Financing and Valuation

In Chapters 5 and 6 we showed how to value a capital investment project by a four-step procedure:

1. Forecast after-tax cash flows, assuming all-equity financing.
2. Assess the project’s risk.
3. Estimate the opportunity cost of capital.
4. Calculate NPV, using the opportunity cost of capital as the discount rate.

There’s nothing wrong with this procedure, but now we’re going to extend it to include value contributed by financing decisions. There are two ways to do this:

1. Adjust the discount rate. The adjustment is typically downward, to account for the value of interest tax shields. This is the most common approach, which is usually implemented via the after-tax weighted-average cost of capital (WACC). We introduced the after-tax WACC in Chapters 9 and 17, but here we provide a lot more guidance on how it is calculated and used.

2. Adjust the present value. That is, start by estimating the firm or project’s base-case value, assuming it is all-equity-financed, and then adjust this base-case value to account for financing.

The adjusted present value (APV)
\[
\text{APV} = \text{base-case value} + \text{value of financing side effects}
\]

Once you identify and value the financing side effects, calculating APV is no more than addition or subtraction.

This is a how-to-do-it chapter. In the first section, we explain and derive the after-tax WACC and use it to value a project and business. Then in Section 19-2 we work through a more complex and realistic valuation problem. Section 19-3 covers some tricks of the trade: helpful hints on how to estimate inputs and on how to adjust WACC when business risk or capital structure changes. Section 19-4 turns to the APV method. The idea behind APV is simple enough, but tracing through all the financing side effects can be tricky. We conclude the chapter with a question-and-answer section designed to clarify points that managers and students often find confusing. The Appendix covers an important special case, namely, the after-tax valuation of safe cash flows.

19-1 The After-Tax Weighted-Average Cost of Capital

We first addressed problems of valuation and capital budgeting in Chapters 2 to 6. In those early chapters we said hardly a word about financing decisions. In fact we proceeded under the simplest possible financing assumption, namely, all-equity financing. We were really assuming a Modigliani–Miller (MM) world in which all financing decisions are irrelevant. In a strict MM world, firms can analyze real investments as if they are all-equity-financed; the actual financing plan is a mere detail to be worked out later.

Under MM assumptions, decisions to spend money can be separated from decisions to raise money. Now we reconsider the capital budgeting decision when investment and financing decisions interact and cannot be wholly separated.
One reason that financing and investment decisions interact is taxes. Interest is a tax-deductible expense. Think back to Chapters 9 and 17 where we introduced the after-tax weighted-average cost of capital:

$$\text{WACC} = r_d(1 - T_c) \frac{D}{V} + r_e \frac{E}{V}$$

Here $D$ and $E$ are the market values of the firm’s debt and equity, $V = D + E$ is the total market value of the firm, $r_d$ and $r_e$ are the costs of debt and equity, and $T_c$ is the marginal corporate tax rate.

Notice that the WACC formula uses the after-tax cost of debt $r_d(1 - T_c)$. That is how the after-tax WACC captures the value of interest tax shields. Notice too that all the variables in the WACC formula refer to the firm as a whole. As a result, the formula gives the right discount rate only for projects that are just like the firm undertaking them. The formula works for the “average” project. It is incorrect for projects that are safer or riskier than the average of the firm’s existing assets. It is incorrect for projects whose acceptance would lead to an increase or decrease in the firm’s target debt ratio.

The WACC is based on the firm’s current characteristics, but managers use it to discount future cash flows. That’s fine as long as the firm’s business risk and debt ratio are expected to remain constant, but when the business risk and debt ratio are expected to change, discounting cash flows by the WACC is only approximately correct.

**EXAMPLE 19.1 Calculating Sangria’s WACC**

Sangria is a U.S.-based company whose products aim to promote happy, low-stress lifestyles. Let’s calculate Sangria’s WACC. Its book and market-value balance sheets are:

<table>
<thead>
<tr>
<th>Sangria Corporation (Book Values, $ millions)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset value $1,000</td>
<td>$500</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Sangria Corporation (Market Values, $ millions)</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>--</td>
</tr>
<tr>
<td>Asset value $1,250</td>
<td>$500</td>
</tr>
</tbody>
</table>

We calculated the market value of equity on Sangria’s balance sheet by multiplying its current stock price ($7.50) by 100 million, the number of its outstanding shares. The company’s future prospects are good, so the stock is trading above book value ($7.50 vs. $5.00 per share). However, interest rates have been stable since the firm’s debt was issued and the book and market values of debt are in this case equal.

Sangria’s cost of debt (the market interest rate on its existing debt and on any new borrowing) is 6%. Its cost of equity (the expected rate of return demanded by investors in Sangria’s stock) is 12.4%.

---

1 Always use an up-to-date interest rate (yield to maturity), not the interest rate when the firm’s debt was first issued and not the coupon rate on the debt’s book value.
The market-value balance sheet shows assets worth $1,250 million. Of course we can’t observe this value directly, because the assets themselves are not traded. But we know what they are worth to debt and equity investors ($500 + 750 = $1,250 million). This value is entered on the left of the market-value balance sheet.

Why did we show the book balance sheet? Only so you could draw a big X through it. Do so now.

When estimating the weighted-average cost of capital, you are not interested in past investments but in current values and expectations for the future. Sangria’s true debt ratio is not 50%, the book ratio, but 40%, because its assets are worth $1,250 million. The cost of equity, \( r_E \), is the expected rate of return from purchase of stock at $7.50 per share, the current market price. It is not the return on book value per share. You can’t buy shares in Sangria for $5 anymore.

Sangria is consistently profitable and pays taxes at the marginal rate of 35%. This tax rate is the final input for Sangria’s WACC. The inputs are summarized here:

<table>
<thead>
<tr>
<th>Cost of debt ( r_D )</th>
<th>.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of equity ( r_E )</td>
<td>.124</td>
</tr>
<tr>
<td>Marginal tax rate ( T_c )</td>
<td>.35</td>
</tr>
<tr>
<td>Debt ratio ( D/V )</td>
<td>500/1,250 = .4</td>
</tr>
<tr>
<td>Equity ratio ( E/V )</td>
<td>750/1,250 = .6</td>
</tr>
</tbody>
</table>

The company’s after-tax WACC is:

\[
WACC = .06 \times (1 - .35) \times .4 + .124 \times .6 = .090, \text{ or } 9.0\%
\]

That’s how you calculate the weighted-average cost of capital. Now let’s see how Sangria would use it.

**EXAMPLE 19.2** Using Sangria’s WACC to value a project

Sangria’s enologists have proposed investing $12.5 million in the construction of a perpetual crushing machine, which (conveniently for us) never depreciates and generates a perpetual stream of earnings and cash flow of $1.731 million per year pretax. The project is average risk, so we can use WACC. The after-tax cash flow is:

<table>
<thead>
<tr>
<th>Pretax cash flow</th>
<th>$1.731 million</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax at 35%</td>
<td>.606</td>
</tr>
<tr>
<td>After-tax cash flow</td>
<td>( C = $1.125 \text{ million} )</td>
</tr>
</tbody>
</table>

Notice: This after-tax cash flow takes no account of interest tax shields on debt supported by the perpetual crusher project. As we explained in Chapter 6, standard capital budgeting practice calculates after-tax cash flows as if the project were all-equity-financed. However, the interest tax shields will not be ignored: We are about to discount the project’s cash flows by Sangria’s WACC, in which the cost of debt is entered after tax. The value of interest tax shields is picked up not as higher after-tax cash flows, but in a lower discount rate.

The crusher generates a perpetual after-tax cash flow of \( C = $1.125 \text{ million} \), so NPV is:

\[
NPV = -12.5 + \frac{1.125}{0.09} = 0
\]
NPV = 0 means a barely acceptable investment. The annual cash flow of $1.125 million per year amounts to a 9% rate of return on investment (1.125/12.5 = .09), exactly equal to Sangria’s WACC.

If project NPV is exactly zero, the return to equity investors must exactly equal the cost of equity, 12.4%. Let’s confirm that Sangria shareholders can actually look forward to a 12.4% return on their investment in the perpetual crusher project.

Suppose Sangria sets up this project as a mini-firm. Its market-value balance sheet looks like this:

<table>
<thead>
<tr>
<th>Perpetual Crusher (Market Values, $ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset value</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Equity</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Calculate the expected dollar return to shareholders:

After-tax interest = \( r_D (1 - T_c)D = .06 \times (1 - .35) \times 5 = .195 \)

Expected equity income = \( C - r_D (1 - T_c)D = 1.125 - .195 = .93 \)

The project’s earnings are level and perpetual, so the expected rate of return on equity is equal to the expected equity income divided by the equity value:

\[
\text{Expected equity return} = r_E = \frac{\text{expected equity income}}{\text{equity value}} = \frac{.93}{7.5} = .124, \text{ or } 12.4\%
\]

The expected return on equity equals the cost of equity, so it makes sense that the project’s NPV is zero.

**Review of Assumptions**

When discounting the perpetual crusher’s cash flows at Sangria’s WACC, we assume that:

- The project’s business risks are the same as those of Sangria’s other assets and remain so for the life of the project.
- The project supports the same fraction of debt to value as in Sangria’s overall capital structure, which remains constant for the life of the project.

You can see the importance of these two assumptions: If the perpetual crusher had greater business risk than Sangria’s other assets, or if the acceptance of the project would lead to a permanent, material change in Sangria’s debt ratio, then Sangria’s shareholders would not be content with a 12.4% expected return on their equity investment in the project.

We have illustrated the WACC formula only for a project offering perpetual cash flows. But the formula works for any cash-flow pattern if the firm adjusts its borrowing

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2 Users of WACC need not worry about small or temporary fluctuations in debt-to-value ratios. Suppose that Sangria management decides for convenience to borrow $12.5 million to allow immediate construction of the crusher. This does not necessarily change Sangria’s long-term financing policy. If the crusher supports only $5.0 million of debt, Sangria would have to pay down debt to restore its overall debt ratio to 40%. For example, it could fund later projects with less debt and more equity.
to maintain a constant debt ratio over time.\(^3\) When the firm departs from this borrowing policy, WACC is only approximately correct.

### 19.2 Valuing Businesses

On most workdays the financial manager concentrates on valuing projects, arranging financing, and helping run the firm more effectively. The valuation of the business as a whole is left to investors and financial markets. But on some days the financial manager has to take a stand on what an entire business is worth. When this happens, a big decision is typically in the offing. For example:

- If firm A is about to make a takeover offer for firm B, then A’s financial managers have to decide how much the combined business A + B is worth under A’s management. This task is particularly difficult if B is a private company with no observable share price.
- If firm C is considering the sale of one of its divisions, it has to decide what the division is worth in order to negotiate with potential buyers.
- When a firm goes public, the investment bank must evaluate how much the firm is worth in order to set the issue price.

