at the end of year 1 of

\[ P_1 = \frac{5.50 + 121}{1.15} = \$110 \]

Today’s price can then be computed either from our original formula

\[ P_0 = \frac{D_1 + P_1}{1 + r} = \frac{5.00 + 110}{1.15} = \$100 \]

or from our expanded formula

\[ P_0 = \frac{D_1}{1 + r} + \frac{D_2 + P_2}{(1 + r)^2} = \frac{5.00}{1.15} + \frac{5.50 + 121}{(1.15)^2} = \$100 \]

We have succeeded in relating today’s price to the forecasted dividends for two years ($D_1$ and $D_2$) plus the forecasted price at the end of the second year ($P_2$). You will not be surprised to learn that we could go on to replace $P_2$ by $(D_3 + P_3)/(1 + r)$ and relate today’s price to the forecasted dividends for three years ($D_1$, $D_2$, and $D_3$) plus the forecasted price at the end of the third year ($P_3$). In fact we can look as far out into the future as we like, removing $P$s as we go. Let us call this final period $H$. This gives us a general stock price formula:

\[ P_0 = \frac{D_1}{1 + r} + \frac{D_2 + P_2}{(1 + r)^2} + \cdots + \frac{D_H + P_H}{(1 + r)^H} \]

\[ = \sum_{t=1}^{H} \frac{D_t}{(1 + r)^t} + \frac{P_H}{(1 + r)^H} \]

The expression $\sum_{t=1}^{H}$ indicates the sum of the discounted dividends from year 1 to year $H$.

Table 4.2 continues the Fledgling Electronics example for various time horizons, assuming that the dividends are expected to increase at a steady 10% compound rate. The expected price $P_t$ increases at the same rate each year. Each line in the table represents an application
TABLE 4.2 Applying the stock valuation formula to Fledgling Electronics.

Assumptions:
1. Dividends increase at 10% per year, compounded.
2. Capitalization rate is 15%.

of our general formula for a different value of $H$. Figure 4.1 is a graph of the table. Each column shows the present value of the dividends up to the time horizon and the present value of the price at the horizon. As the horizon recedes, the dividend stream accounts for an increasing proportion of present value, but the total present value of dividends plus terminal price always equals $100.

How far out could we look? In principle, the horizon period $H$ could be infinitely distant. Common stocks do not expire of old age. Barring such corporate hazards as bankruptcy or acquisition, they are immortal. As $H$ approaches infinity, the present value of the terminal price ought to approach zero, as it does in the final column of Figure 4.1. We can, therefore, forget about the terminal price entirely and express today’s price as the present value of a perpetual stream of cash dividends. This is usually written as

$$P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1 + r)^t}$$

where $\infty$ indicates infinity.

This discounted-cash-flow (DCF) formula for the present value of a stock is just the same as it is for the present value of any other asset. We just discount the cash flows—in this case the dividend stream—by the return that can be earned in the capital market on securities of equivalent risk. Some find the DCF formula implausible because it seems to ignore capital gains. But we know that the formula was derived from the assumption that price in any period is determined by expected dividends and capital gains over the next period.

Notice that it is not correct to say that the value of a share is equal to the sum of the discounted stream of earnings per share. Earnings are generally larger than dividends because part of those earnings is reinvested in new plant, equipment, and working capital. Discounting earnings would recognize the rewards of that investment (a higher future dividend) but not the sacrifice (a lower dividend today). The correct formulation states that share value is equal to the discounted stream of dividends per share.
These days many growth companies do not pay dividends. Any cash that is not plowed back into the company is used to buy back stock. Take Cisco, for example. Cisco has never paid a dividend. Yet it is a successful company with a market capitalization of $100 billion. How can this be consistent with the dividend discount model?

If it were the case that Cisco’s shareholders could never look forward to receiving a cash dividend or being bought out by another company, then it would indeed be difficult to explain the price of the stock. But sometime in the future profitable investment opportunities for Cisco are likely to become less plentiful, releasing cash that can be paid out as dividends. It is this prospect that accounts for the $100 billion that shareholders are prepared to pay for the company.

4-3 Estimating the Cost of Equity Capital

In Chapter 2 we encountered some simplified versions of the basic present value formula. Let us see whether they offer any insights into stock values. Suppose, for example, that we forecast a constant growth rate for a company’s dividends. This does not preclude year-to-year deviations from the trend: It means only that expected dividends grow at a constant rate. Such an investment would be just another example of the growing perpetuity that we valued in Chapter 2. To find its present value we must divide the first year’s cash payment by the difference between the discount rate and the growth rate:

\[
P_0 = \frac{\text{DIV}_1}{r - g}
\]

Remember that we can use this formula only when \( g \), the anticipated growth rate, is less than \( r \), the discount rate. As \( g \) approaches \( r \), the stock price becomes infinite. Obviously \( r \) must be greater than \( g \) if growth really is perpetual.

\[\text{\textsuperscript{1}}\] If Cisco were taken over, any cash payment to Cisco’s shareholders would be equivalent to a bumper dividend.
Our growing perpetuity formula explains $P_0$ in terms of next year’s expected dividend $DIV_1$, the projected growth trend $g$, and the expected rate of return on other securities of comparable risk $r$. Alternatively, the formula can be turned around to obtain an estimate of $r$ from $DIV_1$, $P_0$, and $g$:

$$r = \frac{DIV_1}{P_0} + g$$

The expected return equals the dividend yield ($DIV_1/P_0$) plus the expected rate of growth in dividends ($g$).

These two formulas are much easier to work with than the general statement that “price equals the present value of expected future dividends.” Here is a practical example.

**Using the DCF Model to Set Gas and Electricity Prices**

In the United States the prices charged by local electric and gas utilities are regulated by state commissions. The regulators try to keep consumer prices down but are supposed to allow the utilities to earn a fair rate of return. But what is fair? It is usually interpreted as $r$, the market capitalization rate for the firm’s common stock. In other words the fair rate of return on equity for a public utility ought to be the cost of equity, that is, the rate offered by securities that have the same risk as the utility’s common stock.5

Small variations in estimates of this return can have large effects on the prices charged to the customers and on the firm’s profits. So both utilities and regulators work hard to estimate the cost of equity accurately. They’ve noticed that utilities are mature, stable companies that are tailor-made for application of the constant-growth DCF formula.6

Suppose you wished to estimate the cost of equity for Northwest Natural Gas, a local natural gas distribution company. Its stock was selling for $42.45 per share at the start of 2009. Dividend payments for the next year were expected to be $1.68 a share. Thus it was a simple matter to calculate the first half of the DCF formula:

$$\text{Dividend yield} = \frac{DIV_1}{P_0} = \frac{1.68}{42.45} = .040, \text{ or 4.0\%}$$

The hard part is estimating $g$, the expected rate of dividend growth. One option is to consult the views of security analysts who study the prospects for each company. Analysts are rarely prepared to stick their necks out by forecasting dividends to kingdom come, but they often forecast growth rates over the next five years, and these estimates may provide an indication of the expected long-run growth path. In the case of Northwest, analysts in 2009 were forecasting an annual growth of 6.1%.7 This, together with the dividend yield, gave an estimate of the cost of equity capital:

$$r = \frac{DIV_1}{P_0} + g = .040 + .061 = .101, \text{ or 10.1\%}$$

---

4 These formulas were first developed in 1938 by Williams and were rediscovered by Gordon and Shapiro. See J. B. Williams, *The Theory of Investment Value* (Cambridge, MA: Harvard University Press, 1938); and M. J. Gordon and E. Shapiro, “Capital Equipment Analysis: The Required Rate of Profit,” Management Science 3 (October 1956), pp. 102–110.

