Forward Markets and Transaction Exchange Risk

Commercial Mexicana, Mexico’s third largest retailer and a competitor of Walmart, sells many goods imported from the United States. Because Commercial’s revenues are in Mexican pesos, a strengthening of the dollar relative to the Mexican peso increases Commercial’s costs and lowers its earnings. In general, when the delivery of and payment for goods takes some time, future fluctuations in exchange rates give rise to potential losses, and possible gains, for the parties involved. The possibility of taking a loss in such a transaction is called transaction exchange risk.

In Chapter 2, we examined the organization of the spot foreign exchange market, in which the exchange of currencies typically happens in 2 business days. This chapter examines the forward foreign exchange market (or the forward market, for short). It is the market for exchanges of currencies in the future. One of the major reasons for the existence of forward markets is to manage foreign exchange risk in general and transaction exchange risk in particular.

The forward markets for foreign exchange allow corporations, such as Commercial Mexicana, to protect themselves against transaction exchange risks by hedging. To hedge against such risks, the corporation enters into an additional contract that provides profits when the underlying transaction produces losses. To evaluate the costs and benefits of hedging for a future transaction involving foreign currencies, the hedging party must have some way to quantify the degree of uncertainty it faces about future spot exchange rates. It accomplishes this by figuring out the likelihood of observing various ranges for future exchange rates.

Unfortunately, prior to the global financial crisis, Commercial Mexicana neither assessed nor hedged its transaction exchange risk properly. Instead, it dabbled excessively in complex foreign exchange derivatives contracts. As the dollar strengthened in the fall of 2008, Commercial lost $1.4 billion and was forced into bankruptcy. Numerous other companies throughout the developing world took enormous losses on foreign exchange contracts, including CITIC Pacific of Hong Kong, an infrastructure firm, which lost $1.89 billion, and Aracruz Celulose SA of Brazil, the world’s biggest eucalyptus pulp maker, which lost $0.92 billion.

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1This chapter studies the over-the-counter forward markets. The other type of market for the exchange of currencies in the future is the organized futures foreign exchange market, which is discussed in Chapter 20.
2In Chapter 17, we explore more generally why firms might want to hedge currency risk.
3See Euromoney (2008); many of these losses were related to option-like derivatives, which are also discussed in Chapter 20.
We begin the chapter by defining transaction exchange risk and continue by formalizing how to think about the uncertain future exchange rate movements that cause it. Next, we introduce forward contracts and discuss how transaction exchange risk can be hedged using these contracts. We then provide more details about the conventions and trading practices of the forward exchange market. Finally, we introduce the concept of a forward premium, which describes how forward rates are related to spot rates, a relationship that we will come back to many times throughout the book.

3.1 Transaction Exchange Risk

Corporations, institutional investors, and individuals incur transaction exchange risk if they enter into a transaction in which they are required to pay or to receive a specific amount of foreign currency at a particular date in the future. Because the future spot exchange rate cannot be known with certainty, and the exchange rate can move in an unfavorable direction, such a transaction could lead to a loss. Our next task is to determine the precise nature of the risks associated with these transactions.

Suppose Motorola, a U.S. firm, is importing some electronic equipment from Hitachi, a Japanese company. Motorola orders the equipment and promises to pay a certain amount of yen in, say, 90 days. Suppose that Motorola does nothing between the time that it enters into the transaction and the time that the payment of yen is scheduled to occur. Motorola consequently will be required to purchase the amount of yen that it owes Hitachi with dollars in the future spot market. If the dollar weakens unexpectedly relative to the yen, Motorola will end up paying more dollars than it expected to pay.

Analogously, suppose Oracle, a U.S. firm, exports some Sun SPARC Enterprise Servers to Europe and agrees to receive euro payments in the future, when it delivers the servers. If Oracle does nothing between the time that it enters into the contracts and the date of delivery and payment, Oracle will convert the euros into dollars in the future spot market. If the euro depreciates unexpectedly, Oracle will receive fewer dollars for the transaction than it had anticipated receiving.

Whenever you engage in an international financial transaction that involves an exchange of currencies in the future, you will almost always be unsure about what the spot exchange rate will be in the future when you conduct this transaction. This is true even under regimes of fixed exchange rates because political and economic events can always trigger devaluation or revaluation of the domestic currency relative to foreign currencies. Under the flexible exchange rate system that has characterized the foreign exchange markets for the major currencies for nearly 40 years, exchange rates fluctuate a good deal from day to day. As a financial manager, you must be able to gauge where the exchange rate might head and how likely such fluctuations may be. This range of possible future values for the exchange rate and the likelihood of their occurring will give you an idea of the foreign exchange risk your firm faces and whether it’s a good idea to hedge.

Often, people in corporations discuss the possibility or magnitude of a potential foreign exchange loss by valuing the foreign currency that is scheduled to be paid or received in the future at today’s spot exchange rate. However, this is not the proper way to think about transaction exchange risk unless there is no expected change to the exchange rate. The potential loss or the possible gain from uncertain future exchange rates is appropriately measured relative to the expected future spot rate.

To see why, let’s look at an example regarding transaction exchange rate risk at a fictitious company, Fancy Foods. We return to this example in the next section, after we have discussed how to formally describe uncertainty in future spot rates.
3.2 DESCRIBING UNCERTAIN FUTURE EXCHANGE RATES

To quantify the potential losses or gains due to a transaction exchange risk, we must think more about describing the uncertainty surrounding future spot exchange rates. Although we do not know exactly what value exchange rates will have in the future, we can quantify the possible changes that may occur and thus quantify how much risk we are bearing in international financial transactions. In doing so, we use some statistical concepts that you probably know, but if not, the appendix “A Statistics Refresher,” at the end of this chapter, should bring you up to speed.

Assessing Exchange Rate Uncertainty Using Historical Data

Historical data provide insight not only to what has happened in the past but what might happen in the future. Exhibit 3.1 presents a histogram of monthly percentage changes in the exchange rate of the U.S. dollar per British pound ($/£). The exhibit also superimposes on the graph a normal distribution curve, with the same mean and standard deviation as the data. We will explore this in more detail shortly.
The data in Exhibit 3.1 cover January 1975 to November 2010, or 431 observations. With the spot exchange rate at time \( t \) denoted \( S(t) \), the percentage change in the exchange rate between time \( t-1 \) and time \( t \) is

\[
s(t) = \frac{S(t) - S(t-1)}{S(t-1)}.
\]

Chapter 2 notes that these percentage rates of change are *appreciations* of the pound (if positive) and *depreciations* of the pound (if negative).