In addition, thousands of analysts in stockbrokers’ offices and investment firms spend every workday burrowing away in the hope of finding undervalued firms. Many of these analysts use the valuation tools we are about to cover.

In Chapter 4 we took a first pass at valuing an entire business. We assumed then that the business was financed solely by equity. Now we will show how WACC can be used to value a company that is financed by a mixture of debt and equity as long as the debt ratio is expected to remain approximately constant. You just treat the company as if it were one big project. You forecast the company’s cash flows (the hardest part of the

\(^3\) We can prove this statement as follows. Denote expected after-tax cash flows (assuming all-equity financing) as \(C_1, C_2, \ldots, C_T\). With all-equity financing, these flows would be discounted at the opportunity cost of capital \(r\). But we need to value the cash flows for a firm that is financed partly with debt.

Start with value in the next to last period: \(V_{T-1} = D_{T-1} + E_{T-1}\). The total cash payoff to debt and equity investors is the cash flow plus the interest tax shield. The expected total return to debt and equity investors is:

\[
\text{Expected cash payoff in } T = C_T + T_T D_{T-1}
\]

\[
= V_{T-1} \left(1 + \frac{D_{T-1}}{V_{T-1}} + \frac{E_{T-1}}{V_{T-1}}\right)
\]

Assume the debt ratio is constant at \(L = D/V\). Equate (1) and (2) and solve for \(V_{T-1}\):

\[
V_{T-1} = \frac{C_T}{1 + (1 - T_T) D + T_T (1 - L)} = \frac{C_T}{1 + \text{WACC}}
\]

The logic repeats for \(V_{T-2}\). Note that the next period’s payoff includes \(V_{T-1}\):

\[
V_{T-2} = \frac{C_T + V_{T-1}}{1 + (1 - T_T) D + T_T (1 - L)} = \frac{C_T + V_{T-1}}{1 + \text{WACC}} = \frac{C_T}{1 + \text{WACC}} = \frac{C_T}{(1 + \text{WACC})^2}
\]

We can continue all the way back to date 0:

\[
V_0 = \sum_{t=1}^{T} \frac{C_t}{(1 + \text{WACC})^t}
\]
exercise) and discount back to present value. But be sure to remember three important points:

1. If you discount at WACC, cash flows have to be projected just as you would for a capital investment project. Do not deduct interest. Calculate taxes as if the company were all-equity-financed. (The value of interest tax shields is not ignored, because the after-tax cost of debt is used in the WACC formula.)

2. Unlike most projects, companies are potentially immortal. But that does not mean that you need to forecast every year’s cash flow from now to eternity. Financial managers usually forecast to a medium-term horizon and add a terminal value to the cash flows in the horizon year. The terminal value is the present value at the horizon of all subsequent cash flows. Estimating the terminal value requires careful attention because it often accounts for the majority of the company’s value.

3. Discounting at WACC values the assets and operations of the company. If the object is to value the company’s equity, that is, its common stock, don’t forget to subtract the value of the company’s outstanding debt.

Here’s an example.

**Valuing Rio Corporation**

Sangria is tempted to acquire the Rio Corporation, which is also in the business of promoting relaxed, happy lifestyles. Rio has developed a special weight-loss program called the Brazil Diet, based on barbecues, red wine, and sunshine. The firm guarantees that within three months you will have a figure that will allow you to fit right in at Ipanema or Copacabana beach in Rio de Janeiro. But before you head for the beach, you’ve got the job of working out how much Sangria should pay for Rio.

Rio is a U.S. company. It is privately held, so Sangria has no stock-market price to rely on. Rio has 1.5 million shares outstanding and debt with a market and book value of $36 million. Rio is in the same line of business as Sangria, so we will assume that it has the same business risk as Sangria and can support the same proportion of debt. Therefore we can use Sangria’s WACC.

Your first task is to forecast Rio’s free cash flow (FCF). Free cash flow is the amount of cash that the firm can pay out to investors after making all investments necessary for growth. Free cash flow is calculated assuming the firm is all-equity-financed. Discounting the free cash flows at the after-tax WACC gives the total value of Rio (debt plus equity). To find the value of its equity, you will need to subtract the $36 million of debt.

We will forecast each year’s free cash flow out to a valuation horizon ($H$) and predict the business’s value at that horizon ($PV_H$). The cash flows and horizon value are then discounted back to the present:

$$PV = \frac{FCF_1}{1 + WACC} + \frac{FCF_2}{(1 + WACC)^2} + \ldots + \frac{FCF_H}{(1 + WACC)^H} + \frac{PV_H}{(1 + WACC)^H}$$

Of course, the business will continue after the horizon, but it’s not practical to forecast free cash flow year by year to infinity. $PV_H$ stands in for the value in year $H$ of free cash flow in periods $H + 1$, $H + 2$, etc.

Free cash flow and net income are not the same. They differ in several important ways:

- Income is the return to shareholders, calculated after interest expense. Free cash flow is calculated before interest.
- Income is calculated after various noncash expenses, including depreciation. Therefore we will add back depreciation when we calculate free cash flow.
- Capital expenditures and investments in working capital do not appear as expenses on the income statement, but they do reduce free cash flow.
Free cash flow can be negative for rapidly growing firms, even if the firms are profitable, because investment exceeds cash flow from operations. Negative free cash flow is normally temporary, fortunately for the firm and its stockholders. Free cash flow turns positive as growth slows down and the payoffs from prior investments start to roll in.

Table 19.1 sets out the information that you need to forecast Rio’s free cash flows. We will follow common practice and start with a projection of sales. In the year just ended Rio had sales of $83.6 million. In recent years sales have grown by between 5% and 8% a year. You forecast that sales will grow by about 7% a year for the next three years. Growth will then slow to 4% for years 4 to 6 and to 3% starting in year 7.

The other components of cash flow in Table 19.1 are driven by these sales forecasts. For example, you can see that costs are forecasted at 74% of sales in the first year with a gradual increase to 76% of sales in later years, reflecting increased marketing costs as Rio’s competitors gradually catch up.

Increasing sales are likely to require further investment in fixed assets and working capital. Rio’s net fixed assets are currently about $0.79 for each dollar of sales. Unless Rio has surplus capacity or can squeeze more output from its existing plant and equipment, its investment in fixed assets will need to grow along with sales. Therefore we assume that

<table>
<thead>
<tr>
<th>Assumptions:</th>
<th>Sales growth, %</th>
<th>6.7</th>
<th>7.0</th>
<th>7.0</th>
<th>7.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs (percent of sales)</td>
<td>75.5</td>
<td>74.0</td>
<td>74.5</td>
<td>74.5</td>
<td>75.0</td>
<td>75.0</td>
<td>75.0</td>
<td>75.5</td>
<td>76.0</td>
</tr>
<tr>
<td>Working capital (percent of sales)</td>
<td>13.3</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
</tr>
<tr>
<td>Net fixed assets (percent of sales)</td>
<td>79.2</td>
<td>79.0</td>
<td>79.0</td>
<td>79.0</td>
<td>79.0</td>
<td>79.0</td>
<td>79.0</td>
<td>79.0</td>
<td>79.0</td>
</tr>
<tr>
<td>Depreciation (percent of net fixed assets)</td>
<td>5.0</td>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
<td>14.0</td>
</tr>
</tbody>
</table>

| Tax rate, % | 35.0 |
| WACC, % | 9.0 |
| Long-term growth forecast, % | 3.0 |

| Fixed assets and working capital |
| Gross fixed assets | 95.0 | 109.6 | 125.1 | 141.8 | 156.8 | 172.4 | 188.6 | 204.5 |
| Less accumulated depreciation | 29.0 | 38.9 | 49.5 | 60.8 | 72.6 | 84.9 | 97.6 | 110.7 |
| Net fixed assets | 66.0 | 70.7 | 75.6 | 80.9 | 84.2 | 87.5 | 91.0 | 93.8 |
| Net working capital | 11.1 | 11.6 | 12.4 | 13.3 | 13.9 | 14.4 | 15.0 | 15.4 |
every dollar of sales growth requires an increase of $.79 in net fixed assets. We also assume that working capital grows in proportion to sales.

Rio’s free cash flow is calculated in Table 19.1 as profit after tax, plus depreciation, minus investment. Investment is the change in the stock of (gross) fixed assets and working capital from the previous year. For example, in year 1:

\[
\text{Free cash flow} = \text{Profit after tax} + \text{depreciation} - \text{investment in fixed assets} - \text{investment in working capital}
\]

\[
= 8.7 + 9.9 - (109.6 - 95.0) - (11.6 - 11.1) = \$3.5 \text{ million}
\]

**Estimating Horizon Value**

We will forecast cash flows for each of the first six years. After that, Rio’s sales are expected to settle down to stable, long-term growth starting in year 7. To find the present value of the cash flows in years 1 to 6, we discount at the 9% WACC:

\[
PV = \frac{3.5}{1.09} + \frac{3.2}{1.09^2} + \frac{3.4}{1.09^3} + \frac{5.9}{1.09^4} + \frac{6.1}{1.09^5} + \frac{6.0}{1.09^6} = \$20.3 \text{ million}
\]

Now we need to find the value of the cash flows from year 7 onward. In Chapter 4 we looked at several ways to estimate horizon value. Here we will use the constant-growth DCF formula. This requires a forecast of the free cash flow for year 7, which we have worked out in the final column of Table 19.1, assuming a long-run growth rate of 3% per year. The free cash flow is $6.8 million, so

\[
PV_H = \frac{\text{FCF}_{H+1}}{\text{WACC} - g} = \frac{6.8}{.09 - .03} = \$113.4 \text{ million}
\]

\[
\text{PV at year 0} = \frac{1}{1.09^6} \times 113.4 = \$67.6 \text{ million}
\]

We now have all we need to value the business:

\[
\text{PV (company)} = \text{PV (cash flow years 1–6)} + \text{PV (horizon value)}
\]

\[
= 20.3 + 67.6 = \$87.9 \text{ million}
\]

This is the total value of Rio. To find the value of the equity, we simply subtract the value of the debt:

\[
\text{Total value of equity} = 87.9 - 36.0 = \$51.9 \text{ million}
\]

And to find the value per share, we divide by the total number of shares outstanding:

\[
\text{Value per share} = \frac{51.9}{1.5} = \$34.60
\]

Thus Sangria could afford to pay up to $34.60 per share for Rio.

You now have an estimate of the value of Rio Corporation. But how confident can you be in this figure? Notice that less than a quarter of Rio’s value comes from cash flows in the first six years. The rest comes from the horizon value. Moreover, this horizon value can change in response to only minor changes in assumptions. For example, if the long-run growth rate is 4% rather than 3%, Rio needs to invest more to support this higher growth, but firm value increases from $87.9 million to $89.9 million.