5 This is the accepted interpretation of the U.S. Supreme Court’s directive in 1944 that “the returns to the equity owner [of a regulated business] should be commensurate with returns on investments in other enterprises having corresponding risks.” *Federal Power Commission v. Hope Natural Gas Company*, 302 U.S. 591 at 603.

6 There are many exceptions to this statement. For example, Pacific Gas & Electric (PG&E), which serves northern California, used to be a mature, stable company until the California energy crisis of 2000 sent wholesale electric prices sky-high. PG&E was not allowed to pass these price increases on to retail customers. The company lost more than $3.5 billion in 2000 and was forced to declare bankruptcy in 2001. PG&E emerged from bankruptcy in 2004, but we may have to wait a while before it is again a suitable subject for the constant-growth DCF formula.

7 In this calculation we’re assuming that earnings and dividends are forecasted to grow forever at the same rate $g$. We show how to relax this assumption later in this chapter. The growth rate was based on the average earnings growth forecasted by Value Line and IBES. IBES compiles and averages forecasts made by security analysts. Value Line publishes its own analysts’ forecasts.
An alternative approach to estimating long-run growth starts with the payout ratio, the ratio of dividends to earnings per share (EPS). For Northwest, this was forecasted at 60%. In other words, each year the company was plowing back into the business about 40% of earnings per share:

\[
\text{Plowback ratio} = 1 - \text{payout ratio} = 1 - \frac{\text{DIV}}{\text{EPS}} = 1 - .60 = .40
\]

Also, Northwest’s ratio of earnings per share to book equity per share was about 11%. This is its return on equity, or ROE:

\[
\text{Return on equity} = \text{ROE} = \frac{\text{EPS}}{\text{book equity per share}} = .11
\]

If Northwest earns 11% of book equity and reinvests 40% of income, then book equity will increase by \(.40 \times .11 = .044\), or 4.4%. Earnings and dividends per share will also increase by 4.4%:

\[
\text{Dividend growth rate} = g = \text{plowback ratio} \times \text{ROE} = .40 \times .11 = .044
\]

That gives a second estimate of the market capitalization rate:

\[
\hat{r} = \frac{\text{DIV}_1}{P_0} + g = .040 + .044 = .084, \text{ or } 8.4\%
\]

Although these estimates of Northwest’s cost of equity seem reasonable, there are obvious dangers in analyzing any single firm’s stock with the constant-growth DCF formula. First, the underlying assumption of regular future growth is at best an approximation. Second, even if it is an acceptable approximation, errors inevitably creep into the estimate of \(g\).

Remember, Northwest’s cost of equity is not its personal property. In well-functioning capital markets investors capitalize the dividends of all securities in Northwest’s risk class at exactly the same rate. But any estimate of \(r\) for a single common stock is “noisy” and subject to error. Good practice does not put too much weight on single-company estimates of the cost of equity. It collects samples of similar companies, estimates \(r\) for each, and takes an average. The average gives a more reliable benchmark for decision making.

The next-to-last column of Table 4.3 gives DCF cost-of-equity estimates for Northwest and seven other gas distribution companies. These are all stable, mature companies for which the constant-growth DCF formula ought to work. Notice the variation in the cost-of-equity estimates. Some of the variation may reflect differences in the risk, but some is just noise. The average estimate is 10.2%.

Table 4.4 gives another example of DCF cost-of-equity estimates, this time for U.S. railroads in 2009.

Estimates of this kind are only as good as the long-term forecasts on which they are based. For example, several studies have observed that security analysts are subject to behavioral biases and their forecasts tend to be over-optimistic. If so, such DCF estimates of the cost of equity should be regarded as upper estimates of the true figure.

**Dangers Lurk in Constant-Growth Formulas**

The simple constant-growth DCF formula is an extremely useful rule of thumb, but no more than that. Naive trust in the formula has led many financial analysts to silly conclusions.

We have stressed the difficulty of estimating \(r\) by analysis of one stock only. Try to use a large sample of equivalent-risk securities. Even that may not work, but at least it gives the analyst a fighting chance, because the inevitable errors in estimating \(r\) for a single security tend to balance out across a broad sample.
In addition, resist the temptation to apply the formula to firms having high current rates of growth. Such growth can rarely be sustained indefinitely, but the constant-growth DCF formula assumes it can. This erroneous assumption leads to an overestimate of \( r \).

**DCF Valuation with Varying Growth Rates** Consider Growth-Tech, Inc., a firm with \( \text{DIV}_1 = .50 \) and \( P_0 = 50 \). The firm has plowed back 80% of earnings and has had a return on equity (ROE) of 25%. This means that *in the past*

\[
\text{Dividend growth rate} = \text{plowback ratio} \times \text{ROE} = .80 \times .25 = .20
\]

The temptation is to assume that the future long-term growth rate \( g \) also equals .20. This would imply...
But this is silly. No firm can continue growing at 20% per year forever, except possibly under extreme inflationary conditions. Eventually, profitability will fall and the firm will respond by investing less.

In real life the return on equity will decline gradually over time, but for simplicity let’s assume it suddenly drops to 16% at year 3 and the firm responds by plowing back only 50% of earnings. Then $g$ drops to $0.50/0.16 = 0.08$.

Table 4.5 shows what’s going on. Growth-Tech starts year 1 with book equity of $10.00 per share. It earns $2.50, pays out 50 cents as dividends, and plows back $2. Thus it starts year 2 with book equity of $10 + 2 = 12$. After another year at the same ROE and payout, it starts year 3 with equity of $14.40$. However, ROE drops to .16, and the firm earns only $2.30$. Dividends go up to $1.15$, because the payout ratio increases, but the firm has only $1.15$ to plow back. Therefore subsequent growth in earnings and dividends drops to 8%.

Now we can use our general DCF formula:

$$P_0 = \frac{\text{DIV}_1}{1 + r} + \frac{\text{DIV}_2}{(1 + r)^2} + \frac{\text{DIV}_3 + P_3}{(1 + r)^3}$$

Investors in year 3 will view Growth-Tech as offering 8% per year dividend growth. So we can use the constant-growth formula to calculate $P_3$:

$$P_3 = \frac{\text{DIV}_4}{r - 0.08}$$

$$P_0 = \frac{\text{DIV}_1}{1 + r} + \frac{\text{DIV}_2}{(1 + r)^2} + \frac{\text{DIV}_3}{(1 + r)^3} + \frac{1}{(1 + r)^3} \times \frac{\text{DIV}_4}{r - 0.08}$$

$$= \frac{0.50}{1 + r} + \frac{0.60}{(1 + r)^2} + \frac{1.15}{(1 + r)^3} + \frac{1.24}{r - 0.08}$$