The horizontal axis in Exhibit 3.1 describes the percentage changes historically observed for the $/£ rate, which range from about \(-12\%\) to \(+14.5\%\). To create the histogram, we create ranges (bins) of equal width. The dots on the curve are the midpoints of the bins. The vertical axis represents the percentage frequency of occurrence of the rates of exchange rate change for each bin. The average (mean) monthly percentage change was \(-0.05\%\) for the dollar–pound. Because the mean “centers” the distribution, and because the distribution is bell shaped, observations near the mean are likely to occur. The standard deviation is a measure of the dispersion of possibilities *around* the mean. For the monthly percentage changes in the exchange

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**Notes:** We compute monthly percentage changes in the dollar–pound exchange rate as

\[
s(t) = \frac{S(t) - S(t-1)}{S(t-1)},
\]

where \( S(t) \) represents the exchange rate at time \( t \) (the end of a particular month). If \( s(t) \) is a negative (positive) number, the pound depreciated (appreciated) that month. The graph creates a histogram of the \( s(t) \) data. We consider small ranges (bins) of possible percentage changes (for example, between \(-0.167\%\) and \(0.167\%\)) and compute the number of observations within the bin. The dots on the graph represent the midpoint of the bin and its frequency (the number of observations divided by the total number of observations). The curve connecting them is the histogram. The smooth curve is the density corresponding to a normal distribution.
rates, the standard deviation was 3.03%. Exchange rate changes within 1 standard deviation of the mean (between \(-0.05\% - 3.03\% = -3.08\%\) and \(-0.05\% + 3.03\% = 2.98\%\)) occur more frequently than changes further away from the mean. For the curve in Exhibit 3.1, exchange rate changes 2 standard deviations away from the mean (either smaller than \(-0.05\% - (2 \times 3.03\%) = -6.12\%\) or larger than 6.01%) occurred very infrequently as the vertical distances become very small. For example, our detailed data reveal that exchange rate changes higher than 7.42% have occurred less than 1% of the time.

If we think that the histogram is a useful guide for the future, we can translate it into a probability distribution of future exchange rate changes. You have no doubt encountered probability distributions in other financial applications, such as describing the uncertainty regarding returns on investments in equity. Here, we use a probability distribution to summarize our ignorance about what will happen to future exchange rate changes.

The second curve in Exhibit 3.1 represents a normal probability distribution with the same mean and standard deviation as the historical data. Exhibit 3.1 reveals that the assumption of a normal distribution, characterized by its classic bell-shaped curve, is very reasonable for the dollar–pound rate, as it is for exchange rate changes between all major currencies for monthly rates of change. However, many emerging market currencies exhibit probability distributions that are distinctively non-normal. An example is Exhibit 3.2, which shows the distribution for monthly percentage changes of the Mexican peso relative to the U.S. dollar (MXN/USD) and the normal distribution with the same mean and standard deviation.

The historical distribution in Exhibit 3.2 is obviously not symmetric. Using historical data, we calculate a mean of 0.79% and a standard deviation of 4.76%. But, the most prominent feature of the historical distribution is the long right-hand tail. Statisticians say the distribution is skewed to the right. This indicates that large depreciations or devaluations of the peso relative to the dollar have occurred, and the absence of a large left-hand tail indicates

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**Exhibit 3.2** Peso/Dollar Monthly Exchange Rate: 1994–2010

![Graph showing the distribution of peso to dollar monthly exchange rates from 1994 to 2010. The graph compares the empirical data with the normal curve.]

*Notes:* We perform the same exercise as in Exhibit 3.1, but using peso per dollar exchange rates.
that there have been no analogously large appreciations or revaluations of the peso. Also, many more of the observations are centered around the mean (relative to the normal distribution), which was also true for the pound in Exhibit 3.1. This is always true when distributions have more observations in the tails (both left and right) than the normal, as the area underneath the distribution must add up to 1. This phenomenon is called “fat tails” or leptokurtosis. For now, you should remember that a normal probability distribution is a reasonable description of monthly percentage changes for the major floating currencies, but it is not a good description of emerging market currencies.

**The Probability Distribution of Future Exchange Rates**

Financial managers are also interested in the probability distribution of future spot exchange rates. Given that we observe an exchange rate of $S(t)$ today, we can find the probability distribution of future exchange rates in, say, 90 days from the probability distribution of the percentage change in the exchange rate. From Equation (3.1), we see that the possible future spot exchange rates are

$$S(t+90) = S(t) \times [1 + s(t+90)]$$

(3.2)

where $s(t+90)$ denotes the percentage change in the exchange rate over the next 90 days, $s(t+90) = [S(t+90) - S(t)]/S(t)$.

Exhibit 3.3 provides an example of a normal probability distribution for the dollar–pound spot exchange rate at time $t + 90$, which is 90 days in the future relative to today.

**Conditional Means and Volatilities**

Because the probability distribution of the future exchange rate depends on all the information available at time $t$, we say that it is a **conditional probability distribution** (see the appendix to this chapter). Consequently, the mean, which is the expected value of this distribution, is also referred to as the **conditional mean**, or the **conditional expectation**, of the

**Exhibit 3.3**  Probability Distribution of $S(t+90)$

![Normal probability distribution graph](image-url)
future exchange rate. Because the conditional expectation of the future exchange rate plays an important role in what is to follow, we use the following symbolic notation to represent it:

\[
\text{Conditional expectation at time } t \text{ of the future spot exchange rate at time } t + 90 = E_t[S(t+90)]
\]

One nice feature of the normal distribution is that the probability of any range of possible future exchange rates is completely summarized by its mean and the standard deviation, which is also often referred to as volatility. The conditional mean ties down the location of the probability distribution; the conditional standard deviation describes how spread out the distribution is. Notice that if the mean and the standard deviation of \( s(t+90) \) are denoted \( \mu \) and \( \sigma \), then from Equation (3.2), we see that the conditional mean and conditional standard deviation of \( S(t+90) \) are \( [S(t) (1 + \mu)] \) and \( [S(t) \sigma] \), respectively.