In Chapter 4 we stressed that wise managers won’t stop at this point. They will check their calculations by identifying comparable companies and comparing their price–earnings multiples and ratios of market to book value.\(^5\)

\(^4\) Notice that expected free cash flow increases by about 14% from year 6 to year 7 because the transition from 4% to 3% sales growth reduces required investment. But sales, investment, and free cash flow will all increase at 3% once the company settles into stable growth. Recall that the first cash flow in the constant-growth DCF formula occurs in the next year, year 7 in this case. Growth progresses at a steady-state 3% from year 7 onward. Therefore it’s OK to use the 3% growth rate in the horizon-value formula.

\(^5\) See Section 4-5.
When you forecast cash flows, it is easy to become mesmerized by the numbers and just do it mechanically. As we pointed out in Chapter 11, it is important to take a strategic view. Are the revenue figures consistent with what you expect your competitors to do? Are the costs you have predicted realistic? Probe the assumptions behind the numbers to make sure they are sensible. Be particularly careful about the growth rates and profitability assumptions that drive horizon values. Don’t assume that the business you are valuing will grow and earn more than the cost of capital in perpetuity. This would be a nice outcome for the business, but not an outcome that competition will tolerate.

You should also check whether the business is worth more dead than alive. Sometimes a company’s liquidation value exceeds its value as a going concern. Smart financial analysts sometimes ferret out idle or underexploited assets that would be worth much more if sold to someone else. You may end up counting these assets at their likely sale price and valuing the rest of the business without them.

**WACC vs. the Flow-to-Equity Method**

When valuing Rio we forecast the cash flows assuming all-equity financing and we used the WACC to discount these cash flows. The WACC formula picked up the value of the interest tax shields. Then to find equity value, we subtracted the value of debt from the total value of the firm.

If our task is to value a firm’s equity, there’s an obvious alternative to discounting company cash flows at the firm’s WACC: Discount cash flows to equity, after interest and after taxes, at the cost of equity capital. This is called the flow-to-equity method. If the company’s debt ratio is constant over time, the flow-to-equity method should give the same answer as discounting cash flows at the WACC and then subtracting debt.

The flow-to-equity method seems simple, and it is simple if the proportions of debt and equity financing stay reasonably close to constant for the life of the company. But the cost of equity depends on financial leverage; in other words, it depends on financial risk as well as business risk. If financial leverage is expected to change significantly, discounting flows to equity at today’s cost of equity will not give the right answer.

**Some Tricks of the Trade**

Sangria had just one asset and two sources of financing. A real company’s market-value balance sheet has many more entries, for example:

4 Table 19.1 is too optimistic in this respect, because the horizon value increases with the assumed long-run growth rate. This implies that Rio has valuable growth opportunities (PVGO) even after the horizon in year 6. A more sophisticated spreadsheet would add an intermediate growth stage, say from years 7 through 10, and gradually reduce profitability to competitive levels. See Problem 26 at the end of this chapter.

7 This balance sheet is for exposition and should not be confused with a real company’s books. It includes the value of growth opportunities, which accountants do not recognize, though investors do. It excludes certain accounting entries, for example, deferred taxes.

Deferred taxes arise when a company uses faster depreciation for tax purposes than it uses in reports to investors. That means the company reports more in taxes than it pays. The difference is accumulated as a liability for deferred taxes. In a sense there is a liability, because the Internal Revenue Service “catches up,” collecting extra taxes, as assets age. But this is irrelevant in capital investment analysis, which focuses on actual after-tax cash flows and uses accelerated tax depreciation. Deferred taxes should not be regarded as a source of financing or an element of the weighted-average cost of capital formula. The liability for deferred taxes is not a security held by investors. It is a balance sheet entry created for accounting purposes.

Deferred taxes can be important in regulated industries, however. Regulators take deferred taxes into account in calculating allowed rates of return and the time patterns of revenues and consumer prices.
Several questions immediately arise:

**How does the formula change when there are more than two sources of financing?** Easy: There is one cost for each element. The weight for each element is proportional to its market value. For example, if the capital structure includes both preferred and common shares,

\[
WACC = r_p (1 - T_c) \frac{D}{V} + r_p \frac{P}{V} + r_E \frac{E}{V}
\]

where \( r_p \) is investors’ expected rate of return on the preferred stock, \( P \) is the amount of preferred stock outstanding, and \( V = D + P + E \).

**What about short-term debt?** Many companies consider only long-term financing when calculating WACC. They leave out the cost of short-term debt. In principle this is incorrect. The lenders who hold short-term debt are investors who can claim their share of operating earnings. A company that ignores this claim will misstate the required return on capital investments.

But “zeroing out” short-term debt is not a serious error if the debt is only temporary, seasonal, or incidental financing or if it is offset by holdings of cash and marketable securities. Suppose, for example, that one of your foreign subsidiaries takes out a six-month loan to finance its inventory and accounts receivable. The dollar equivalent of this loan will show up as a short-term debt. At the same time headquarters may be lending money by investing surplus dollars in short-term securities. If this lending and borrowing offset, there is no point in including the cost of short-term debt in the weighted-average cost of capital, because the company is not a net short-term borrower.

**What about other current liabilities?** Current liabilities are usually “netted out” by subtracting them from current assets. The difference is entered as net working capital on the left-hand side of the balance sheet. The sum of long-term financing on the right is called total capitalization.

When net working capital is treated as an asset, forecasts of cash flows for capital investment projects must treat increases in net working capital as a cash outflow and decreases as an inflow. This is standard practice, which we followed in Section 6-2. We also did so when we estimated the future investments that Rio would need to make in working capital.

Since current liabilities include short-term debt, netting them out against current assets excludes the cost of short-term debt from the weighted-average cost of capital. We have just explained why this can be an acceptable approximation. But when short-term debt is an important, permanent source of financing—as is common for small firms and firms outside the United States—it should be shown explicitly on the right-hand side of the balance sheet,
not netted out against current assets. The interest cost of short-term debt is then one element of the weighted-average cost of capital.

**How are the costs of financing calculated?** You can often use stock market data to get an estimate of $r_E$, the expected rate of return demanded by investors in the company’s stock. With that estimate, WACC is not too hard to calculate, because the borrowing rate $r_D$ and the debt and equity ratios $D/V$ and $E/V$ can be directly observed or estimated without too much trouble. Estimating the value and required return for preferred shares is likewise usually not too complicated.

Estimating the required return on other security types can be troublesome. Convertible debt, where the investors’ return comes partly from an option to exchange the debt for the company’s stock, is one example. We leave convertibles to Chapter 24.

Junk debt, where the risk of default is high, is likewise difficult. The higher the odds of default, the lower the market price of the debt, and the higher is the promised rate of interest. But the weighted-average cost of capital is an expected, that is average, rate of return, not a promised one. For example, in July 2009, MGM Mirage bonds maturing in 2015 sold at only 66% of face value and offered a 15% promised yield, nearly 13 percentage points above yields on the highest-quality debt issues maturing at the same time. The price and yield on the MGM bond demonstrated investors’ concern about the company’s chronic financial ill-health. But the 15% yield was not an expected return, because it did not average in the losses to be incurred if MGM were to default. Including 15% as a “cost of debt” in a calculation of WACC would therefore have overstated MGM’s true cost of capital.

This is bad news: There is no easy or tractable way of estimating the expected rate of return on most junk debt issues. The good news is that for most debt the odds of default are small. That means the promised and expected rates of return are close, and the promised rate can be used as an approximation in the weighted-average cost of capital.

**Company vs. Industry WACCs** Of course you want to know what your company’s WACC is. Yet industry WACCs are sometimes more useful. Here’s an example. Kansas City Southern used to be a portfolio of (1) the Kansas City Southern Railroad, with operations running from the U.S. Midwest south to Texas and Mexico, and (2) Stillwell Financial, an investment-management business that included the Janus mutual funds. It’s hard to think of two more dissimilar businesses. Kansas City Southern’s overall WACC was not right for either of them. The company would have been well advised to use a railroad industry WACC for its railroad operations and an investment management WACC for Stillwell.

KCS spun off Stillwell in 2000 and is now a pure-play railroad. But even now the company would be wise to check its WACC against a railroad industry WACC. Industry WACCs are less exposed to random noise and estimation errors. Fortunately for Kansas City Southern, there are several large, pure-play U.S. railroads from which a railroad industry WACC can be estimated. Of course, use of an industry WACC for a particular company’s investments assumes that the company and industry have approximately the same business risk and financing.

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8 Financial practitioners have rules of thumb for deciding whether short-term debt is worth including in WACC. One rule checks whether short-term debt is at least 10% of total liabilities and net working capital is negative. If so, then short-term debt is almost surely being used to finance long-term assets and is explicitly included in WACC.

9 Most corporate debt is not actively traded, so its market value cannot be observed directly. But you can usually value a nontraded debt security by looking to securities that are traded and that have approximately the same default risk and maturity. See Chapter 23.

For healthy firms the market value of debt is usually not too far from book value, so many managers and analysts use book value for $D$ in the weighted-average cost of capital formula. However, be sure to use market, not book, values for $E$.

10 When betas can be estimated for the junk issue or for a sample of similar issues, the expected return can be calculated from the capital asset pricing model. Otherwise the yield should be adjusted for the probability of default. Evidence on historical default rates on junk bonds is described in Chapter 23.

11 See Tables 4.4 and 9.1.
Mistakes People Make in Using the Weighted-Average Formula

The weighted-average formula is very useful but also dangerous. It tempts people to make logical errors. For example, manager Q, who is campaigning for a pet project, might look at the formula

$$\text{WACC} = r_D(1 - T_c)\frac{D}{V} + r_E\frac{E}{V}$$

and think, “Aha! My firm has a good credit rating. It could borrow, say, 90% of the project’s cost if it likes. That means $D/V = .9$ and $E/V = .1$. My firm’s borrowing rate $r_D$ is 8%, and the required return on equity, $r_E$, is 15%. Therefore

$$\text{WACC} = .08(1 - .35)(.9) + .15(.1) = .062$$

or 6.2%. When I discount at that rate, my project looks great.”

Manager Q is wrong on several counts. First, the weighted-average formula works only for projects that are carbon copies of the firm. The firm isn’t 90% debt-financed.

Second, the immediate source of funds for a project has no necessary connection with the hurdle rate for the project. What matters is the project’s overall contribution to the firm’s borrowing power. A dollar invested in Q’s pet project will not increase the firm’s debt capacity by $.90. If the firm borrows 90% of the project’s cost, it is really borrowing in part against its existing assets. Any advantage from financing the new project with more debt than normal should be attributed to the old projects, not to the new one.