We have to use trial and error to find the value of $r$ that makes $P_0$ equal $50$. It turns out that the $r$ implicit in these more realistic forecasts is approximately $0.099$, quite a difference from our “constant-growth” estimate of $0.21$.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book equity</td>
<td>10.00</td>
<td>12.00</td>
<td>14.40</td>
<td>15.55</td>
</tr>
<tr>
<td>Earnings per share, EPS</td>
<td>2.50</td>
<td>3.00</td>
<td>2.30</td>
<td>2.49</td>
</tr>
<tr>
<td>Return on equity, ROE</td>
<td>.25</td>
<td>.25</td>
<td>.16</td>
<td>.16</td>
</tr>
<tr>
<td>Payout ratio</td>
<td>.20</td>
<td>.20</td>
<td>.50</td>
<td>.50</td>
</tr>
<tr>
<td>Dividends per share, DIV</td>
<td>.50</td>
<td>.60</td>
<td>1.15</td>
<td>1.24</td>
</tr>
<tr>
<td>Growth rate of dividends (%)</td>
<td>—</td>
<td>20</td>
<td>92</td>
<td>8</td>
</tr>
</tbody>
</table>

**TABLE 4.5** Forecasted earnings and dividends for Growth-Tech. Note the changes in year 3: ROE and earnings drop, but payout ratio increases, causing a big jump in dividends. However, subsequent growth in earnings and dividends falls to 8% per year. Note that the increase in equity equals the earnings not paid out as dividends.
Our present value calculations for Growth-Tech used a *two-stage* DCF valuation model. In the first stage (years 1 and 2), Growth-Tech is highly profitable (ROE = 25%), and it plows back 80% of earnings. Book equity, earnings, and dividends increase by 20% per year. In the second stage, starting in year 3, profitability and plowback decline, and earnings settle into long-term growth at 8%. Dividends jump up to $1.15 in year 3, and then also grow at 8%.

Growth rates can vary for many reasons. Sometimes growth is high in the short run not because the firm is unusually profitable, but because it is recovering from an episode of low profitability. Table 4.6 displays projected earnings and dividends for Phoenix Corp., which is gradually regaining financial health after a near meltdown. The company’s equity is growing at a moderate 4%. ROE in year 1 is only 4%, however, so Phoenix has to reinvest all its earnings, leaving no cash for dividends. As profitability increases in years 2 and 3, an increasing dividend can be paid. Finally, starting in year 4, Phoenix settles into steady-state growth, with equity, earnings, and dividends all increasing at 4% per year.

Assume the cost of equity is 10%. Then Phoenix shares should be worth $9.13 per share:

\[
P_0 = \frac{0}{1.1} + \frac{.31}{(1.1)^2} + \frac{.65}{(1.1)^3} + \frac{1}{(1.1)^3} \times \frac{.67}{(.10 - .04)} = $9.13
\]

You could go on to valuation models with three or more stages. For example, the far right columns of Tables 4.3 and 4.4 present multistage DCF estimates of the cost of equity for our local gas distribution companies and railroads. In this case the long-term growth rates reported in the table do not continue forever. After five years, each company’s growth rate gradually adjusts to an estimated long-term growth rate for Gross Domestic Product (GDP). The resulting cost-of-equity estimates for the gas distribution companies are fairly similar to the estimates from the simple, perpetual-growth model. The estimates for the railroads are substantially different.

We must leave you with two more warnings about DCF formulas for valuing common stocks or estimating the cost of equity. First, it’s almost always worthwhile to lay out a simple spreadsheet, like Table 4.5 or 4.6, to ensure that your dividend projections are consistent with the company’s earnings and required investments. Second, be careful about using DCF valuation formulas to test whether the market is correct in its assessment of a stock’s value. If your estimate of the value is different from that of the market, it is probably because you have used poor dividend forecasts. Remember what we said at the beginning of this chapter about simple ways of making money on the stock market: there aren’t any.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book equity at start of year</td>
<td>10.00</td>
<td>10.40</td>
<td>10.82</td>
<td>11.25</td>
</tr>
<tr>
<td>Earnings per share, EPS</td>
<td>.40</td>
<td>.73</td>
<td>1.08</td>
<td>1.12</td>
</tr>
<tr>
<td>Return on equity, ROE</td>
<td>.04</td>
<td>.07</td>
<td>.10</td>
<td>.10</td>
</tr>
<tr>
<td>Dividends per share, DIV</td>
<td>0</td>
<td>.31</td>
<td>.65</td>
<td>.67</td>
</tr>
<tr>
<td>Growth rate of dividends (%)</td>
<td>—</td>
<td>—</td>
<td>110</td>
<td>4</td>
</tr>
</tbody>
</table>

**TABLE 4.6** Forecasted earnings and dividends for Phoenix Corp. The company can initiate and increase dividends as profitability (ROE) recovers. Note that the increase in book equity equals the earnings not paid out as dividends.
Investors separate *growth stocks* from *income stocks*. They buy growth stocks primarily for the expectation of capital gains, and they are interested in the future growth of earnings rather than in next year’s dividends. They buy income stocks primarily for the cash dividends. Let us see whether these distinctions make sense.

Imagine first the case of a company that does not grow at all. It does not plow back any earnings and simply produces a constant stream of dividends. Its stock would resemble the perpetual bond described in Chapter 2. Remember that the return on a perpetuity is equal to the yearly cash flow divided by the present value. So the expected return on our share would be equal to the yearly dividend divided by the share price (i.e., the dividend yield). Since all the earnings are paid out as dividends, the expected return is also equal to the earnings per share divided by the share price (i.e., the earnings–price ratio). For example, if the dividend is $10 a share and the stock price is $100, we have

\[
\text{Expected return} = \frac{\text{dividend yield}}{\text{earnings–price ratio}} = \frac{\text{DIV}_1}{P_0} = \frac{\text{EPS}_1}{P_0} = \frac{10.00}{100} = .10
\]

The price equals

\[
P_0 = \frac{\text{DIV}_1}{r} = \frac{\text{EPS}_1}{r} = \frac{10.00}{.10} = 100
\]

The expected return for *growing* firms can also equal the earnings–price ratio. The key is whether earnings are reinvested to provide a return equal to the market capitalization rate. For example, suppose our monotonous company suddenly hears of an opportunity to invest $10 a share next year. This would mean no dividend at \( t = 1 \). However, the company expects that in each subsequent year the project would earn $1 per share, and therefore the dividend could be increased to $11 a share.

Let us assume that this investment opportunity has about the same risk as the existing business. Then we can discount its cash flow at the 10% rate to find its net present value at year 1:

\[
\text{Net present value per share at year 1} = -10 + \frac{1}{.10} = 0
\]

Thus the investment opportunity will make no contribution to the company’s value. Its prospective return is equal to the opportunity cost of capital.

What effect will the decision to undertake the project have on the company’s share price? Clearly none. The reduction in value caused by the nil dividend in year 1 is exactly offset by the increase in value caused by the extra dividends in later years. Therefore, once again the market capitalization rate equals the earnings–price ratio:

\[
r = \frac{\text{EPS}_1}{P_0} = \frac{10}{100} = .10
\]

Table 4.7 repeats our example for different assumptions about the cash flow generated by the new project. Note that the earnings–price ratio, measured in terms of \( \text{EPS}_1 \), next year’s expected earnings, equals the market capitalization rate \( r \) only when the new project’s \( \text{NPV} = 0 \). This is an extremely important point—managers frequently make poor financial decisions because they confuse earnings–price ratios with the market capitalization rate.
In general, we can think of stock price as the capitalized value of average earnings under a no-growth policy, plus PVGO, the net present value of growth opportunities:

\[ P_0 = \frac{EPS_1}{r} + PVGO \]

The earnings–price ratio, therefore, equals

\[ \frac{EPS}{P_0} = r \left( 1 - \frac{PVGO}{P_0} \right) \]

It will underestimate \( r \) if PVGO is positive and overestimate it if PVGO is negative. The latter case is less likely, since firms are rarely forced to take projects with negative net present values.