Let’s look at how Exhibit 3.3 is constructed. Suppose, as in Example 3.1, that the current exchange rate is $1.50/£, and that people expect the pound to appreciate relative to the dollar by 2% over the next 90 days. The conditional expectation of the future spot rate in 90 days is then $1.53/£ = (1.50/£) \times (1 + 0.02). Suppose that the standard deviation of the rate of appreciation over the next 90 days is 4%. Because 4% of $1.50/£ is $0.06/£, the standard deviation of the conditional distribution of the expected future spot exchange rate is $0.06/£. To summarize,

<table>
<thead>
<tr>
<th>Formula</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional expectation of the future exchange rate (mean)</td>
<td>( S(t) \times (1 + \mu) )</td>
</tr>
<tr>
<td>Conditional volatility of the future expected exchange rate (standard deviation)</td>
<td>( S(t) \times \sigma )</td>
</tr>
</tbody>
</table>

Armed with the conditional mean and conditional standard deviation of the future exchange rate, we can determine the probability that the future exchange rate will fall within any given range of exchange rates. For example, for the normal distribution, slightly more than two-thirds, or 68.27%, of the probability distribution is within plus or minus 1 standard deviation of the mean. In our example, this range is from

\[
$1.47/£ = $1.53/£ - $0.06/£
\]
to

\[
$1.59/£ = $1.53/£ + $0.06/£
\]

Consequently, the area under the curve between the two vertical lines emanating from $1.47/£ and $1.59/£ represents 68.27% of the total area. Also, for the normal distribution, 95.45% of the probability distribution is within plus or minus 2 standard deviations of the mean. Thus, the range of future exchange rates that encompasses all but 4.55% of the future possible values of dollar–pound exchange rates is $1.41/£ to $1.65/£.

**Assessing the Likelihood of Particular Future Exchange Rate Ranges**

Given a probability distribution of future exchange rates, we can also determine the probability that the exchange rate in the future will be greater or less than a particular future spot rate. For example, suppose we want to know how likely it is that the pound will strengthen over the next 90 days to at least an exchange rate of $1.60/£. Because $1.60/£ is $0.07/£ greater than the conditional mean of $1.53/£ and the standard deviation is $0.06/£, we want to know how likely it is that we will be 0.07/£ = 1.167 standard deviations above the
mean. For the normal distribution, this probability is 12.16%—that is, the probability of the exchange rate rising to $1.60/£ or higher from $1.50/£ is 12.16%.

Now that you can describe the possible changes in exchange rates that you may experience, you are in a better position to define and understand the concept of transaction exchange risk, so let’s revisit the Fancy Foods example.

**Example 3.2 Transaction Exchange Risk at Fancy Foods Revisited**

Fancy Foods must pay Porky Pies £1,000,000 in 90 days, and the current exchange rate is $1.50/£. The conditional distribution of future $/£ rates is based on the information that the firm has when it is making its decision. Let’s assume that the firm bases its decision on the probability distribution in Exhibit 3.3. Our calculations of the range of possible future exchange rates calculated earlier tell us that with 95.45% probability, the exchange rate will fall between $1.41/£ and $1.65/£. Hence, there is a 95.45% chance that Fancy Foods will pay between $1,410,000 = $1.41/£ × £1,000,000 and $1,650,000 = $1.65/£ × £1,000,000 to offset its pound liability. Remember that Fancy Foods expects to pay $1,530,000. If the dollar weakens to $1.65/£, we can think of Fancy Foods as losing

\[ 1,650,000 - 1,530,000 = 120,000, \]

compared to what it expected to pay. In contrast, if the dollar strengthens to $1.41/£, we can think of Fancy Foods as gaining

\[ 1,530,000 - 1,410,000 = 120,000, \]

compared to what it expected to pay. Of course, Fancy Foods is exposed to potentially larger losses and possibly bigger gains because something more extreme than this range of exchange rates could happen, but the probability of such extreme events is less than 4.55% if our probability distribution accurately reflects rational beliefs about the future.

### 3.3 Hedging Transaction Exchange Risk

Fancy Foods can totally eliminate the risk of loss due to a change in the exchange rate if it uses a *forward contract*. Let’s see why.

**Forward Contracts and Hedging**

A forward contract between a bank and a customer calls for delivery, at a fixed future date, of a specified amount of one currency against payment in another currency. The exchange rate specified in the contract, called the *forward rate*, is fixed at the time the parties enter into the contract. If you owe someone foreign currency at some date in the future, you can “buy the foreign currency forward” by contracting to have a bank deliver a specific amount of foreign currency to you on the date that you need it. At that time, you must pay the bank an amount of domestic currency equal to the forward rate (domestic currency per foreign currency) multiplied by the amount of foreign currency. Because the total amount you would owe the bank is determined today, it does not depend in any way on the actual value of the future exchange rate. Thus, using a forward contract eliminates transaction exchange risk.
Similarly, if you are scheduled to receive some foreign currency on a specific date in the future, you can “sell it forward” and entirely eliminate the foreign exchange risk. You contract to have the bank buy from you the amount of foreign currency you will receive in the future on that date in the future. Your forward contract establishes today the amount of domestic currency that you will receive in the future, which is equal to the forward exchange rate (domestic currency per foreign currency) multiplied by the amount of foreign currency you will be selling. The amount of domestic currency that you receive in the future consequently does not depend in any way on the future spot exchange rate.

Notice that in both cases, you have completely hedged your transaction exchange risk. Basically, you eliminate your risk by acquiring a foreign currency asset or liability that exactly offsets the foreign currency liability or asset that is given to you by your business.