Third, even if the firm were willing and able to lever up to 90% debt, its cost of capital would not decline to 6.2% (as Q’s naive calculation predicts). You cannot increase the debt ratio without creating financial risk for stockholders and thereby increasing $r_E$, the expected rate of return they demand from the firm’s common stock. Going to 90% debt would certainly increase the borrowing rate, too.

Adjusting WACC when Debt Ratios and Business Risks Differ

The WACC formula assumes that the project or business to be valued will be financed in the same debt–equity proportions as the company (or industry) as a whole. What if that is not true? For example, what if Sangria’s perpetual crusher project supports only 20% debt, versus 40% for Sangria overall?

Moving from 40% to 20% debt may change all the inputs to the WACC formula.\(^\text{12}\) Obviously the financing weights change. But the cost of equity $r_E$ is less, because financial risk is reduced. The cost of debt may be lower too.

Take another look at Figure 17.4 on page 433, which plots WACC and the costs of debt and equity as a function of the debt–equity ratio. The flat line is $r$, the opportunity cost of capital. Remember, this is the expected rate of return that investors would want from the project if it were all-equity-financed. The opportunity cost of capital depends only on business risk and is the natural reference point.

Suppose Sangria or the perpetual crusher project were all-equity-financed ($D/V = 0$). At that point WACC equals cost of equity, and both equal the opportunity cost of capital. Start from that point in Figure 19.1. As the debt ratio increases, the cost of equity increases, because of financial risk, but notice that WACC declines. The decline is not caused by use of “cheap” debt in place of “expensive” equity. It falls because of the tax shields on debt interest payments. If there were no corporate income taxes, the weighted-average cost of

\(^\text{12}\) Even the tax rate could change. For example, Sangria might have enough taxable income to cover interest payments at 20% debt but not at 40% debt. In that case the effective marginal tax rate would be higher at 20% than 40% debt.
capital would be constant, and equal to the opportunity cost of capital, at all debt ratios. We showed this in Chapter 17.

Figure 19.1 shows the shape of the relationship between financing and WACC, but initially we have numbers only for Sangria’s current 40% debt ratio. We want to recalculate WACC at a 20% ratio.

Here is the simplest way to do it. There are three steps.

**Step 1** Calculate the opportunity cost of capital. In other words, calculate WACC and the cost of equity at zero debt. This step is called *unlevering* the WACC. The simplest unlevering formula is

\[
\text{Opportunity cost of capital} = r = r_D D/V + r_E E/V
\]

This formula comes directly from Modigliani and Miller’s proposition 1 (see Section 17-1). If taxes are left out, the weighted-average cost of capital equals the opportunity cost of capital and is independent of leverage.

**Step 2** Estimate the cost of debt, \( r_D \), at the new debt ratio, and calculate the new cost of equity.

\[
r_E = r + (r - r_D)D/E
\]

This formula is Modigliani and Miller’s proposition 2 (see Section 17-2). It calls for \( D/E \), the ratio of debt to equity, not debt to value.

**Step 3** Recalculate the weighted-average cost of capital at the new financing weights.

Let’s do the numbers for Sangria at \( D/V = .20 \), or 20%.

**Step 1.** Sangria’s current debt ratio is \( D/V = .4 \). So

\[
r = .06(.4) + .124(.6) = .0984, \text{ or } 9.84%
\]
Step 2. We will assume that the debt cost stays at 6% when the debt ratio is 20%. Then
\[ r_E = .0984 + (0.0984 - 0.06)(0.25) = 0.108, \text{ or } 10.8\% \]
Note that the debt–equity ratio is \( 0.2/0.8 = 0.25 \).

Step 3. Recalculate WACC.

\[ \text{WACC} = 0.06(1 - 0.35)(0.2) + 0.108(0.8) = 0.0942, \text{ or } 9.42\% \]
Figure 19.1 enters these numbers on the plot of WACC versus the debt–equity ratio.

**Unlevering and Relevering Betas**

Our three-step procedure (1) unlevers and then (2) relevers the cost of equity. Some financial managers find it convenient to (1) unlever and then (2) relever the equity beta. Given the beta of equity at the new debt ratio, the cost of equity is determined from the capital asset pricing model. Then WACC is recalculated.

The formula for unlevering beta was given in Section 17-2.

\[ \beta_A = \beta_D (D/V) + \beta_E (E/V) \]
This equation says that the beta of a firm’s assets is revealed by the beta of a portfolio of all of the firm’s outstanding debt and equity securities. An investor who bought such a portfolio would own the assets free and clear and absorb only business risks.

The formula for relevering beta closely resembles MM’s proposition 2, except that betas are substituted for rates of return:

\[ \beta_E = \beta_A + (\beta_A - \beta_D)D/E \]
Use this formula to recalculate \( \beta_E \) when \( D/E \) changes.

**The Importance of Rebalancing**

The formulas for WACC and for unlevering and relevering expected returns are simple, but we must be careful to remember underlying assumptions. The most important point is rebalancing.

Calculating WACC for a company at its existing capital structure requires that the capital structure not change; in other words, the company must rebalance its capital structure to maintain the same market-value debt ratio for the relevant future. Take Sangria Corporation as an example. It starts with a debt-to-value ratio of 40% and a market value of $1,250 million. Suppose that Sangria’s products do unexpectedly well in the marketplace and that market value increases to $1,500 million. Rebalancing means that it will then increase debt to \( 0.4 \times 1,500 = 600 \text{ million} \), thus regaining a 40% ratio. If market value instead falls, Sangria would have to pay down debt proportionally.

Of course real companies do not rebalance capital structure in such a mechanical and compulsive way. For practical purposes, it’s sufficient to assume gradual but steady adjustment toward a long-run target. But if the firm plans significant changes in capital structure (for example, if it plans to pay off its debt), the WACC formula won’t work. In such cases, you should turn to the APV method, which we describe in the next section.

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13 The proceeds of the additional borrowing would be paid out to shareholders or used, along with additional equity investment, to finance Sangria’s growth.
Our three-step procedure for recalculating WACC makes a similar rebalancing assumption. Whatever the starting debt ratio, the firm is assumed to rebalance to maintain that ratio in the future.

The Modigliani–Miller Formula, Plus Some Final Advice

What if the firm does not rebalance to keep its debt ratio constant? In this case the only general approach is adjusted present value, which we cover in the next section. But sometimes financial managers turn to other discount-rate formulas, including one derived by Modigliani and Miller (MM). MM considered a company or project generating a level, perpetual stream of cash flows financed with fixed, perpetual debt, and derived a simple after-tax discount rate:

\[ r_{MM} = r(1 - T_D/V) \]

14 Similar, but not identical. The basic WACC formula is correct whether rebalancing occurs at the end of each period or continuously. The unlevering and relevering formulas used in steps 1 and 2 of our three-step procedure are exact only if rebalancing is continuous so that the debt ratio stays constant day-to-day and week-to-week. However, the errors introduced from annual rebalancing are very small and can be ignored for practical purposes.

15 Here’s why the formulas work with continuous rebalancing. Think of a market-value balance sheet with assets and interest tax shields on the left and debt and equity on the right, with \( D/V \) and \( E/V \).

\[ \beta_{x,\text{shields}} = \beta_D = \beta_D + \beta_E \]  

where \( \alpha \) is the proportion of the total firm value from its assets and \( 1 - \alpha \) is the proportion from interest tax shields. If the firm readjusts its capital structure to keep \( D/V \) constant, then the beta of the tax shield must be the same as the beta of the assets. With rebalancing, an \( x\% \) change in firm value \( V \) changes debt \( D \) by \( x\% \). So the interest tax shield \( T_c \) will change by \( x\% \) as well. Thus the risk of the tax shield must be the same as the risk of the firm as a whole:

\[ \beta_{x,\text{shields}} = \beta_D = \beta_D + \beta_E \]

This is our unlevering formula expressed in terms of beta. Since expected returns depend on beta:

\[ r_a = r_D + r_E \]

Rearrange formulas (2) and (3) to get the relevering formulas for \( r_a \) and \( r_b \):

\[ r_a = \beta_D + (\beta_D - \beta_E)D/E \]

\[ r_b = r_a + (r_b - r_a)D/E \]

All this assumes continuous rebalancing. Suppose instead that the firm rebalances once a year, so that the next year’s interest tax shield, which depends on this year’s debt, is known. Then you can use a formula developed by Miles and Ezzell:

\[ r_{\text{Miles-Ezzell}} = r_a - (D/V)r_D (\frac{1 + r_D}{1 + r_E}) \]


Given perpetual fixed debt,

\[ V = \frac{C}{r} + \frac{T_c D}{r} \]

\[ V = \frac{C}{r(1 - T_c D/V)} = \frac{C}{r_{\text{MM}}} \]
Here it’s easy to unlever: just set the debt-capacity parameter \((D/V)\) equal to zero.\(^{17}\)

MM’s formula is still used in practice, but the formula is exact only in the special case where there is a level, perpetual stream of cash flows and fixed, perpetual debt. However, the formula is not a bad approximation for shorter-lived projects when debt is issued in a fixed amount.\(^{18}\)

So which team do you want to play with, the fixed-debt team or the rebalancers? If you join the fixed-debt team you will be outnumbered. Most financial managers use the plain, after-tax WACC, which assumes constant market-value debt ratios and therefore assumes rebalancing. That makes sense, because the debt capacity of a firm or project must depend on its future value, which will fluctuate.

At the same time, we must admit that the typical financial manager doesn’t care much if his or her firm’s debt ratio drifts up or down within a reasonable range of moderate financial leverage. The typical financial manager acts as if a plot of WACC against the debt ratio is “flat” (constant) over this range. This too makes sense, if we just remember that interest tax shields are the only reason why the after-tax WACC declines in Figure 17.4 or 19.1. The WACC formula doesn’t explicitly capture costs of financial distress or any of the other nontax complications discussed in Chapter 18.\(^{19}\) All these complications may roughly cancel the value added by interest tax shields (within a range of moderate leverage). If so, the financial manager is wise to focus on the firm’s operating and investment decisions, rather than on fine-tuning its debt ratio.