### Calculating the Present Value of Growth Opportunities for Fledgling Electronics

In our last example both dividends and earnings were expected to grow, but this growth made no net contribution to the stock price. The stock was in this sense an “income stock.” Be careful not to equate firm performance with the growth in earnings per share. A company that reinvests earnings at below the market capitalization rate \( r \) may increase earnings but will certainly reduce the share value.

Now let us turn to that well-known growth stock, Fledgling Electronics. You may remember that Fledgling’s market capitalization rate, \( r_1 \), is 15%. The company is expected to pay a dividend of $5 in the first year, and thereafter the dividend is predicted to increase indefinitely by 10% a year. We can use the simplified constant-growth formula to work out Fledgling’s price:

\[ P_0 = \frac{DIV_1}{r - g} = \frac{5}{.15 - .10} = $100 \]

Suppose that Fledgling has earnings per share of \( EPS_1 = 8.33 \). Its payout ratio is then

\[ \text{Payout ratio} = \frac{DIV_1}{EPS_1} = \frac{5.00}{8.33} = .6 \]

In other words, the company is plowing back \( 1 - .6 \), or 40% of earnings. Suppose also that Fledgling’s ratio of earnings to book equity is ROE = .25. This explains the growth rate of 10%:

\[ g = \text{plowback ratio} \times \text{ROE} = .4 \times .25 = .10 \]
The capitalized value of Fledgling's earnings per share if it had a no-growth policy would be

\[ \frac{\text{EPS}_1}{r} = \frac{8.33}{.15} = \$55.56 \]

But we know that the value of Fledgling stock is $100. The difference of $44.44 must be the amount that investors are paying for growth opportunities. Let's see if we can explain that figure.

Each year Fledgling plows back 40% of its earnings into new assets. In the first year Fledgling invests $3.33 at a permanent 25% return on equity. Thus the cash generated by this investment is .25 \times 3.33 = \$8.33 per year starting at \( t = 2 \). The net present value of the investment as of \( t = 1 \) is

\[ \text{NPV}_1 = -3.33 + \frac{.83}{.15} = \$2.22 \]

Everything is the same in year 2 except that Fledgling will invest $3.67, 10% more than in year 1 (remember \( g = .10 \)). Therefore at \( t = 2 \) an investment is made with a net present value of

\[ \text{NPV}_2 = -3.67 + \frac{.83 \times 1.10}{.15} = \$2.44 \]

Thus the payoff to the owners of Fledgling Electronics stock can be represented as the sum of (1) a level stream of earnings, which could be paid out as cash dividends if the firm did not grow, and (2) a set of tickets, one for each future year, representing the opportunity to make investments having positive NPVs. We know that the first component of the value of the share is

\[ \text{Present value of level stream of earnings} = \frac{\text{EPS}_1}{r} = \frac{8.33}{.15} = \$55.56 \]

The first ticket is worth $2.22 in \( t = 1 \), the second is worth $2.22 \times 1.10 = \$2.44 in \( t = 2 \), the third is worth $2.44 \times 1.10 = \$2.69 in \( t = 3 \). These are the forecasted cash values of the tickets. We know how to value a stream of future cash values that grows at 10% per year: Use the constant-growth DCF formula, replacing the forecasted dividends with forecasted ticket values:

\[ \text{Present value of growth opportunities} = \text{PVGO} = \frac{\text{NPV}_1}{r - g} = \frac{2.22}{.15 - .10} = \$44.44 \]

Now everything checks:

\[ \text{Share price} = \text{present value of level stream of earnings} + \text{present value of growth opportunities} = \frac{\text{EPS}_1}{r} + \text{PVGO} = \$55.56 + \$44.44 = \$100 \]

Why is Fledgling Electronics a growth stock? Not because it is expanding at 10% per year. It is a growth stock because the net present value of its future investments accounts for a significant fraction (about 44%) of the stock's price.

Today's stock price reflects investor expectations about the earning power of the firm's current and future assets. Take Google, for example. All its earnings are plowed back into new investments and the stock sells at 26 times current earnings of $13.31 a share. Suppose that the earnings from Google's existing business are expected to stay constant in real terms. In this case the value of the business is equal to the real earnings divided by an estimated 7.4% real cost of equity:

\[ \text{PV assets in place} = \frac{13.31}{.074} = \$180 \]
But, as we write this, Google’s stock price is $344. So it looks as if investors are valuing Google’s future investment opportunities at $344/1.80 = $164. Google is a growth stock because roughly 50% of the stock price comes from the value that investors place on its future investment opportunities.

### Valuing a Business by Discounted Cash Flow

Investors routinely buy and sell shares of common stock. Companies frequently buy and sell entire businesses or major stakes in businesses. For example, in 2009 *The New York Times* announced that it had retained Goldman Sachs to explore the possible sale of its interest in the Boston Red Sox baseball team. You can be sure that *The New York Times*, Goldman Sachs, and potential purchasers all burned a lot of midnight oil to estimate the value of the business.

Do the discounted-cash-flow formulas we presented in this chapter work for entire businesses as well as for shares of common stock? Sure: It doesn’t matter whether you forecast dividends per share or the total free cash flow of a business. Value today always equals future cash flow discounted at the opportunity cost of capital.

### Valuing the Concatenator Business

Rumor has it that Establishment Industries is interested in buying your company’s concatenator manufacturing operation. Your company is willing to sell if it can get the full value of this rapidly growing business. The problem is to figure out what its true present value is.

Table 4.8 gives a forecast of free cash flow (FCF) for the concatenator business. Free cash flow is the amount of cash that a firm can pay out to investors after paying for all investments necessary for growth. As we will see, free cash flow can be negative for rapidly growing businesses.

Table 4.8 is similar to Table 4.5, which forecasted earnings and dividends per share for Growth-Tech, based on assumptions about Growth-Tech’s equity per share, return on equity,

<table>
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<tr>
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<th>3</th>
<th>4</th>
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<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<tbody>
<tr>
<td>Asset value</td>
<td>10.00</td>
<td>12.00</td>
<td>14.40</td>
<td>17.28</td>
<td>20.74</td>
<td>23.43</td>
<td>26.47</td>
<td>28.05</td>
<td>29.73</td>
<td>31.51</td>
</tr>
<tr>
<td>Earnings</td>
<td>1.20</td>
<td>1.44</td>
<td>1.73</td>
<td>2.07</td>
<td>2.49</td>
<td>2.81</td>
<td>3.18</td>
<td>3.36</td>
<td>3.57</td>
<td>3.78</td>
</tr>
<tr>
<td>Net investment</td>
<td>2.00</td>
<td>2.40</td>
<td>2.88</td>
<td>3.46</td>
<td>2.69</td>
<td>3.04</td>
<td>1.59</td>
<td>1.68</td>
<td>1.78</td>
<td>1.89</td>
</tr>
<tr>
<td>Free cash flow</td>
<td>-.80</td>
<td>-.96</td>
<td>-1.15</td>
<td>-1.39</td>
<td>-2.0</td>
<td>-2.3</td>
<td>1.59</td>
<td>1.68</td>
<td>1.79</td>
<td>1.89</td>
</tr>
<tr>
<td>Earnings growth from</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>13</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

**TABLE 4.8** Forecasts of free cash flow, in $ millions, for the Concatenator Manufacturing Division. Rapid expansion in years 1–6 means that free cash flow is negative, because required additional investment outstrips earnings. Free cash flow turns positive when growth slows down after year 6.