**Hedging Currency Risk of Fancy Foods**

Consider again Example 3.1, in which Fancy Foods owes Porky Pies £1,000,000 in 90 days. Let the forward rate at which Fancy Foods can contract to buy and sell pounds be $1.53/£. Fancy Foods can wait to transact in 90 days, but it risks losing money if the pound strengthens against the dollar. Contracting with a bank in the forward market to buy £1,000,000 at $1.53/£ gives Fancy Foods a foreign currency asset that is equivalent to its foreign currency liability. Fancy Foods’s £1,000,000 liability from its business transaction is offset by a £1,000,000 asset, which is the bank’s promise to pay Fancy Foods on the forward contract. Fancy Foods is left with an offsetting dollar liability of $1,530,000 = ($1.53/£) × (£1,000,000). We can summarize this position using the asset and liability accounts on Fancy Foods’s balance sheet:

<table>
<thead>
<tr>
<th>Fancy Foods Partial Balance Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>£1,000,000 due from the bank in 90 days</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

**Hedging at Nancy Foods**

Now let’s consider Nancy Foods, which is scheduled to receive £1,000,000 from Quirky Pies in 90 days. The sale of the quiches gives Nancy Foods a foreign currency asset. Entering into a forward contract to sell £1,000,000 to the bank provides Nancy Foods with an equivalent foreign currency liability and a domestic currency asset. This hedges its foreign exchange risk. In this example, Nancy Foods’s asset and liability positions would look like this:

<table>
<thead>
<tr>
<th>Nancy Foods Partial Balance Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>£1,000,000 receivable from Quirky Pies in 90 days</td>
</tr>
<tr>
<td>$1,530,000 receivable from the bank in 90 days</td>
</tr>
</tbody>
</table>

These asset and liability accounts demonstrate that using forward contracts can turn the underlying British pound asset or liability that arises in the course of a U.S. firm’s normal business transactions into a dollar asset or liability that has no foreign exchange risk associated with it.


**Exposure of Hedged Versus Unhedged Strategies**

Exhibit 3.4 summarizes the exposures to transaction exchange risk of various strategies for buying or selling foreign currency. On the horizontal axis of Exhibit 3.4 (Panel A) are the future spot rates that can be realized in terms of the domestic currency (for example, dollars) per unit of foreign currency (for example, pounds). As you move to the right, the price of the foreign currency (pounds) in terms of the domestic currency (dollars) rises. In other words, the foreign currency is appreciating in value. On the vertical axis are the domestic currency costs per unit of foreign currency (if you must buy the foreign currency in the future) or the domestic currency revenue per unit of foreign currency (if you must sell the foreign currency in the future). Hence, we can represent the domestic currency revenue or cost of hedging or not hedging as a function of the actual value of the future spot exchange rate using simple lines.

**Exhibit 3.4  Gains and Losses Associated with Hedged Versus Unhedged Strategies**

Panel A: General Case

Panel B: Fancy Foods/Nancy Foods
The 45-degree line represents the unhedged strategy. If you must buy foreign currency in the future and you are unhedged, your cost will fluctuate one-for-one with the domestic currency price of foreign currency that is realized in the future. As the domestic currency weakens, your cost rises, and as the domestic currency strengthens, your cost declines. Your risk is unlimited in the sense that your cost keeps rising one-for-one with the future exchange rate. Conversely, your costs decline directly with any strengthening of the domestic currency relative to the foreign currency. Theoretically, your costs could fall to zero, although it’s highly unlikely that the domestic currency would strengthen to that extent.

The horizontal line in Exhibit 3.4 represents the strategy of hedging with a forward contract. If an international transaction requires you to buy foreign currency in the future, and you completely hedge by buying a forward contract today, your cost will be the same (equal to the forward rate) no matter what spot exchange rate is realized in the future. You bear no risk because the price you will pay is fixed, even if the domestic currency weakens relative to the foreign currency. But the price you pay also cannot decline if the domestic currency strengthens relative to the foreign currency.

In Panel B, we consider the cases of Fancy Foods and Nancy Foods. Suppose that after 90 days, when the contracts must be settled, the spot rate is $1.55/£. If the companies entered a forward contract at $1.53/£, this is entirely immaterial. Fancy Foods will avoid paying $1.55/£ as it has locked in $1.53/£, and Nancy Foods will receive only $1.53/£, even though it could have done better in the spot market by selling its pounds at $1.55/£.

The Costs and Benefits of a Forward Hedge

In light of the discussion of hedging transaction exchange risk, what is the appropriate way to think about the cost of a forward hedge? First, it is important to ascertain when the cost is computed. Are we looking ex post (after the fact) and examining whether we paid more or less with our forward contract than we would have paid had we waited to transact at the realized future spot rate? Or are we thinking of cost in an ex ante (before the fact) sense, in which case we have to examine the expected cost? In the latter case, you should remember that if you do not hedge, you will bear the foreign exchange risk, and the actual exchange rate at which you will transact in the future is very likely not going to be the expected future spot rate.

If you are buying foreign currency with domestic currency because your underlying transaction gives you a foreign currency liability, you will be glad to have hedged ex post if the future spot rate (domestic currency per foreign currency) is below the forward rate. You will have regrets ex post if the future spot rate is below the forward rate. These costs and benefits are summarized in Exhibit 3.5.

When you are trying to determine whether to hedge, how the forward rate relates to the expected future spot exchange rate dictates whether there is an expected cost or an expected benefit to hedging. If you are buying foreign currency because your underlying transaction gives you a foreign currency liability, you will think that there is an expected cost to hedging if the expected future spot rate of domestic currency per unit of foreign currency is below the forward rate (domestic currency per foreign currency). Hedging would require you to

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**Exhibit 3.5 Costs and Benefits of Hedging**

<table>
<thead>
<tr>
<th></th>
<th>( F(t, k) &lt; S(t+k) )</th>
<th>( F(t, k) &gt; S(t+k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign currency asset</td>
<td>Cost of hedging</td>
<td>Benefit of hedging</td>
</tr>
<tr>
<td>Foreign currency liability</td>
<td>Benefit of hedging</td>
<td>Cost of hedging</td>
</tr>
</tbody>
</table>

*Notes*: The spot rate and the forward rate are in domestic currency per unit of foreign currency. \( F(t, k) \) is the forward rate at time \( t \) for delivery at time \( t+k \). The costs/benefits are calculated ex post, after the realization of \( S(t+k) \). If we replace \( S(t+k) \) by \( E_t[S(t+k)] \), they become expected costs/benefits.
transact at a domestic currency price higher than you expect to have to pay if you do not hedge. Conversely, you will think there is an expected benefit to hedging if the expected future spot rate (domestic currency per foreign currency) is above the forward rate. In this case, hedging allows you to purchase foreign currency with domestic currency more cheaply than you would have expected to have to pay. Of course, complete hedging removes all potential benefits as well as all possible losses.

**Examples of Using Forward Contracts to Hedge Transaction Risk**

Let’s look at some examples to see the nature of different exposures, the extent of the possible losses, and how the exposures might be fully hedged with forward contracts.