The idea behind **adjusted present value (APV)** is to divide and conquer. APV does not attempt to capture taxes or other effects of financing in a WACC or adjusted discount rate. A series of present value calculations is made instead. The first establishes a base-case value for the project or firm: its value as a separate, all-equity-financed venture. The discount rate for the base-case value is just the opportunity cost of capital. Once the base-case value is set, then each financing side effect is traced out, and the present value of its cost or benefit to the firm is calculated. Finally, all the present values are added together to estimate the project’s total contribution to the value of the firm:

\[
APV = \text{base-case NPV} + \text{sum of PVs of financing side effects}^{20}
\]

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\(^{17}\) In this case the relevering formula for the cost of equity is:

\[
r_e = r_d + (1 - T_c)(r_d - r_f)D/E
\]

The unlevering and relevering formulas for betas are

\[
\beta_A = \frac{\beta_E(1 - T_c)D/E + \beta_U}{1 + (1 - T_c)D/E}
\]

and

\[
\beta_E = \beta_A + (1 - T_c)(\beta_A - \beta_U)D/E
\]


\(^{19}\) Costs of financial distress can show up as rapidly increasing costs of debt and equity, especially at high debt ratios. The costs of financial distress could “flatten out” the WACC curve in Figures 17.4 and 19.1, and finally increase WACC as leverage climbs. Thus some practitioners calculate an industry WACC and take it as constant, at least within the range of debt ratios observed for healthy companies in the industry.

Personal taxes could also generate a flatter curve for after-tax WACC as a function of leverage. See Section 18-2.

The most important financing side effect is the interest tax shield on the debt supported by the project (a plus). Other possible side effects are the issue costs of securities (a minus) or financing packages subsidized by a supplier or government (a plus).

APV gives the financial manager an explicit view of the factors that are adding or subtracting value. APV can prompt the manager to ask the right follow-up questions. For example, suppose that base-case NPV is positive but less than the costs of issuing shares to finance the project. That should prompt the manager to look around to see if the project can be rescued by an alternative financing plan.

**APV for the Perpetual Crusher**

APV is easiest to understand in simple numerical examples. Let’s apply it to Sangria’s perpetual crusher project. We start by showing that APV is equivalent to discounting at WACC if we make the same assumptions about debt policy.

We used Sangria’s WACC (9%) as the discount rate for the crusher’s projected cash flows. The WACC calculation assumed that debt will be maintained at a constant 40% of the future value of the project or firm. In this case, the risk of interest tax shields is the same as the risk of the project. Therefore we will discount the tax shields at the opportunity cost of capital ($r$).

The first step is to calculate base-case NPV. We discount after-tax project cash flows of $1.125 million at the opportunity cost of capital of 9.84% and subtract the $12.5 million outlay. The cash flows are perpetual, so:

$$\text{Base-case NPV} = -12.5 + \frac{1.125}{.0984} = -$1.067 \text{ million}$$

Thus the project would not be worthwhile with all-equity financing. But it actually supports debt of $5 million. At a 6% borrowing rate ($r_D = .06$) and a 35% tax rate ($T_c = .35$), annual tax shields are $.35 \times .06 \times 5 = .105$, or $105,000$.

What are those tax shields worth? If the firm is constantly rebalancing its debt, we discount at $r = 9.84\%$:

$$\text{PV(interest tax shields, debt rebalanced)} = \frac{105,000}{.0984} = 1.067 \text{ million}$$

APV is the sum of base-case value and PV(interest tax shields)

$$\text{APV} = -1.067 \text{ million} + 1.067 \text{ million} = 0$$

This is exactly the same as we obtained by one-step discounting with WACC. The perpetual crusher is a break-even project by either valuation method.

But with APV, we don’t have to hold debt at a constant proportion of value. Suppose Sangria plans to keep project debt fixed at $5 million. In this case we assume the risk of the tax shields is the same as the risk of the debt and we discount at the 6% rate on debt:

$$\text{PV(tax shields, debt fixed)} = \frac{105,000}{.06} = 1.75 \text{ million}$$

$$\text{APV} = -1.067 + 1.75 = 5.683 \text{ million}$$

Now the project is more attractive. With fixed debt, the interest tax shields are safe and therefore worth more. (Whether the fixed debt is safer for Sangria is another matter. If the perpetual crusher project fails, the $5 million of fixed debt may end up as a burden on Sangria’s other assets.)

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21 That is, $\beta_d = \beta_{\text{tax shields}}$. See footnote 15 above.
Other Financing Side Effects

Suppose Sangria has to finance the perpetual crusher by issuing debt and equity. It issues $7.5 million of equity with issue costs of 7% ($525,000) and $5 million of debt with issue costs of 2% ($100,000). Assume the debt is fixed once issued, so that interest tax shields are worth $1.75 million. Now we can recalculate APV, taking care to subtract the issue costs:

\[
\text{APV} = -1.067 + 1.75 - .525 - .100 = .058 \text{ million, or $58,000}
\]

The issue costs would reduce APV to nearly zero.

Sometimes there are favorable financing side effects that have nothing to do with taxes. For example, suppose that a potential manufacturer of crusher machinery offers to sweeten the deal by leasing it to Sangria on favorable terms. Then you could calculate APV as the sum of base-case NPV plus the NPV of the lease. Or suppose that a local government offers to lend Sangria $5 million at a very low interest rate if the crusher is built and operated locally. The NPV of the subsidized loan could be added in to APV. (We cover leases in Chapter 25 and subsidized loans in the Appendix to this chapter.)

APV for Businesses

APV can also be used to value businesses. Let's take another look at the valuation of Rio. In Table 19.1, we assumed a constant 40% debt ratio and discounted free cash flow at Sangria's WACC. Table 19.2 runs the same analysis, but with a fixed debt schedule.

We'll suppose that Sangria has decided to make an offer for Rio. If successful, it plans to finance the purchase with $51 million of debt. It intends to pay down the debt to $45 million in year 6. Recall Rio's horizon value of $113.4 million, which is calculated in Table 19.1 and shown again in Table 19.2. The debt ratio at the horizon is therefore projected at 45/113.4 = .397, about 40%. Thus Sangria plans to take Rio back to a normal 40% debt ratio at the horizon. But Rio will be carrying a heavier debt load before the horizon. For example, the $51 million of initial debt is about 58% of company value as calculated in Table 19.1.

Let's see how Rio's APV is affected by this more aggressive borrowing schedule. Table 19.2 shows projections of free cash flows from Table 19.1. Now we need Rio's base-case value, so we discount these flows at the opportunity cost of capital (9.84%), not at WACC. The resulting base-case value for Rio is $84.3 million. Table 19.2 also projects debt levels, interest, and interest tax shields. If the debt levels are taken as fixed, then the tax shields should be discounted back at the 6% borrowing rate. The resulting PV of interest tax shields is $5.0 million. Thus,

\[
\text{APV} = \text{base-case NPV} + \text{PV(interest tax shields)} \\
= \$84.3 + 5.0 = \$89.3 \text{ million}
\]

an increase of $1.4 million from NPV in Table 19.1. The increase can be traced to the higher early debt levels and to the assumption that the debt levels and interest tax shields are fixed and relatively safe.

Now a difference of $1.4 million is not a big deal, considering all the lurking risks and pitfalls in forecasting Rio's free cash flows. But you can see the advantage of the flexibility that APV provides. The APV spreadsheet allows you to explore the implications of different

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22 Therefore we still calculate the horizon value in year 6 by discounting subsequent free cash flows at WACC. The horizon value in year 6 is discounted back to year 0 at the opportunity cost of capital, however.

23 Many of the assumptions and calculations in Table 19.1 have been hidden in Table 19.2. The hidden rows can be recalled in the "live" version of Table 19.2, which is available on this book’s Web site (www.mhhe.com/bma).

24 But will Rio really support debt at the levels shown in Table 19.2? If not, then the debt must be partly supported by Sangria’s other assets, and only part of the $5 million in PV(interest tax shields) can be attributed to Rio itself.
financing strategies without locking into a fixed debt ratio or having to calculate a new WACC for every scenario.

APV is particularly useful when the debt for a project or business is tied to book value or has to be repaid on a fixed schedule. For example, Kaplan and Ruback used APV to analyze the prices paid for a sample of leveraged buyouts (LBOs). LBOs are takeovers, typically of mature companies, financed almost entirely with debt. However, the new debt is not intended to be permanent. LBO business plans call for generating extra cash by selling assets, shaving costs, and improving profit margins. The extra cash is used to pay down the LBO debt. Therefore you can’t use WACC as a discount rate to evaluate an LBO because its debt ratio will not be constant.

APV works fine for LBOs. The company is first evaluated as if it were all-equity-financed. That means that cash flows are projected after tax, but without any interest tax shields generated by the LBO’s debt. The tax shields are then valued separately and added to the all-equity value. Any other financing side effects are added also. The result is an APV valuation for the company.\(^{25}\) Kaplan and Ruback found that APV did a pretty good job explaining prices paid in these hotly contested takeovers, considering that not all the information available to bidders had percolated into the public domain. Kaplan and Ruback were restricted to publicly available data.

### APV for International Investments

APV is most useful when financing side effects are numerous and important. This is frequently the case for large international investments, which may have custom-tailored project

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financing and special contracts with suppliers, customers, and governments. Here are a few examples of financing side effects encountered in international finance.

We explain project finance in Chapter 24. It typically means very high debt ratios to start, with most or all of a project’s early cash flows committed to debt service. Equity investors have to wait. Since the debt ratio will not be constant, you have to turn to APV.

Project financing may include debt available at favorable interest rates. Most governments subsidize exports by making special financing packages available, and manufacturers of industrial equipment may stand ready to lend money to help close a sale. Suppose, for example, that your project requires construction of an on-site electricity generating plant. You solicit bids from suppliers in various countries. Don’t be surprised if the competing suppliers sweeten their bids with offers of low interest rate project loans or if they offer to lease the plant on favorable terms. You should then calculate the NPVs of these loans or leases and include them in your project analysis.

Sometimes international projects are supported by contracts with suppliers or customers. Suppose a manufacturer wants to line up a reliable supply of a crucial raw material—powdered magnoosium, say. The manufacturer could subsidize a new magnoosium smelter by agreeing to buy 75% of production and guaranteeing a minimum purchase price. The guarantee is clearly a valuable addition to project APV: if the world price of powdered magnoosium falls below the minimum, the project doesn’t suffer. You would calculate the value of this guarantee (by the methods explained in Chapters 20 to 22) and add it to APV.

Sometimes local governments impose costs or restrictions on investment or disinvestment. For example, Chile, in an attempt to slow down a flood of short-term capital inflows in the 1990s, required investors to “park” part of their incoming money in non-interest-bearing accounts for a period of two years. An investor in Chile during this period would calculate the cost of this requirement and subtract it from APV.

Question: All these cost of capital formulas—which ones do financial managers actually use?
Answer: The after-tax weighted-average cost of capital, most of the time. WACC is estimated for the company, or sometimes for an industry. We recommend industry WACCs when data are available for firms with similar assets, operations, business risks, and growth opportunities.

Of course, conglomerate companies, with divisions operating in two or more unrelated industries, should not use a single company or industry WACC. Such firms should try to estimate a different industry WACC for each operating division.