**Notes:**
1. Starting asset value is $10 million. Assets required for the business grow initially at 20% per year, then at 13%, and finally at 6%.
2. Profitability (earnings/asset values) is constant at 12%.
3. Free cash flow equals earnings minus net investment. Net investment equals total capital expenditures less depreciation. Note that earnings are also calculated net of depreciation.
and the growth of its business. For the concatenator business, we also have assumptions about
assets, profitability—in this case, after-tax operating earnings relative to assets—and growth.
Growth starts out at a rapid 20% per year, then falls in two steps to a moderate 6% rate for
the long run. The growth rate determines the net additional investment required to expand
assets, and the profitability rate determines the earnings thrown off by the business.\(^8\)

Free cash flow, the next to last line in Table 4.8, is equal to the firm’s earnings less any
new investment expenditures. Free cash flow is negative in years 1 through 6. The concat-
enator business is paying a negative dividend to the parent company; it is absorbing more
cash than it is throwing off.

Is that a bad sign? Not really: The business is running a cash deficit not because it is
unprofitable, but because it is growing so fast. Rapid growth is good news, not bad, so
long as the business is earning more than the opportunity cost of capital. Your company,
or Establishment Industries, will be happy to invest an extra $800,000 in the concatenator
business next year, so long as the business offers a superior rate of return.

Valuation Format
The value of a business is usually computed as the discounted value of free cash flows out
to a valuation horizon \( H \), plus the forecasted value of the business at the horizon, also
discounted back to present value. That is,

\[
PV = \frac{FCF_1}{1 + r} \left( \frac{1}{(1 + r)^H} \right) + \frac{FCF_2}{(1 + r)^2} \left( \frac{1}{(1 + r)^H} \right) + \cdots + \frac{FCF_H}{(1 + r)^H} \left( \frac{1}{(1 + r)^H} \right) + \frac{PV_{H+1}}{(1 + r)^H}
\]

Of course, the concatenator business will continue after the horizon, but it’s not practical
to forecast free cash flow year by year to infinity. \( PV_{H+1} \) stands in for free cash flow in periods
\( H + 1, H + 2, \) etc.

Valuation horizons are often chosen arbitrarily. Sometimes the boss tells everybody to
use 10 years because that’s a round number. We will try year 6, because growth of the con-
catenator business seems to settle down to a long-run trend after year 7.

Estimating Horizon Value
There are several common formulas or rules of thumb for estimating horizon value. First,
let us try the constant-growth DCF formula. This requires free cash flow for year 7, which
we have from Table 4.8, a long-run growth rate, which appears to be 6%; and a discount
rate, which some high-priced consultant has told us is 10%. Therefore,

\[
PV \text{ (horizon value)} = \frac{1}{(1.1)^6} \left( \frac{1.59}{.10 - .06} \right) = 22.4
\]

The present value of the near-term free cash flows is

\[
PV \text{ (cash flows)} = \frac{-.80}{1.1} - \frac{.96}{(1.1)^2} - \frac{1.15}{(1.1)^3} - \frac{1.39}{(1.1)^4} - \frac{.20}{(1.1)^5} - \frac{.23}{(1.1)^6}
\]

\[= -3.6\]

and, therefore, the present value of the business is

\[
PV \text{ (business)} = PV \text{ (free cash flow)} + PV \text{ (horizon value)}
\]

\[= -3.6 + 22.4\]

\[= $18.8 \text{ million}\]

---

\(^{8}\) Table 4.8 shows net investment, which is total investment less depreciation. We are assuming that investment for replacement of
existing assets is covered by depreciation and that net investment is devoted to growth.
Now, are we done? Well, the mechanics of this calculation are perfect. But doesn’t it make you just a little nervous to find that 119% of the value of the business rests on the horizon value? Moreover, a little checking shows that the horizon value can change dramatically in response to apparently minor changes in assumptions. For example, if the long-run growth rate is 8% rather than 6%, the value of the business increases from $18.8 to $26.3 million.\(^9\)

In other words, it’s easy for a discounted-cash-flow business valuation to be mechanically perfect and practically wrong. Smart financial managers try to check their results by calculating horizon value in different ways.

**Horizon Value Based on P/E Ratios** Suppose you can observe stock prices for mature manufacturing companies whose scale, risk, and growth prospects today roughly match those projected for the concatenator business in year 6. Suppose further that these companies tend to sell at price—earnings ratios of about 11. Then you could reasonably guess that the price—earnings ratio of a mature concatenator operation will likewise be 11. That implies:

\[
\begin{align*}
\text{PV(horizon value)} &= \frac{1}{(1.1)^6}(11 \times 3.18) = 19.7 \\
\text{PV(business)} &= -3.6 + 19.7 = $16.1 \text{ million}
\end{align*}
\]

**Horizon Value Based on Market–Book Ratios** Suppose also that the market–book ratios of the sample of mature manufacturing companies tend to cluster around 1.4. If the concatenator business market–book ratio is 1.4 in year 6,

\[
\begin{align*}
\text{PV(horizon value)} &= \frac{1}{(1.1)^6}(1.4 \times 23.43) = 18.5 \\
\text{PV(business)} &= -3.6 + 18.5 = $14.9 \text{ million}
\end{align*}
\]

It’s easy to poke holes in these last two calculations. Book value, for example, is often a poor measure of the true value of a company’s assets. It can fall far behind actual asset values when there is rapid inflation, and it often entirely misses important intangible assets, such as your patents for concatenator design. Earnings may also be biased by inflation and a long list of arbitrary accounting choices. Finally, you never know when you have found a sample of truly similar companies.

But remember, the purpose of discounted cash flow is to estimate market value—to estimate what investors would pay for a stock or business. When you can observe what they actually pay for similar companies, that’s valuable evidence. Try to figure out a way to use it. One way to use it is through valuation by comparables, based on price—earnings or market–book ratios. Valuation rules of thumb, artfully employed, sometimes beat a complex discounted-cash-flow calculation hands down.

**A Further Reality Check**

Here is another approach to valuing a business. It is based on what you have learned about price—earnings ratios and the present value of growth opportunities.

Suppose the valuation horizon is set not by looking for the first year of stable growth, but by asking when the industry is likely to settle into competitive equilibrium. You might go to the operating manager most familiar with the concatenator business and ask:

\[
\text{PV(horizon value)} = \frac{1}{(1.1)^6}(1.06) = $29.9
\]

\[
\text{PV(business)} = -3.6 + 29.9 = $26.3 \text{ million}
\]

\(^9\) If long-run growth is 8% rather than 6%, an extra 2% of period-7 assets will have to be plowed back into the concatenator business. This reduces free cash flow by $5.3 million to $1.06 million. So,

\[
\begin{align*}
\text{PV(horizon value)} &= \frac{1}{(1.1)^6}(1.06) = $29.9 \\
\text{PV(business)} &= -3.6 + 29.9 = $26.3 \text{ million}
\end{align*}
\]
Sooner or later you and your competitors will be on an equal footing when it comes to major new investments. You may still be earning a superior return on your core business, but you will find that introductions of new products or attempts to expand sales of existing products trigger intense resistance from competitors who are just about as smart and efficient as you are. Give a realistic assessment of when that time will come.