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**Example 3.3 Hedging Import Payments**

Assume that you are the financial manager of Zachy’s, a wine store in Scarsdale, New York, that imports wine from France. You have just contracted to import some Chateau Margaux wine, and your invoice is for €4 million. You have agreed to pay this number of euros when you have received the wine and determined that it is in good condition. Payment of the euros and delivery of the wine are scheduled for 90 days in the future. The following data are available:

\[
\begin{align*}
\text{Today’s spot rate} & = \$1.10/\€ \\
\text{Today’s 90-day forward rate} & = \$1.08/\€ 
\end{align*}
\]

What is the source of your transaction exchange risk, and how much could you lose? First, as the U.S. importer, you have a euro-denominated liability because you have agreed to pay euros in the future. You are exposed to losses if the euro strengthens relative to the dollar unexpectedly to, say, \( \$1.12/\€ \). In this case, the dollar cost of the euros would be higher. If you do nothing to hedge your risk, your loss is theoretically unlimited in the sense that the dollar cost of the euros could go to infinity because the dollar amount that you will pay is \( S(t+90, $/\€) \times €4 \text{ million} \). Although this extreme loss is very unlikely, there is always some downside risk due to possible weakening, or depreciation, of the dollar relative to the euro.

You can eliminate the transaction exchange risk completely by buying €4 million in the forward market. The dollars that will be paid in 90 days are

\[
(€4,000,000) \times (1.08/\€) = $4,320,000
\]

Notice that the cash inflow of euros that you generate from the forward contract (€4,000,000) exactly matches the cash outflow of euros that you have from your underlying transaction. In other words, you have neutralized the euro liability that arises from your business by acquiring an equivalent euro asset, which is the promise by the bank to deliver euros to you. Hence, as long as you trust the bank that is your counterparty, you are not exposed to the risk of loss from fluctuations in exchange rates.

Of course, if you buy euros forward and the dollar strengthens substantially over the next 90 days (for example, to \$1.05/\€), you will still have to buy your euros from the bank at the forward price of $1.08/\€ because that is the price you agreed to in the contract with the bank. In this sense, the forward contract eliminates your risk of loss, but it does so by keeping you from participating in possible gains in the future.
Example 3.4  Hedging Export Receipts

Now, place yourself in the position of Shetland Sweaters, a British manufacturer. Consider your transaction exchange risk if you agree to ship sweaters to Japan and are willing to accept ¥500,000,000 in payment from the Japanese sweater importer Nobu Inc. Delivery of the goods and receipt of the yen are scheduled for 30 days from now, and the following data are available:

Today’s spot rate = ¥176/£

Today’s 30-day forward rate = ¥180/£

What are the nature and extent of your transaction exchange risk? Because you have agreed to accept yen in payment for your sweaters, you have a yen-denominated asset. You are exposed to losses if you wait to sell the yen in the future spot market and the yen depreciates, or weakens, unexpectedly relative to the pound. In this case, the yen you receive in payment for your sweaters will purchase fewer pounds than you expect. If you do nothing between the time you enter into the contract and the time you receive your yen, you risk everything in the sense that, theoretically, the pound value of your yen receivable could go to zero. Although that is very unlikely, there certainly is a downside risk due to a possible weakening of the yen relative to the pound. Of course, there is also a possible gain if the yen strengthens relative to the pound.

How can you fully hedge, or eliminate, this transaction risk from your business? You can eliminate the risk of loss by selling ¥500,000,000 in the forward market for pounds. The pounds that will be received in 30 days are

\[ ¥500,000,000 / (¥180/£) = £2,777,778 \]

Notice again that your contractual yen cash outflow (¥500,000,000) to pay the bank for the forward purchase of pounds in 30 days exactly matches the cash inflow of yen that you will have from your underlying transaction. You have neutralized the foreign exchange exposure of your business by acquiring a foreign currency liability that is exactly equivalent to your foreign currency asset. Your promise to deliver yen to the bank is your yen liability. Hence, as long as you are willing to trust that the bank will be able to deliver pounds to you in the future and that Nobu Inc. will pay yen for the goods, you are not exposed to risk of loss due to an unanticipated change in the exchange rate.

Of course, if the yen strengthens relative to the pound over the next 30 days, you will still have to sell your yen at the forward price specified by your agreement with the bank because the forward contract is not contingent on the future exchange rate. The rate is carved in stone, so to speak, by your contract with the bank. In this sense, the forward contract eliminates your risk of loss, but it does so by not allowing you to participate in possible gains in the future.

**Point–Counterpoint**

“Refining” a Hedging Strategy

With the Financial Times in hand, Ante Handel bursts into his brother’s room, shouting, “I told you non-financial companies should stay out of the forex markets! Another Japanese company has been pounded in the forward market. Kashima Oil has just announced a loss of ¥61.9 billion. At least it is only half the loss that other Japanese oil refinery, Showa Shell, had to swallow last year. I wonder what the stock market will think of this baby. Showa’s equity value dropped in half when the news of their foreign exchange loss broke!”
Ante’s brother, Freedy, responded surprisingly fast. “Come off it. Kashima is an oil refinery. They were just trying to hedge their currency risk. Oil is priced in dollars, and they were buying dollars in the forward market, and the exchange rate moved against them. It’s just bad luck. It could have gone the other way.”

Fortunately, their cousin, Suttle Trooth, had overheard everything through the thin walls of their dorm rooms, and he was intrigued. “This is not so simple,” he thought. “Should an oil company be hedging in the foreign exchange market? What really happened? Did they simply get a bad shock?” Rather than disturb the raucous discourse of the two brothers, Suttle put on his headphones, cranked up his iPod, and started searching the Internet. The facts soon became clear.

Suttle quickly learned that the Japanese oil refineries, Showa Shell and Kashima, are exposed to foreign exchange risk. All contracts in the oil business are settled in dollars, implying that these companies have dollar costs because they import crude oil, and they have yen revenues because they sell their refined oil in Japan. Showa Shell and Kashima face the risk that their yen costs will escalate if the dollar appreciates unexpectedly. To hedge that risk, both companies routinely buy dollars in the forward market for several months and sometimes years ahead. It happened to be the case that the forward yen price of the dollar was usually lower than the prevailing spot rate when most of these contracts were struck. So the forward contracts reduced the cost of the dollars relative to the prevailing spot rate and protected the companies against the risk of a dollar appreciation. However, the relevant comparison rate to judge the ex post benefit of the hedge is the future exchange rate at which crude oil would have been bought had the oil refineries not hedged. There were quite a few instances where the dollar did not appreciate relative to the yen; and, in fact, the actual yen price of the dollar in the future turned out to be lower than the forward rate the companies had agreed to. In such cases, the companies would have been better off, ex post, not to hedge. They would have had lower yen costs by buying the dollars they needed in the spot market with the stronger yen.