Question: But WACC is the correct discount rate only for “average” projects. What if the project’s financing differs from the company’s or industry’s?
Answer: Remember, investment projects are usually not separately financed. Even when they are, you should focus on the project’s contribution to the firm’s overall debt capacity, not on its immediate financing. (Suppose it’s convenient to raise all the money for a particular project with a bank loan. That doesn’t mean the project itself supports 100% debt financing. The company is borrowing against its existing assets as well as the project.)

But if the project’s debt capacity is materially different from the company’s existing assets, or if the company’s overall debt policy changes, WACC should be adjusted. The adjustment can be done by the three-step procedure explained in Section 19-3.

Question: Could we do one more numerical example?
Answer: Sure. Suppose that WACC has been estimated as follows at a 30% debt ratio:

\[
\text{WACC} = r_d (1 - T_c) \frac{D}{V} + r_e \frac{E}{V} \\
= .09(1 - .35)(.3) + .15(.7) = .1226, \text{ or } 12.26\% 
\]
What is the correct discount rate at a 50% debt ratio?

Step 1. Calculate the opportunity cost of capital.

\[
r = r_D D/V + r_E E/V
\]

\[
= .09 (.3) + .15 (.7) = .132, \text{ or } 13.2\%
\]

Step 2. Calculate the new costs of debt and equity. The cost of debt will be higher at 50% debt than 30%. Say it is \(r_D = .095\). The new cost of equity is:

\[
r_E = r + (r - r_D) D/E
\]

\[
= .132 + (.132 - .095) 50/50
\]

\[
= .169, \text{ or } 16.9\%
\]

Step 3. Recalculate WACC.

\[
WACC = r_D (1 - T_c) D/V + r_E E/V
\]

\[
= .095 (1 - .35) (.5) + .169 (.5) = .1154, \text{ or about } 11.5\%
\]

**Question:** How do I use the capital asset pricing model to calculate the after-tax weighted-average cost of capital?

**Answer:** First plug the equity beta into the capital asset pricing formula to calculate \(r_E\), the expected return to equity. Then use this figure, along with the after-tax cost of debt and the debt-to-value and equity-to-value ratios, in the WACC formula.

Of course the CAPM is not the only way to estimate the cost of equity. For example, you might be able to use the dividend-discount model (see Section 4-3).

**Question:** But suppose I do use the CAPM? What if I have to recalculate the equity beta for a different debt ratio?

**Answer:** The formula for the equity beta is:

\[
\beta_E = \beta_A + (\beta_A - \beta_D) D/E
\]

where \(\beta_E\) is the equity beta, \(\beta_A\) is the asset beta, and \(\beta_D\) is the beta of the company’s debt.

The asset beta is a weighted average of the debt and equity betas:

\[
\beta_A = \beta_D (D/V) + \beta_E (E/V)
\]

Suppose you needed the opportunity cost of capital \(r\). You could calculate \(\beta_A\) and then \(r\) from the capital asset pricing model.

**Question:** I think I understand how to adjust for differences in debt capacity or debt policy. How about differences in business risk?

**Answer:** If business risk is different, then \(r\), the opportunity cost of capital, is different.

Figuring out the right \(r\) for an unusually safe or risky project is never easy. Sometimes the financial manager can use estimates of risk and expected return for companies similar to the project. Suppose, for example, that a traditional pharmaceutical company is considering a major commitment to biotech research. The financial manager could pick a sample of biotech companies, estimate their average beta and cost of capital, and use these estimates as benchmarks for the biotech investment.

But in many cases it’s difficult to find a good sample of matching companies for an unusually safe or risky project. Then the financial manager has to adjust the opportunity cost of capital by judgment. Section 9-3 may be helpful in such cases.

**Question:** When do I need adjusted present value (APV)?

**Answer:** The WACC formula picks up only one financing side effect: the value of interest tax shields on debt supported by a project. If there are other side effects—subsidized financing tied to a project, for example—you should use APV.

You can also use APV to break out the value of interest tax shields:

\[
APV = \text{base-case NPV} + \text{PV(tax shield)}
\]
Suppose, for example, that you are analyzing a company just after a leveraged buyout. The company has a very high initial debt level but plans to pay down the debt as rapidly as possible. APV could be used to obtain an accurate valuation.

**Question:** When should personal taxes be incorporated into the analysis?

**Answer:** Always use \( T_c \), the marginal corporate tax rate, when calculating WACC as a weighted average of the costs of debt and equity. The discount rate is adjusted only for corporate taxes.

In principle, APV can be adjusted for personal taxes by replacing the marginal corporate rate \( T_c \) with an effective tax rate that combines corporate and personal taxes and reflects the net tax advantage per dollar of interest paid by the firm. We provided back-of-the-envelope calculations of this advantage in Section 18-2. The effective tax rate is almost surely less than \( T_c \), but it is very difficult to pin down the numerical difference. Therefore, in practice \( T_c \) is almost always used as an approximation.

**Question:** Are taxes really that important? Do financial managers really fine-tune the debt ratio to minimize WACC?

**Answer:** As we saw in Chapter 18, financing decisions reflect many forces beyond taxes, including costs of financial distress, differences in information, and incentives for managers. There may not be a sharply defined optimal capital structure. Therefore most financial managers don’t fine-tune their companies’ debt ratios, and they don’t rebalance financing to keep debt ratios strictly constant. In effect they assume that a plot of WACC for different debt ratios is “flat” over a reasonable range of moderate leverage.

In this chapter we considered how financing can be incorporated into the valuation of projects and ongoing businesses. There are two ways to take financing into account. The first is to calculate NPV by discounting at an adjusted discount rate, usually the after-tax weighted-average cost of capital (WACC). The second approach discounts at the opportunity cost of capital and then adds or subtracts the present values of financing side effects. The second approach is called adjusted present value, or APV.

The formula for the after-tax WACC is:

\[
WACC = \frac{r_D(1 - T_c)D}{V} + \frac{r_EE}{V}
\]

where \( r_D \) and \( r_E \) are the expected rates of return demanded by investors in the firm’s debt and equity securities, \( D \) and \( E \) are the current market values of debt and equity, and \( V \) is the total market value of the firm \((V = D + E)\). Of course, the WACC formula expands if there are other sources of financing, for example, preferred stock.

Strictly speaking, discounting at WACC works only for projects that are carbon copies of the existing firm—projects with the same business risk that will be financed to maintain the firm’s current, market debt ratio. But firms can use WACC as a benchmark rate to be adjusted for differences in business risk or financing. We gave a three-step procedure for adjusting WACC for different debt ratios.

Discounting cash flows at the WACC assumes that debt is rebalanced to keep a constant ratio of debt to market value. The amount of debt supported by a project is assumed to rise or fall with the project’s after-the-fact success or failure. The WACC formula also assumes that financing matters only because of interest tax shields. When this or other assumptions are violated, only APV will give an absolutely correct answer.

APV is, in concept at least, simple. First calculate the base-case NPV of the project or business on the assumption that financing doesn’t matter. (The discount rate is not WACC, but the opportunity cost of capital.) Then calculate the present values of any relevant financing side effects and add or subtract from base-case value. A capital investment project is worthwhile if

\[
APV = \text{base-case NPV} + \text{PV(financing side effects)}
\]
is positive. Common financing side effects include interest tax shields, issue costs, and special financing packages offered by suppliers or governments.

For firms or going-concern businesses, value depends on free cash flow. Free cash flow is the amount of cash that can be paid out to all investors, debt as well as equity, after deducting cash needed for new investment or increases in working capital. Free cash flow does not include the value of interest tax shields, however. The WACC formula accounts for interest tax shields by using the after-tax cost of debt. APV adds PV(interest tax shields) to base-case value.

Businesses are usually valued in two steps. First free cash flow is forecasted out to a valuation horizon and discounted back to present value. Then a horizon value is calculated and also discounted back. The horizon value is usually estimated by using the perpetual-growth DCF formula or by multiplying forecasted EBIT or EBITDA\(^ {26} \) by multiples observed for similar firms. Be particularly careful to avoid unrealistically high horizon values. By the time the horizon arrives, competitors will have had several years to catch up. Also, when you are done valuing the business, don’t forget to subtract its debt to get the value of the firm’s equity.

All of this chapter’s examples reflect assumptions about the amount of debt supported by a project or business. Remember not to confuse “supported by” with the immediate source of funds for investment. For example, a firm might, as a matter of convenience, borrow $1 million for a $1 million research program. But the research is unlikely to contribute $1 million in debt capacity; a large part of the $1 million new debt would be supported by the firm’s other assets.

Also remember that debt capacity is not meant to imply an absolute limit on how much the firm can borrow. The phrase refers to how much it chooses to borrow against a project or ongoing business.

The Harvard Business Review has published a popular account of APV:

There have been dozens of articles on the weighted-average cost of capital and other issues discussed in this chapter. Here are three:


Two books that provide detailed explanations of how to value companies are:

The valuation rule for safe, nominal cash flows is developed in:

26 Recall that EBIT = earnings before interest and taxes and EBITDA = EBIT plus depreciation and amortization.
PROBLEM SETS

BASIC

1. Calculate the weighted-average cost of capital (WACC) for Federated Junkyards of America, using the following information:
   • Debt: $75,000,000 book value outstanding. The debt is trading at 90% of book value. The yield to maturity is 9%.
   • Equity: 2,500,000 shares selling at $42 per share. Assume the expected rate of return on Federated’s stock is 18%.
   • Taxes: Federated’s marginal tax rate is \( T_c = .35 \).

2. Suppose Federated Junkyards decides to move to a more conservative debt policy. A year later its debt ratio is down to 15% (\( D/V = .15 \)). The interest rate has dropped to 8.6%. Recalculate Federated’s WACC under these new assumptions. The company’s business risk, opportunity cost of capital, and tax rate have not changed. Use the three-step procedure explained in Section 19-3.

3. True or false? Use of the WACC formula assumes
   a. A project supports a fixed amount of debt over the project’s economic life.
   b. The ratio of the debt supported by a project to project value is constant over the project’s economic life.
   c. The firm rebalances debt each period, keeping the debt-to-value ratio constant.

4. What is meant by the flow-to-equity valuation method? What discount rate is used in this method? What assumptions are necessary for this method to give an accurate valuation?

5. True or false? The APV method
   a. Starts with a base-case value for the project.
   b. Calculates the base-case value by discounting project cash flows, forecasted assuming all-equity financing, at the WACC for the project.
   c. Is especially useful when debt is to be paid down on a fixed schedule.

6. A project costs $1 million and has a base-case NPV of exactly zero (NPV = 0). What is the project’s APV in the following cases?
   a. If the firm invests, it has to raise $500,000 by a stock issue. Issue costs are 15% of net proceeds.
   b. If the firm invests, its debt capacity increases by $500,000. The present value of interest tax shields on this debt is $76,000.