“That time” is the horizon after which PVGO, the net present value of subsequent growth opportunities, is zero. After all, PVGO is positive only when investments can be expected to earn more than the cost of capital. When your competition catches up, that happy prospect disappears.

We know that present value in any period equals the capitalized value of next period’s earnings, plus PVGO:

$$PV_t = \frac{\text{earnings}_{t+1}}{r} + \text{PVGO}$$

But what if PVGO = 0? At the horizon period $H$, then,

$$PV_H = \frac{\text{earnings}_{H+1}}{r}$$

In other words, when the competition catches up, the price-earnings ratio equals $1/r$, because PVGO disappears.

Suppose that competition is expected to catch up in period 9. Then we can calculate the horizon value at period 8 as the present value of a level stream of earnings starting in period 9 and continuing indefinitely. The resulting value for the concatenator business is:

$$PV(\text{horizon value}) = \frac{1}{(1 + r)^8} \left( \frac{\text{earnings in period } 9}{r} \right)$$

$$= \frac{1}{(1.1)^8} \left( \frac{3.57}{.10} \right)$$

$$= \$16.7\text{ million}$$

$$PV(\text{business}) = -2.0 + 16.7 = \$14.7\text{ million}$$

We now have four estimates of what Establishment Industries ought to pay for the concatenator business. The estimates reflect four different methods of estimating horizon value. There is no best method, although in many cases we put most weight on the last method, which sets the horizon date at the point when management expects PVGO to disappear. The last method forces managers to remember that sooner or later competition catches up.

Our calculated values for the concatenator business range from $14.7$ to $18.8$ million, a difference of about $4$ million. The width of the range may be disquieting, but it is not unusual. Discounted-cash-flow formulas only estimate market value, and the estimates change as forecasts and assumptions change. Managers cannot know market value for sure until an actual transaction takes place.

10 In other words, we can calculate horizon value as if earnings will not grow after the horizon date, because growth will add no value. But what does “no growth” mean? Suppose that the concatenator business maintains its assets and earnings in real (inflation-adjusted) terms. Then nominal earnings will growth at the inflation rate. This takes us back to the constant-growth formula: earnings in period $H + 1$ should be valued by dividing by $r - g$, where $g$ in this case equals the inflation rate.

11 Three additional points about this calculation: First, the PV of free cash flow before the horizon improves to $-2.0$ million because inflows in years 7 and 8 are now included. Second, if competition really catches up by year 9, then the earnings shown for year 10 in Table 4.8 are too high, since they include a 12% return on the investment in year 9. Competition would allow only the 10% cost of capital. Third, we assume earnings in year 9 of $3.57$, 12% on assets of $29.73$. But competition might force down the rate of return on existing assets in addition to returns on new investment. That is, earnings in year 9 could be only $2.97$ (10% of $29.73$). Problem 26 explores these possibilities.
In this chapter we have used our newfound knowledge of present values to examine the market price of common stocks. The value of a stock is equal to the stream of cash payments discounted at the rate of return that investors expect to receive on other securities with equivalent risks.

Common stocks do not have a fixed maturity; their cash payments consist of an indefinite stream of dividends. Therefore, the present value of a common stock is

$$\text{PV} = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1 + r)^t}$$

However, we did not just assume that investors purchase common stocks solely for dividends. In fact, we began with the assumption that investors have relatively short horizons and invest for both dividends and capital gains. Our fundamental valuation formula is, therefore,

$$P_0 = \frac{\text{DIV}_1 + P_1}{1 + r}$$

This is a condition of market equilibrium. If it did not hold, the share would be overpriced or underpriced, and investors would rush to sell or buy it. The flood of sellers or buyers would force the price to adjust so that the fundamental valuation formula holds.

We also made use of the formula for a growing perpetuity presented in Chapter 2. If dividends are expected to grow forever at a constant rate of $g$, then

$$P_0 = \frac{\text{DIV}_1}{r - g}$$

It is often helpful to twist this formula around and use it to estimate the market capitalization rate $r$, given $P_0$ and estimates of $\text{DIV}_1$ and $g$:

$$r = \frac{\text{DIV}_1}{P_0} + g$$

Remember, however, that this formula rests on a very strict assumption: constant dividend growth in perpetuity. This may be an acceptable assumption for mature, low-risk firms, but for many firms, near-term growth is unsustainably high. In that case, you may wish to use a two-stage DCF formula, where near-term dividends are forecasted and valued, and the constant-growth DCF formula is used to forecast the value of the shares at the start of the long run. The near-term dividends and the future share value are then discounted to present value.

The general DCF formula can be transformed into a statement about earnings and growth opportunities:

$$P_0 = \frac{\text{EPS}_1}{r} + \text{PVGO}$$

The ratio $\text{EPS}_1/r$ is the capitalized value of the earnings per share that the firm would generate under a no-growth policy. PVGO is the net present value of the investments that the firm will make in order to grow. A growth stock is one for which PVGO is large relative to the capitalized value of EPS. Most growth stocks are stocks of rapidly expanding firms, but expansion alone does not create a high PVGO. What matters is the profitability of the new investments.

The same formulas that we used to value common shares can also be used to value entire businesses. In that case, we discount not dividends per share but the entire free cash flow generated by the business. Usually a two-stage DCF model is deployed. Free cash flows are forecasted out to a horizon and discounted to present value. Then a horizon value is forecasted, discounted, and added to the value of the free cash flows. The sum is the value of the business.

Valuing a business is simple in principle but not so easy in practice. Forecasting reasonable horizon values is particularly difficult. The usual assumption is moderate long-run growth after the horizon, which allows use of the growing-perpetuity DCF formula at the horizon. Horizon
values can also be calculated by assuming “normal” price–earnings or market-to-book ratios at the horizon date.

In earlier chapters you should have acquired—we hope painlessly—a knowledge of the basic principles of valuing assets and a facility with the mechanics of discounting. Now you know something of how common stocks are valued and market capitalization rates estimated. In Chapter 5 we can begin to apply all this knowledge in a more specific analysis of capital budgeting decisions.

For a comprehensive review of company valuation, see:


Leibowitz and Kogelman call PVGO the “franchise factor.” They analyze it in detail in:


BASIC

1. True or false?
   a. All stocks in an equivalent-risk class are priced to offer the same expected rate of return.
   b. The value of a share equals the PV of future dividends per share.

2. Respond briefly to the following statement:
   “You say stock price equals the present value of future dividends? That’s crazy! All the investors I know are looking for capital gains.”

3. Company X is expected to pay an end-of-year dividend of $5 a share. After the dividend its stock is expected to sell at $110. If the market capitalization rate is 8%, what is the current stock price?

4. Company Y does not plow back any earnings and is expected to produce a level dividend stream of $5 a share. If the current stock price is $40, what is the market capitalization rate?

5. Company Z’s earnings and dividends per share are expected to grow indefinitely by 5% a year. If next year’s dividend is $10 and the market capitalization rate is 8%, what is the current stock price?

6. Company Z-prime is like Z in all respects save one: Its growth will stop after year 4. In year 5 and afterward, it will pay out all earnings as dividends. What is Z-prime’s stock price? Assume next year’s EPS is $15.

7. If company Z (see Problem 5) were to distribute all its earnings, it could maintain a level dividend stream of $15 a share. How much is the market actually paying per share for growth opportunities?