Unfortunately, as Suttle read on, he learned that these companies did not just hedge. People in the companies’ finance departments who were authorized to make forward contracts expected the dollar to appreciate. They thought they could profit from this outlook, and they agreed to forward contracts for much more than the actual currency exposure the companies had from their underlying oil businesses. In other words, people at both companies were speculating in an effort to make a profit! When the yen continued to appreciate and the speculators’ losses mounted, they did not disclose these losses to their superiors. They instead hid the losses from the companies’ accounting statements and simply entered into additional forward contracts with their banks, hoping that the yen would eventually fall in value. Showa’s total losses finally amounted to ¥125 billion and Kashima’s to ¥152.5 billion.

**Hedging Versus Speculating.** Suttle Trooth decided to analyze this case step by step. The first thing to do is to separate the hedging part from the speculation part. Pure speculation in the currency markets does not seem to be a great idea for any corporate finance department. In addition, not disclosing mounting losses to your shareholders is illegal in most countries. So on that part, Ante is right, Suttle mused. Kashima should not have dabbled in foreign exchange markets the way it did. Not surprisingly, Japan’s regulatory authorities cracked down on the practice of non-disclosure, and new disclosure rules regarding unrealized losses or profits from forward contracts in the foreign exchange markets were instituted in the wake of the oil companies’ debacles.

**To Hedge or Not to Hedge?** Now, Suttle wondered whether hedging made sense in this case. Why was Freedy so convinced this was absolutely a normal thing to do? Certainly, if Kashima has a number of contracts to buy oil in the future with dollars, and we view this as a source of transaction exposure, it makes sense to hedge, right? After all, Kashima
3.4 The Forward Foreign Exchange Market

Now that you understand how forward contracts can be used to manage foreign exchange risk, let’s examine the organization of the forward market in more detail.

Market Organization

The organization of trading for future purchase or delivery of foreign currency in the forward foreign exchange market is similar to the spot market discussed in Chapter 2. Whereas some traders focus on spot contracts, other traders focus on forward contracts. As mentioned previously, forward contracts greatly facilitate corporate risk management, and bank traders happily quote forward exchange rates for their corporate and institutional customers. However, such simple forward contracts, called outright forward contracts, are a relatively unimportant component of the foreign exchange market. In fact, a Bank for International Settlements (2010) survey found that only 12% of all transactions in the foreign exchange market are outright forward contracts. The survey also found that forward contracts are much more often part of a package deal, called a swap. In fact, about 44% of forex market transactions are swaps. A swap transaction involves the simultaneous purchase and sale of a certain amount of foreign currency for two different dates in the future. Given the importance of swaps, we discuss the swap market after we describe some of the details regarding the trading of forward contracts.
Forward Contract Maturities and Value Dates

Forward exchange rates are contractual prices, quoted today, at which trade will be conducted in the future. The parties agree to the price today, but no monies change hands until the maturity of the contract, which is called the forward value date, or forward settlement date.

The most active maturities in the forward market tend to be the even maturities of 30, 60, 90, and 180 days. Because the forward market is an over-the-counter market, however, it is possible for the corporate and institutional customers of banks and traders at other banks to arrange odd-date forward contracts with maturities of, say, 46 or 67 days.

The exchange of currencies in a forward contract takes place on the forward value date. Determination of the value date for a forward contract begins by finding today’s spot value date. As we saw in Chapter 2, this is 2 business days in the future for trades between U.S. dollars and European currencies or the Japanese yen. Exchange of monies in a 30-day forward contract occurs on the calendar day in the next month that corresponds to today’s spot value date, assuming that it is a legitimate business day. So, if today is July 28 and the spot value date is July 30, the forward value date for a 30-day contract is August 30. If the forward value date is a weekend or a bank holiday in either country, settlement of the forward contract occurs on the next business day. If the next business day moves the settlement of the forward contract into a new month, the forward value day becomes the previous business day. For example, in our previous example, it is possible that August 30 and 31 are weekend days. In that case, the value date would be August 29. This rule is followed except when the spot value day is the last business day of the current month, in which case the forward value day is the last business day of the next month (this is referred to as the end–end rule).

Let’s consider an example.

Example 3.5  Finding the Forward Value Date

Suppose we purchase euros with dollars in the spot market on Friday, November 11, 2011. The dollars will come from our Citibank account in New York, and the euros will be paid into our Deutsche Bank account in Germany. The spot value day for such a trade is Tuesday, November 15, 2011, a legitimate business day in both countries. If we also initiated a 30-day forward contract to buy euros with dollars on Friday, November 11, 2011, when would the exchange of currencies take place? We can find the forward value date by following the logic just described. Because the spot value date is November 15, 2011, the forward value date is Thursday, December 15, 2011, a legitimate business day in both countries. Notice that the exchange of currencies on the 30-day forward contract is actually 34 days in the future in this example.

Of course, you don’t have to actually own the currency that you contract to deliver when entering into a forward contract. It may be that you expect to receive the currency in the future in the normal course of your business, or you may plan to acquire the currency in the spot market sometime between when the forward contract is made and when the exchange of monies takes place on the forward value date. Suppose you have contracted to deliver euros as part of a forward contract (as in the previous example), but you do not own any euros. When is the last day that you could purchase euros in the spot market? We know that you must have euros on Thursday, December 15, 2011. Thus, you could buy the euros in the spot market 2 business days before this day, or on Tuesday, December 13, 2011, which is 32 days in the future relative to the date the forward contract was initiated.
**Forward Market Bid–Ask Spreads**

We noted in Chapter 2 that bid–ask spreads are quite narrow in the spot market. In the forward market, however, they tend to widen as the maturity of the forward contract increases. Yet forward bid–ask spreads for active maturities remain small and are typically less than 0.05% for the major currencies. In particular, for 90-day forward contracts, spreads are mostly less than a pip wider than the spot spread. For very long-dated contracts, especially extending beyond 1 year, bid–ask spreads are wider.4

**Liquidity in the Forward Market**

The bid–ask spreads are larger in the forward market than in the spot market because the forward market is less liquid than the spot market. Liquid markets allow traders to buy and sell something without incurring large transaction costs and without significantly influencing the market price. The liquidity of the market depends on the number of people who are actively trading in the market and on the sizes of the positions they are willing to take. In very liquid markets, it is easy to find a buyer if you want to be a seller and vice versa. It is also easy to conduct large transactions without having to provide concessions to the party taking the opposite side of the transaction. Illiquid markets are sometimes referred to as thin markets.