7. Whispering Pines, Inc., is all-equity-financed. The expected rate of return on the company’s shares is 12%.
   a. What is the opportunity cost of capital for an average-risk Whispering Pines investment?
   b. Suppose the company issues debt, repurchases shares, and moves to a 30% debt-to-value ratio (\( D/V = .30 \)). What will the company’s weighted-average cost of capital be at the new capital structure? The borrowing rate is 7.5% and the tax rate is 35%.

8. Consider a project lasting one year only. The initial outlay is $1,000 and the expected inflow is $1,200. The opportunity cost of capital is \( r = .20 \). The borrowing rate is \( r_D = .10 \), and the tax shield per dollar of interest is \( T_c = .35 \).
   a. What is the project’s base-case NPV?
   b. What is its APV if the firm borrows 30% of the project’s required investment?

9. The WACC formula seems to imply that debt is “cheaper” than equity—that is, that a firm with more debt could use a lower discount rate. Does this make sense? Explain briefly.
10. Suppose KCS Corp. buys out Patagonia Trucking, a privately owned business, for $50 million. KCS has only $5 million cash in hand, so it arranges a $45 million bank loan. A normal debt-to-value ratio for a trucking company would be 50% at most, but the bank is satisfied with KCS’s credit rating.

Suppose you were valuing Patagonia by APV in the same format as Table 19.2. How much debt would you include? Explain briefly.

INTERMEDIATE
11. Table 19.3 shows a book balance sheet for the Wishing Well Motel chain. The company’s long-term debt is secured by its real estate assets, but it also uses short-term bank financing. It pays 10% interest on the bank debt and 9% interest on the secured debt. Wishing Well has 10 million shares of stock outstanding, trading at $90 per share. The expected return on Wishing Well’s common stock is 18%.

Calculate Wishing Well’s WACC. Assume that the book and market values of Wishing Well’s debt are the same. The marginal tax rate is 35%.

12. Suppose Wishing Well is evaluating a new motel and resort on a romantic site in Madison County, Wisconsin. Explain how you would forecast the after-tax cash flows for this project. (Hints: How would you treat taxes? Interest expense? Changes in working capital?)

13. To finance the Madison County project, Wishing Well will have to arrange an additional $80 million of long-term debt and make a $20 million equity issue. Underwriting fees, spreads, and other costs of this financing will total $4 million. How would you take this into account in valuing the proposed investment?

14. Table 19.4 shows a simplified balance sheet for Rensselaer Felt. Calculate this company’s weighted-average cost of capital. The debt has just been refinanced at an interest rate of 6% (short term) and 8% (long term). The expected rate of return on the company’s shares is 15%. There are 7.46 million shares outstanding, and the shares are trading at $46. The tax rate is 35%.

15. How will Rensselaer Felt’s WACC and cost of equity change if it issues $50 million in new equity and uses the proceeds to retire long-term debt? Assume the company’s borrowing rates are unchanged. Use the three-step procedure from Section 19-3.

16. Digital Organics (DO) has the opportunity to invest $1 million now \( t = 0 \) and expects after-tax returns of $600,000 in \( t = 1 \) and $700,000 in \( t = 2 \). The project will last for two years only. The appropriate cost of capital is 12% with all-equity financing, the
part five  payout policy and capital structure

borrowing rate is 8%, and DO will borrow $300,000 against the project. This debt must
be repaid in two equal installments. Assume debt tax shields have a net value of $0.30
per dollar of interest paid. Calculate the project’s APV using the procedure followed in
Table 19.2.

17. Consider another perpetual project like the crusher described in Section 19-1. Its initial
investment is $1,000,000, and the expected cash inflow is $95,000 a year in perpetuity. The
opportunity cost of capital with all-equity financing is 10%, and the project allows the firm
to borrow at 7%. The tax rate is 35%.

Use APV to calculate this project’s value.

a. Assume first that the project will be partly financed with $400,000 of debt and that the
debt amount is to be fixed and perpetual.

b. Then assume that the initial borrowing will be increased or reduced in proportion to
changes in the market value of this project.

Explain the difference between your answers to (a) and (b).

18. Suppose the project described in Problem 17 is to be undertaken by a university. Funds
for the project will be withdrawn from the university’s endowment, which is invested in a
widely diversified portfolio of stocks and bonds. However, the university can also borrow
at 7%. The university is tax exempt.

The university treasurer proposes to finance the project by issuing $400,000 of perpetual
bonds at 7% and by selling $600,000 worth of common stocks from the endowment. The
expected return on the common stocks is 10%. He therefore proposes to evaluate the proj-

ect by discounting at a weighted-average cost of capital, calculated as:

\[ r = \frac{D}{V} + \frac{E}{V} \]

\[ = 0.07 \left(\frac{400,000}{1,000,000}\right) + 0.10 \left(\frac{600,000}{1,000,000}\right) \]

\[ = 0.088, or 8.8 \%

What’s right or wrong with the treasurer’s approach? Should the university invest? Should
it borrow? Would the project’s value to the university change if the treasurer financed the
project entirely by selling common stocks from the endowment?

19. Consider a project to produce solar water heaters. It requires a $10 million investment and
offers a level after-tax cash flow of $1.75 million per year for 10 years. The opportunity cost
of capital is 12%, which reflects the project’s business risk.

a. Suppose the project is financed with $5 million of debt and $5 million of equity. The
interest rate is 8% and the marginal tax rate is 35%. The debt will be paid off in equal
annual installments over the project’s 10-year life. Calculate APV.

b. How does APV change if the firm incurs issue costs of $400,000 to raise the $5 million
of required equity?

20. Take another look at the valuations of Rio in Tables 19.1 and 19.2. Now use the live
spreadsheets on this book’s Web site (www.mhhe.com/bma) to show how the valuations
depend on:

a. The forecasted long-term growth rate.

b. The required amounts of investment in fixed assets and working capital.

c. The opportunity cost of capital. Note you can also vary the opportunity cost of capital
in Table 19.1.

d. Profitability, that is, cost of goods sold as a percentage of sales.

e. The assumed amount of debt financing.
21. The Bunsen Chemical Company is currently at its target debt ratio of 40%. It is contemplating a $1 million expansion of its existing business. This expansion is expected to produce a cash inflow of $130,000 a year in perpetuity.

The company is uncertain whether to undertake this expansion and how to finance it. The two options are a $1 million issue of common stock or a $1 million issue of 20-year debt. The flotation costs of a stock issue would be around 5% of the amount raised, and the flotation costs of a debt issue would be around 1½%.

Bunsen’s financial manager, Ms. Polly Ethylene, estimates that the required return on the company’s equity is 14%, but she argues that the flotation costs increase the cost of new equity to 19%. On this basis, the project does not appear viable.

On the other hand, she points out that the company can raise new debt on a 7% yield, which would make the cost of new debt 8½%. She therefore recommends that Bunsen should go ahead with the project and finance it with an issue of long-term debt.

Is Ms. Ethylene right? How would you evaluate the project?

22. Nevada Hydro is 40% debt-financed and has a weighted-average cost of capital of 9.7%:

\[
WACC = (1 - T_c)\frac{D}{V} + \frac{E}{V}
\]

\[
= (1 - .35)(.085)(.40) + .125(.60) = .097
\]

Goldensacks Company is advising Nevada Hydro to issue $75 million of preferred stock at a dividend yield of 9%. The proceeds would be used to repurchase and retire common stock. The preferred issue would account for 10% of the preissuance market value of the firm.

Goldensacks argues that these transactions would reduce Nevada Hydro’s WACC to 9.4%:

\[
WACC = (1 -.35)(.085)(.40) + .09(.10) + .125(.50)
\]

\[
= .094, \text{ or } 9.4\%
\]

Do you agree with this calculation? Explain...

23. Chiara Company’s management has made the projections shown in Table 19.5. Use this Excel spreadsheet as a starting point to value the company as a whole. The WACC for

<table>
<thead>
<tr>
<th>Historical</th>
<th>Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year:</strong></td>
<td>0</td>
</tr>
<tr>
<td>1. Sales</td>
<td>35,348</td>
</tr>
<tr>
<td>2. Cost of goods sold</td>
<td>17,834</td>
</tr>
<tr>
<td>3. Other costs</td>
<td>6,968</td>
</tr>
<tr>
<td>4. EBITDA (1–2–3)</td>
<td>10,546</td>
</tr>
<tr>
<td>5. Depreciation</td>
<td>5,671</td>
</tr>
<tr>
<td>6. EBIT (Pretax profit) (4–5)</td>
<td>4,875</td>
</tr>
<tr>
<td>7. Tax at 35%</td>
<td>1,706</td>
</tr>
<tr>
<td>8. Profit after tax (6–7)</td>
<td>3,169</td>
</tr>
<tr>
<td>9. Change in working capital</td>
<td>325</td>
</tr>
<tr>
<td>10. Investment (change in gross fixed assets)</td>
<td>5,235</td>
</tr>
</tbody>
</table>

**TABLE 19.5**  Cash flow projections for Chiara Corp. ($ thousands).
Chiara is 12% and the long-run growth rate after year 5 is 4%. The company has $5 million debt and 865,000 shares outstanding. What is the value per share?

**CHALLENGE**

24. In Footnote 15 we referred to the Miles–Ezzell discount rate formula, which assumes that debt is not rebalanced continuously, but at one-year intervals. Derive this formula. Then use it to unlever Sangria’s WACC and calculate Sangria’s opportunity cost of capital. Your answer will be slightly different from the opportunity cost that we calculated in Section 19-3. Can you explain why?

25. The WACC formula assumes that debt is rebalanced to maintain a constant debt ratio \( D/V \). Rebalancing ties the level of future interest tax shields to the future value of the company. This makes the tax shields risky. Does that mean that fixed debt levels (no rebalancing) are better for stockholders?

26. Modify Table 19.1 on the assumption that competition eliminates any opportunities to earn more than WACC on new investment after year 7 (PVGO = 0). How does the valuation of Rio change?

Table 19.6 is a simplified book balance sheet for Apache Corp. at year-end 2008. Here is some further information:

| Number of outstanding shares \((N)\) | 335.75 million |
| Price per share \((P)\) | $74.0 |
| Beta | 1.21 |
| Treasury bill rate | 0.11% |
| 20-year Treasury bond rate | 2.25% |
| Cost of debt \((r_D)\) | 7.50 |
| Marginal tax rate | 35% |

a. Calculate Apache’s WACC. Use the capital asset pricing model and the additional information given above. Make additional assumptions and approximations as necessary.

b. What is Apache’s opportunity cost of capital?

c. Finally, go to the Standard & Poor’s Market Insight Web site (www.mhhe.com/edumarketinsight) and update your answers to questions (a) and (b).