8. Consider three investors:
   a. Mr. Single invests for one year.
   b. Ms. Double invests for two years.
   c. Mrs. Triple invests for three years.
Assume each invests in company Z (see Problem 5). Show that each expects to earn a rate of return of 8% per year.

   a. The value of a share equals the discounted stream of future earnings per share.
   b. The value of a share equals the PV of earnings per share assuming the firm does not grow, plus the NPV of future growth opportunities.

10. Under what conditions does \( r \), a stock’s market capitalization rate, equal its earnings-price ratio \( \frac{EPS}{P_0} \)?


12. What is meant by the “horizon value” of a business? How can it be estimated?

13. Suppose the horizon date is set at a time when the firm will run out of positive-NPV investment opportunities. How would you calculate the horizon value? (Hint: What is the \( P/E \) ratio when \( PVGO = 0 \)?)

**INTERMEDIATE**

   a. What is the latest price of IBM stock?
   b. What are the annual dividend payment and the dividend yield on IBM stock?
   c. What would the yield be if IBM changed its yearly dividend to $1.50?
   d. What is the P/E on IBM stock?
   e. Use the P/E to calculate IBM’s earnings per share.
   f. Is IBM’s P/E higher or lower than that of Exxon Mobil?
   g. What are the possible reasons for the difference in P/E?

15. Rework Table 4.2 under the assumption that the dividend on Fledgling Electronics is $10 next year and that it is expected to grow by 5% a year. The capitalization rate is 15%.

16. Consider the following three stocks:
   a. Stock A is expected to provide a dividend of $10 a share forever.
   b. Stock B is expected to pay a dividend of $5 next year. Thereafter, dividend growth is expected to be 4% a year forever.
   c. Stock C is expected to pay a dividend of $5 next year. Thereafter, dividend growth is expected to be 20% a year for five years (i.e., until year 6) and zero thereafter.

   If the market capitalization rate for each stock is 10%, which stock is the most valuable? What if the capitalization rate is 7%?

17. Pharmecology is about to pay a dividend of $1.35 per share. It’s a mature company, but future EPS and dividends are expected to grow with inflation, which is forecasted at 2.75% per year.
   a. What is Pharmecology’s current stock price? The nominal cost of capital is 9.5%.
   b. Redo part (a) using forecasted real dividends and a real discount rate.

18. Company Q’s current return on equity (ROE) is 14%. It pays out one-half of earnings as cash dividends (payout ratio = .5). Current book value per share is $50. Book value per share will grow as Q reinvests earnings.

   Assume that the ROE and payout ratio stay constant for the next four years. After that, competition forces ROE down to 11.5% and the payout ratio increases to 0.8. The cost of capital is 11.5%.
   a. What are Q’s EPS and dividends next year? How will EPS and dividends grow in years 2, 3, 4, 5, and subsequent years?
   b. What is Q’s stock worth per share? How does that value depend on the payout ratio and growth rate after year 4?
19. Mexican Motors stock sells for 200 pesos per share and next year’s dividend is 8.5 pesos. Security analysts are forecasting earnings growth of 7.5% per year for the next five years.
   a. Assume that earnings and dividends are expected to grow at 7.5% in perpetuity. What rate of return are investors expecting?
   b. Mexican Motors has generally earned about 12% on book equity (ROE = .12) and paid out 50% of earnings as dividends. Suppose it maintains the same ROE and payout ratio in the long-run future. What is the implication for $g$? For $r$? Should you revise your answer to part (a) of this question?

20. Phoenix Corp. faltered in the recent recession but has recovered since. EPS and dividends have grown rapidly since 2017.

<table>
<thead>
<tr>
<th></th>
<th>2017</th>
<th>2018</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPS</td>
<td>$.75</td>
<td>2.00</td>
<td>2.50</td>
<td>2.60</td>
<td>2.65</td>
</tr>
<tr>
<td>Dividends</td>
<td>$0</td>
<td>1.00</td>
<td>2.00</td>
<td>2.30</td>
<td>2.65</td>
</tr>
<tr>
<td>Dividend growth</td>
<td>—</td>
<td>100%</td>
<td>15%</td>
<td>15%</td>
<td>—</td>
</tr>
</tbody>
</table>

The figures for 2020 and 2021 are of course forecasts. Phoenix’s stock price today in 2019 is $21.75. Phoenix’s recovery will be complete in 2021, and there will be no further growth in EPS or dividends.

A security analyst forecasts next year’s rate of return on Phoenix stock as follows:

$$ r = \frac{\text{DIV}}{P} + g = \frac{2.30}{21.75} + .15 = .256, \text{ about 26%} $$

What’s wrong with the security analyst’s forecast? What is the actual expected rate of return over the next year?

21. Each of the following formulas for determining shareholders’ required rate of return can be right or wrong depending on the circumstances:

   a. $r = \frac{\text{DIV}_1}{P_0} + g$

   b. $r = \frac{\text{EPS}_1}{P_0}$

For each formula construct a simple numerical example showing that the formula can give wrong answers and explain why the error occurs. Then construct another simple numerical example for which the formula gives the right answer.

22. Alpha Corp’s earnings and dividends are growing at 15% per year. Beta Corp’s earnings and dividends are growing at 8% per year. The companies’ assets, earnings, and dividends per share are now (at date 0) exactly the same. Yet PVGO accounts for a greater fraction of Beta Corp’s stock price. How is this possible? (Hint: There is more than one possible explanation.)

23. Look again at the financial forecasts for Growth-Tech given in Table 4.5. This time assume you know that the opportunity cost of capital is $r = .12$ (discard the .099 figure calculated in the text). Assume you do not know Growth-Tech’s stock value. Otherwise follow the assumptions given in the text.

   a. Calculate the value of Growth-Tech stock.
   b. What part of that value reflects the discounted value of $P_3$, the price forecasted for year 3?
   c. What part of $P_3$ reflects the present value of growth opportunities (PVGO) after year 3?
   d. Suppose that competition will catch up with Growth-Tech by year 4, so that it can earn only its cost of capital on any investments made in year 4 or subsequently. What is Growth-Tech stock worth now under this assumption? (Make additional assumptions if necessary.)
24. Compost Science, Inc. (CSI), is in the business of converting Boston’s sewage sludge into fertilizer. The business is not in itself very profitable. However, to induce CSI to remain in business, the Metropolitan District Commission (MDC) has agreed to pay whatever amount is necessary to yield CSI a 10% book return on equity. At the end of the year CSI is expected to pay a $4 dividend. It has been reinvesting 40% of earnings and growing at 4% a year.

a. Suppose CSI continues on this growth trend. What is the expected long-run rate of return from purchasing the stock at $100? What part of the $100 price is attributable to the present value of growth opportunities?

b. Now the MDC announces a plan for CSI to treat Cambridge sewage. CSI’s plant will, therefore, be expanded gradually over five years. This means that CSI will have to reinvest 80% of its earnings for five years. Starting in year 6, however, it will again be able to pay out 60% of earnings. What will be CSI’s stock price once this announcement is made and its consequences for CSI are known?

25. Permian Partners (PP) produces from aging oil fields in west Texas. Production is 1.8 million barrels per year in 2009, but production is declining at 7% per year for the foreseeable future. Costs of production, transportation, and administration add up to $25 per barrel. The average oil price was $65 per barrel in 2009.