The reasons forward markets are less liquid than spot markets are subtle and are best explained in the context of an example.

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**Example 3.6 The Source of Low Liquidity in the Forward Market**

Suppose Canada Beer, a Canadian company, exports beer to the United States and receives regular payments in U.S. dollars. Suppose Canada Beer enters into a 30-day forward contract with Bank of America to sell USD1,000,000 in exchange for Canadian dollars. That is, Canada Beer is selling its dollar revenues forward for Canadian dollars. Assume that the forward rate is $0.90/CAD. We are interested in seeing what risk this transaction creates for Bank of America. Consider Panel A in Exhibit 3.6.

The forward contract implies that Bank of America is now short Canadian dollars in the forward market—that is, it owes Canadian dollars for future delivery. Conversely, in the forward contract, Canada Beer is long Canadian dollars and short U.S. dollars, but Canada Beer expects to receive U.S. dollar revenues from its beer sales, which hedges this position.

What are the risks involved for Bank of America? The most obvious risk is currency risk. In 30 days, Bank of America must deliver CAD1,111,111 = $1,000,000/($0.90/CAD) to Canada Beer in exchange for $1,000,000. In the meantime, the Canadian dollar may increase in value relative to the U.S. dollar, yet Bank of America will receive only the $1,000,000 specified in the forward contract. For example, suppose the spot exchange rate in 30 days moves up to $1.00/CAD. Then the cost of CAD1,111,111 would be $1,111,111, not the $1,000,000 Bank of America is receiving!

It is tempting to think that this position carries more transactions exchange risk than a spot position with delivery 2 days from now because adverse exchange rate movements are more likely over the longer time span. Although it is true that the size of possible adverse exchange rate movements increases over the longer time span,

---

4The relatively high transaction costs in the long-term forward market contributed to the development of an entirely new market, the long-term currency swap market, which is discussed in Chapter 21.
the forward position does not pose a larger currency risk than the spot position as long as the forward market is liquid enough to allow a fast reversal of the forward position. That is, if Bank of America thinks that it may take a loss on the forward contract because of an adverse movement in the Canadian dollar exchange rate, the bank will want to close its position by buying Canadian dollars forward for the remaining life of the contract. Let’s reconsider Exhibit 3.6. In Case 1 (Panel B), Bank of America waits 1 day and sees the spot rate increase. It suddenly feels that the risk of a short position in Canadian dollars is not worth taking and goes long Canadian dollars in the interbank market with a 29-day contract. We assume that the forward rate for this contract is $0.92/CAD, making the dollar equivalent of CAD1,111,111 equal to
There are two main reasons why forward markets are less liquid than spot markets. First, banks are exposed to counterparty default risk for a much longer time interval in a forward contract than in a spot contract. In fact, banks are so worried about counterparty default risk in forward contracts that they impose limits on the total magnitude of the contracts (the “positions”) traders can enter into with their counterparty banks in the interbank market. The limits vary with the creditworthiness and reputation of the other trading bank. In retail transactions, the dealer bank also often requires the non-bank counterparty either to maintain a minimum deposit balance with the dealer bank, to accept a reduction in its normal credit line, or to provide some other form of collateral. Second, because increased counterparty default risk reduces the number of forward transactions banks are willing to do, banks find it more difficult to manage open positions in forward contracts. Because it may take longer to find a counterparty with whom to trade at reasonable prices, forward contracts are more susceptible to foreign exchange risk. The increased inventory risk reduces liquidity even more.

Given these concerns, the lack of liquidity in the interbank forward market and the resulting increase in bid–ask spreads are not so surprising. In addition, some contracts are less heavily traded than others and are therefore less liquid. As a result, the bid–ask spread for these contracts is greater. Odd-maturity forward contracts—that is, contracts that do not have standard value dates 30, 60, or 90 days in the future—are an example.
The 2008 Global Financial Crisis and Forward Market Bid–Ask Spreads

The role of counterparty risk and inventory risk in driving the bid–ask spreads of forward contracts became painfully obvious during the 2008 global financial crisis. When Lehman Brothers declared bankruptcy in September of 2008, there was no longer any doubt that there was substantial credit risk attached to dealing even with major financial institutions. Not only did the volatility of exchange rate changes increase substantially, but so did bid–ask spreads. Bid–ask spreads on spot contracts for the major currencies increased by about 400%. However, the spreads on forward contracts widened much more than the spreads on spot contracts. Three-month forward contract spreads were double those of spot contracts, instead of being just fractionally higher. Foreign exchange dealers did not want to be exposed to counterparties with questionable credit risk for a full 3 months.

Net Settlement

Most outright forward contracts are settled by payment and delivery of the amounts in the contract. It is possible, however, to settle a contract by paying or receiving a net settlement amount that depends on the value of the contract. For example, suppose you think you will owe a Mexican company MXN20,000,000 in 30 days, and you would like to pay with dollars. You could enter into a forward contract to purchase MXN20,000,000 with dollars at a forward rate of, say, MXN10/USD. On the settlement day of the forward contract, you could expect to receive MXN20,000,000 from the bank and expect to pay $2 million for it:

\[
\text{MXN20,000,000} \times \frac{1}{\text{MXN10/USD}} = \text{USD2,000,000}
\]

Suppose that 1 business day before the forward value date, the spot exchange rate is MXN12/USD, and you learn that you no longer need to purchase MXN20,000,000 because the underlying transaction has been cancelled. Must you still follow through with the forward contract, paying the USD2 million and receiving the MXN20,000,000 that you will now have to sell for dollars? It turns out that the bank will let you make a net payment. Notice that the MXN20,000,000 is now worth only

\[
\frac{\text{MXN20,000,000}}{\text{MXN12/USD}} = \text{USD1,666,667}
\]

Hence, if you pay the bank

\[
\text{USD2,000,000} - \text{USD1,666,667} = \text{USD333,333}
\]

this is equivalent to carrying out the original transaction and then entering into a new spot transaction in which you immediately sell the MXN20,000,000 back to the original seller of pesos at the current spot rate.