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**APPENDIX**

**Discounting Safe, Nominal Cash Flows**

Suppose you’re considering purchase of a $100,000 machine. The manufacturer sweetens the deal by offering to finance the purchase by lending you $100,000 for five years, with annual interest payments of 5%. You would have to pay 13% to borrow from a bank. Your marginal tax rate is 35% \((T_c = .35)\).
How much is this loan worth? If you take it, the cash flows, in thousands of dollars, are:

<table>
<thead>
<tr>
<th>Period</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow</td>
<td>100</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-105</td>
</tr>
<tr>
<td>Tax shield</td>
<td>+1.75</td>
<td>+1.75</td>
<td>+1.75</td>
<td>+1.75</td>
<td>+1.75</td>
<td></td>
</tr>
<tr>
<td>After-tax cash flow</td>
<td>100</td>
<td>-3.25</td>
<td>-3.25</td>
<td>-3.25</td>
<td>-3.25</td>
<td>-103.25</td>
</tr>
</tbody>
</table>

What is the right discount rate?

Here you are discounting safe, nominal cash flows—safe because your company must commit to pay if it takes the loan, and nominal because the payments would be fixed regardless of future inflation. Now, the correct discount rate for safe, nominal cash flows is your company’s after-tax, unsubsidized borrowing rate, which is $r_D(1 - T_c)$.

\[
NPV = \frac{+100 - \frac{3.25}{1.0845} - \frac{3.25}{(1.0845)^2} - \frac{3.25}{(1.0845)^3} - \frac{3.25}{(1.0845)^4} - \frac{103.25}{(1.0845)^5}}{1.0845}
\]

\[= +20.52, \text{ or } $20,520\]

The manufacturer has effectively cut the machine’s purchase price from $100,000 to $79,480. You can now go back and recalculate the machine’s NPV using this fire-sale price, or you can use the NPV of the subsidized loan as one element of the machine’s adjusted present value.

**A General Rule**

Clearly, we owe an explanation of why $r_D(1 - T_c)$ is the right discount rate for safe, nominal cash flows. It’s no surprise that the rate depends on $r_D$, the unsubsidized borrowing rate, for that is investors’ opportunity cost of capital, the rate they would demand from your company’s debt. But why should $r_D$ be converted to an after-tax figure?

Let’s simplify by taking a one-year subsidized loan of $100,000 at 5%. The cash flows, in thousands of dollars, are:

<table>
<thead>
<tr>
<th>Period 0</th>
<th>Period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow</td>
<td>100</td>
</tr>
<tr>
<td>Tax shield</td>
<td>+1.75</td>
</tr>
<tr>
<td>After-tax cash flow</td>
<td>100</td>
</tr>
</tbody>
</table>

Now ask, What is the maximum amount $X$ that could be borrowed for one year through regular channels if $103,250 is set aside to service the loan?

“Regular channels” means borrowing at 13% pretax and 8.45% after tax. Therefore you will need 108.45% of the amount borrowed to pay back principal plus after-tax interest charges. If $1.0845X = 103,250$, then $X = 95,205$. Now if you can borrow $100,000 by a subsidized loan, but only $95,205 through normal channels, the difference ($4,795) is money in the bank. Therefore, it must also be the NPV of this one-period subsidized loan.

When you discount a safe, nominal cash flow at an after-tax borrowing rate, you are implicitly calculating the equivalent loan, the amount you could borrow through normal channels, using the cash flow as debt service. Note that:

\[
\text{Equivalent loan} = \frac{\text{PV(cash flow available for debt service)}}{1.0845} = \frac{103,250}{1.0845} = 95,205
\]

27 In theory, safe means literally “risk-free,” like the cash returns on a Treasury bond. In practice, it means that the risk of not paying or receiving a cash flow is small.

28 In Section 13-1 we calculated the NPV of subsidized financing using the pretax borrowing rate. Now you can see that was a mistake. Using the pretax rate implicitly defines the loan in terms of its pretax cash flows, violating a rule promulgated way back in Section 6-1: Always estimate cash flows on an after-tax basis.
In some cases, it may be easier to think of taking the lender’s side of the equivalent loan rather
than the borrower’s. For example, you could ask, How much would my company have to invest
today to cover next year’s debt service on the subsidized loan? The answer is $95,205: If you lend
that amount at 13%, you will earn 8.45% after tax, and therefore have 95,205(1.0845) = $103,250.
By this transaction, you can in effect cancel, or “zero out,” the future obligation. If you can bor-
row $100,000 and then set aside only $95,205 to cover all the required debt service, you clearly
have $4,795 to spend as you please. That amount is the NPV of the subsidized loan.
Therefore, regardless of whether it’s easier to think of borrowing or lending, the correct dis-
count rate for safe, nominal cash flows is an after-tax interest rate.

In some ways, this is an obvious result once you think about it. Companies are free to bor-
lrow or lend money. If they lend, they receive the after-tax interest rate on their investment; if
they borrow in the capital market, they pay the after-tax interest rate. Thus, the opportunity cost
to companies of investing in debt-equivalent cash flows is the after-tax interest rate. This is the
adjusted cost of capital for debt-equivalent cash flows.

Some Further Examples Here are some further examples of debt-equivalent cash flows.

Payout Fixed by Contract Suppose you sign a maintenance contract with a truck leasing firm,
which agrees to keep your leased trucks in good working order for the next two years in exchange
for 24 fixed monthly payments. These payments are debt-equivalent flows.

Depreciation Tax Shields Capital projects are normally valued by discounting the total after-
tax cash flows they are expected to generate. Depreciation tax shields contribute to project cash
flow, but they are not valued separately; they are just folded into project cash flows along with
dozens, or hundreds, of other specific inflows and outflows. The project’s opportunity cost of
capital reflects the average risk of the resulting aggregate.

However, suppose we ask what depreciation tax shields are worth by themselves. For a firm
that’s sure to pay taxes, depreciation tax shields are a safe, nominal flow. Therefore, they should
be discounted at the firm’s after-tax borrowing rate.

Suppose we buy an asset with a depreciable basis of $200,000, which can be depreciated by
the five-year tax depreciation schedule (see Table 6.4). The resulting tax shields are:

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage deductions</td>
<td>20</td>
<td>32</td>
<td>19.2</td>
<td>11.5</td>
<td>11.5</td>
<td>5.8</td>
</tr>
<tr>
<td>Dollar deductions (thousands)</td>
<td>$40</td>
<td>$64</td>
<td>$38.4</td>
<td>$23</td>
<td>$23</td>
<td>$11.6</td>
</tr>
<tr>
<td>Tax shields at $T_c = .35 (thousands)</td>
<td>$14</td>
<td>$22.4</td>
<td>$13.4</td>
<td>$8.1</td>
<td>$8.1</td>
<td>$4.0</td>
</tr>
</tbody>
</table>

The after-tax discount rate is $r_D$($1 - T_c$) = .13(1 - .35) = .0845. (We continue to assume a 13%
pretax borrowing rate and a 35% marginal tax rate.) The present value of these shields is:

\[
PV = \frac{14}{1.0845} + \frac{22.4}{(1.0845)^2} + \frac{13.4}{(1.0845)^3} + \frac{8.1}{(1.0845)^4} + \frac{8.1}{(1.0845)^5} + \frac{4.0}{(1.0845)^6}
\]

\[
= +56.2, \text{ or } $56,200
\]

Borrowing and lending rates should not differ by much if the cash flows are truly safe, that is, if the chance of default is small. Usually your decision will not hinge on the rate used. If it does, ask which offsetting transaction—borrowing or lending—seems most natural and reasonable for the problem at hand. Then use the corresponding interest rate.

All the examples in this section are forward-looking; they call for the value today of a stream of future debt-equivalent cash flows. But similar issues arise in legal and contractual disputes when a past cash flow has to be brought forward in time to a present
value today. Suppose it’s determined that company A should have paid B $1 million 10 years ago. B clearly deserves more than $1 million today, because it has lost the time value of money. The time value of money should be expressed as an after-tax bor-
rowing or lending rate, or if no risk enters, as the after-tax risk-free rate. The time value of money is not equal to B’s overall cost of
capital. Allowing B to “earn” its overall cost of capital on the payment allows it to earn a risk premium without bearing risk. For
a broader discussion of these issues, see F. Fisher and C. Romaine, “Janis Joplin’s Yearbook and Theory of Damages,” Journal of
A Consistency Check  You may have wondered whether our procedure for valuing debt-equivalent cash flows is consistent with the WACC and APV approaches presented earlier in this chapter. Yes, it is consistent, as we will now illustrate.

Let’s look at another very simple numerical example. You are asked to value a $1 million payment to be received from a blue-chip company one year hence. After taxes at 35%, the cash inflow is $650,000. The payment is fixed by contract.

Since the contract generates a debt-equivalent flow, the opportunity cost of capital is the rate investors would demand on a one-year note issued by the blue-chip company, which happens to be 8%. For simplicity, we’ll assume this is your company’s borrowing rate too. Our valuation rule for debt-equivalent flows is therefore to discount at 

\[ r_D(1 - T) = .08(1 - .35) = .052 \]

\[ PV = \frac{650,000}{1.052} = \$617,900 \]

What is the debt capacity of this $650,000 payment? Exactly $617,900. Your company could borrow that amount and pay off the loan completely—principal and after-tax interest—with the $650,000 cash inflow. The debt capacity is 100% of the PV of the debt-equivalent cash flow.

If you think of it that way, our discount rate \( r_D(1 - T) \) is just a special case of WACC with a 100% debt ratio \( D/V = 1 \).

\[ WACC = r_D(1 - T)D/V + r_EE/V \]

\[ = r_D(1 - T) \text{ if } D/V = 1 \text{ and } E/V = 0 \]

Now let’s try an APV calculation. This is a two-part valuation. First, the $650,000 inflow is discounted at the opportunity cost of capital, 8%. Second, we add the present value of interest tax shields on debt supported by the project. Since the firm can borrow 100% of the cash flow’s value, the tax shield is \( r_D T \), APV, and APV is:

\[ APV = \frac{650,000}{1.08} + \frac{.08(.35) \text{APV}}{1.08} \]

Solving for APV, we get $617,900, the same answer we obtained by discounting at the after-tax borrowing rate. Thus our valuation rule for debt-equivalent flows is a special case of APV.

QUESTIONS

1. The U.S. government has settled a dispute with your company for $16 million. It is committed to pay this amount in exactly 12 months. However, your company will have to pay tax on the award at a marginal tax rate of 35%. What is the award worth? The one-year Treasury rate is 5.5%.

2. You are considering a five-year lease of office space for R&D personnel. Once signed, the lease cannot be canceled. It would commit your firm to six annual $100,000 payments, with the first payment due immediately. What is the present value of the lease if your company’s borrowing rate is 9% and its tax rate is 35%? (Note: The lease payments would be tax-deductible.)