PP has 7 million shares outstanding. The cost of capital is 9%. All of PP’s net income is distributed as dividends. For simplicity, assume that the company will stay in business forever and that costs per barrel are constant at $25. Also, ignore taxes.

a. What is the PV of a PP share? Assume that oil prices are expected to fall to $60 per barrel in 2010, $55 per barrel in 2011, and $50 per barrel in 2012. After 2012, assume a long-term trend of oil-price increases at 5% per year.

b. What is PP’s EPS/P ratio and why is it not equal to the 9% cost of capital?

26. Construct a new version of Table 4.8, assuming that competition drives down profitability (on existing assets as well as new investment) to 11.5% in year 6, 11% in year 7, 10.5% in year 8, and 8% in year 9 and all later years. What is the value of the concatenator business?

27. The constant-growth DCF formula:

\[ P_0 = \frac{\text{DIV}_1}{r - g} \]

is sometimes written as:

\[ P_0 = \frac{\text{ROE}(1 - b) \text{BVPS}}{r - b \text{ROE}} \]

where BVPS is book equity value per share, \( b \) is the plowback ratio, and ROE is the ratio of earnings per share to BVPS. Use this equation to show how the price-to-book ratio varies as ROE changes. What is price-to-book when \( \text{ROE} = r \)?

28. Portfolio managers are frequently paid a proportion of the funds under management. Suppose you manage a $100 million equity portfolio offering a dividend yield (\( \text{DIV}/P_0 \)) of 5%. Dividends and portfolio value are expected to grow at a constant rate. Your annual fee for managing this portfolio is .5% of portfolio value and is calculated at the end of each year. Assuming that you will continue to manage the portfolio from now to eternity, what is the present value of the management contract? How would the contract value change if you invested in stocks with a 4% yield?

29. Suppose the concatenator division, which we valued based on Table 4.8, is spun off as an independent company, Concatco, with 1 million shares of common stock outstanding. What would each share sell for? Before answering, notice the negative free cash flows for
years 1 to 6. The PV of these cash flows is $3.6 million. Assume that this shortfall will have to be financed by additional shares issued in the near future. Also assume for simplicity that the $3.6 million earns interest at 10% and is sufficient to cover the negative free cash flows in Table 4.8. Concatco will pay no dividends in years 1 to 6, but will pay out all free cash flow starting in year 7.

Now calculate the value of each of the 1 million existing Concatco shares. Briefly explain your answer. 

Hints: Suppose the existing stockholders, who own 1 million shares, buy newly issued shares to cover the $3.6 million financing requirement. In other words, the $3.6 million comes directly out of existing stockholders’ wallets. What’s the value per share? Now suppose instead that the $3.6 million comes from new investors, who buy shares at a fair price. Does your answer change?

The major stock exchanges have wonderful Web sites. Look at both the NYSE site (www.nyse.com) and the Nasdaq site (www.nasdaq.com). You will find plenty of material on their trading systems, and you can also access quotes and other data.

1. Go to www.nyse.com. Find NYSE MarkeTrac and click on the DJIA ticker tape, which shows trades for the stocks in the Dow Jones Industrial Averages. Stop the tape at GE. What are the latest price, dividend yield, and P/E ratio?

Use data from the Standard & Poor’s Market Insight Database at www.mhhe.com/edumarketinsight or from finance.yahoo.com to answer the following questions.

2. Look up General Mills, Inc., and Kellogg Co. The companies’ ticker symbols are GIS and K.  
   a. What are the current dividend yield and price–earnings ratio (P/E) for each company? How do the yields and P/Es compare with the average for the food industry and for the stock market as a whole? (The stock market is represented by the S & P 500 index.)
   b. What are the growth rates of earnings per share (EPS) and dividends for each company over the last five years? Do these growth rates appear to reflect a steady trend that could be projected for the long-run future?
   c. Would you be confident in applying the constant-growth DCF valuation model to these companies’ stocks? Why or why not?

3. Look up Intel (INTC), Dell Computer (DELL), Dow Chemical (DOW), Harley-Davidson (HOG), and Pfizer, Inc. (PFE). Look at “Financial Highlights” and “Company Profile” for each company. You will note wide differences in these companies’ price–earnings ratios. What are the possible explanations for these differences? Which would you classify as growth (high-PVGO) stocks and which as income stocks?

MINI-CASE

Reeby Sports

Ten years ago, in 2001, George Reeby founded a small mail-order company selling high-quality sports equipment. Since those early days Reeby Sports has grown steadily and been consistently profitable. The company has issued 2 million shares, all of which are owned by George Reeby and his five children.

For some months George has been wondering whether the time has come to take the company public. This would allow him to cash in on part of his investment and would make it easier for the firm to raise capital should it wish to expand in the future.
But how much are the shares worth? George’s first instinct is to look at the firm’s balance sheet, which shows that the book value of the equity is $26.34 million, or $13.17 per share. A share price of $13.17 would put the stock on a P/E ratio of 6.6. That is quite a bit lower than the 13.1 P/E ratio of Reeby’s larger rival, Molly Sports.

George suspects that book value is not necessarily a good guide to a share’s market value. He thinks of his daughter Jenny, who works in an investment bank. She would undoubtedly know what the shares are worth. He decides to phone her after she finishes work that evening at 9 o’clock or before she starts the next day at 6.00 a.m.

Before phoning, George jots down some basic data on the company’s profitability. After recovering from its early losses, the company has earned a return that is higher than its estimated 10% cost of capital. George is fairly confident that the company could continue to grow fairly steadily for the next six to eight years. In fact he feels that the company’s growth has been somewhat held back in the last few years by the demands from two of the children for the company to make large dividend payments. Perhaps, if the company went public, it could hold back on dividends and plow more money back into the business.

There are some clouds on the horizon. Competition is increasing and only that morning Molly Sports announced plans to form a mail-order division. George is worried that beyond the next six or so years it might become difficult to find worthwhile investment opportunities.

George realizes that Jenny will need to know much more about the prospects for the business before she can put a final figure on the value of Reeby Sports, but he hopes that the information is sufficient for her to give a preliminary indication of the value of the shares.

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
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<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011E</th>
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<tr>
<td>Earnings per share, $</td>
<td>-2.10</td>
<td>-0.70</td>
<td>0.23</td>
<td>0.81</td>
<td>1.10</td>
<td>1.30</td>
<td>1.52</td>
<td>1.64</td>
<td>2.00</td>
<td>2.03</td>
</tr>
<tr>
<td>Dividend, $</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.20</td>
<td>0.20</td>
<td>0.30</td>
<td>0.30</td>
<td>0.60</td>
<td>0.60</td>
<td>0.80</td>
</tr>
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<td>Book value per share, $</td>
<td>9.80</td>
<td>7.70</td>
<td>7.00</td>
<td>7.61</td>
<td>8.51</td>
<td>9.51</td>
<td>10.73</td>
<td>11.77</td>
<td>13.17</td>
<td>14.40</td>
</tr>
<tr>
<td>ROE, %</td>
<td>-27.10</td>
<td>-7.1</td>
<td>3.0</td>
<td>11.6</td>
<td>14.5</td>
<td>15.3</td>
<td>16.0</td>
<td>15.3</td>
<td>17.0</td>
<td>15.4</td>
</tr>
</tbody>
</table>

QUESTIONS

1. Help Jenny to forecast dividend payments for Reeby Sports and to estimate the value of the stock. You do not need to provide a single figure. For example, you may wish to calculate two figures, one on the assumption that the opportunity for further profitable investment is reduced in year 6 and another on the assumption that it is reduced in year 8.

2. How much of your estimate of the value of Reeby’s stock comes from the present value of growth opportunities?