Net settlement is often used in the forex futures market, which we discuss in Chapter 20, and for emerging market currencies. In many emerging markets, there are capital controls in place, making it more difficult to trade foreign exchange for non-residents. Foreign exchange dealers have responded by developing offshore markets in forward contracts that do not require physical delivery of currency but are cash settled, mostly in U.S. dollars. These non-deliverable forward contracts (NDFs) have become an important market segment for currencies such as the Korean won, the Chinese yuan, the Indian rupee, the Brazilian real, and the Russia ruble. EBS now even offers electronically traded NDFs in over 10 currencies.

\[5\] See Melvin and Taylor (2009) for further details on this topic.
The Foreign Exchange Swap Market

Most of the trading of forward contracts happens in the swap market. We now discuss in more detail what swap contracts are, how swap rates are quoted, and why swaps are so popular.

A swap simultaneously combines two foreign exchange transactions with different value dates but in opposite directions. The most common example of a swap is the combination of a spot and a forward contract, for example, buying the foreign currency spot against purchasing the foreign currency forward. Other swaps involve the purchase (sale) of foreign currency short-term forward against the sale (purchase) of foreign currency long-term forward. The main reason swaps are so popular is that simultaneous spot and forward transactions in opposite directions occur quite naturally. In Chapter 6, we discuss interest rate arbitrage, and we show that arbitrage transactions in the money markets across two countries involve spot and forward transactions in opposite directions. Similarly, in Part IV, we discuss investments in international bond and equity markets. Many portfolio managers want to invest in the bond and equity markets of foreign countries without being exposed to changes in the values of those countries’ currencies. To buy a foreign equity, these people must first buy the foreign currency in the spot market. To hedge the currency risk, they sell that currency forward. Hence, it is again natural to combine the spot and forward transaction in one trade.

Banks also actively use swaps to manage the maturity structure of their currency exposure. If they think they have too much exposure at one particular maturity, they can conveniently switch their position to another maturity, using a single swap transaction without changing their overall exposure to that currency. For example, when a bank has a short Swiss franc position of CHF1,000,000 (that is, when it sold CHF1,000,000 forward for dollars) with a maturity of 180 days and would like to shorten the maturity of these contracts to 90 days, it can simply enter into a swap to buy CHF1,000,000 at a 180-day value date and sell CHF1,000,000 at a 90-day value date. Because of the existence of the swap market, these transactions can be carried out with one phone call to a swap trader.

How Swap Prices Are Quoted

Before we examine the details of the cash flows associated with a swap, let’s look at how prices are quoted. We focus on swaps involving a spot transaction and a forward transaction. The following is an example of a swap quote:

<table>
<thead>
<tr>
<th>Spot</th>
<th>30-day</th>
</tr>
</thead>
<tbody>
<tr>
<td>¥/$ 104.30–35</td>
<td>15/20</td>
</tr>
</tbody>
</table>

A quote mentions the spot rates (first column) and the swap points (second column). The spot rates quoted by a bank in this example are ¥104.30/$ bid and ¥104.35/$ ask. Remember that the bank’s bid price is the rate at which the bank buys dollars from someone in exchange for yen. In contrast, the bank’s ask or offer price is the rate at which the bank sells dollars to someone and receives yen from them. The swap points are a set of pips that must be either added to or subtracted from the current spot bid and ask prices to yield the actual 30-day bid and ask forward prices.

A Rule for Using Swap Points

A confusing aspect of moving from swap quotes to outright forward quotes is knowing whether to add the swap points to or to subtract the points from the bid and ask prices. Here’s the rule: If the first number in the swap quote is smaller than the second, you add the points to the spot bid and ask prices to get the outright forward quotes; if the first number in the swap points is larger than the second, you subtract the points.
Let’s examine the logic behind this rule, using the sample prices. With the swap points quoted as 15/20, the points should be added, so the outright forward quotes for 30 days would be

\[ ¥104.30/\$ \text{ spot bid} + ¥0.15/\$ = ¥104.45/\$ \text{ forward bid for dollars} \]

and

\[ ¥104.35/\$ \text{ spot ask} + ¥0.20/\$ = ¥104.55/\$ \text{ forward ask for dollars} \]

Notice that adding the swap points in this case makes the bid–ask spread in the forward market larger than the bid–ask spread in the spot market, which it should be.

When the first swap point quote is larger than the second, the points must be subtracted. Traders could quote negative numbers to indicate subtraction, but they follow a different convention. Rather than quote negative numbers when they want to indicate that the forward exchange rates are less than the spot prices, traders are assumed to understand that a swap quote of, say, 20/15 indicates that the swap points must be subtracted from the spot bid and ask rates. In this second example, the outright forward quotes for 30 days would be

\[ ¥104.30/\$ \text{ spot bid} - ¥0.20/\$ = ¥104.10/\$ \text{ forward bid for dollars} \]

and

\[ ¥104.35/\$ \text{ spot ask} - ¥0.15/\$ = ¥104.20/\$ \text{ forward ask for dollars} \]

Notice that in both of these examples, the bid–ask spread in the forward market is 10 points (or pips), which is larger than the 5-point spread in the spot market. If we had, in error, added the points in the second example, the forward market bid–ask spread would have fallen to 0 points, which is less than the 5-point spot bid–ask spread. This would tell us that we made an error because we know that the forward market is less liquid than the spot market. Hence, if you are having trouble remembering the rule and are trying to determine whether to add the swap points or to subtract them, you can always check to make sure that the forward bid–ask spread is larger than the spot bid–ask spread.

**Cash Flows in a Swap**

Let’s consider an example of a swap to see what the cash flows look like.

---

**Example 3.7 Swapping Out of Dollars and into Yen**

Nomura, a Japanese investment bank, quotes the spot rates ¥104.30/\$ bid and ¥104.35/\$ ask and swap points of 20/15. Suppose that IBM wants to swap out of $10,000,000 and into yen for 30 days. To do so, IBM sells dollars in the spot market in exchange for yen, but also wants to buy dollars for yen 30 days from now using a forward transaction. Both transactions can be combined in a swap. IBM swaps out of $10,000,000 and into an equivalent amount of yen for 30 days. The swap diagram in Exhibit 3.7 summarizes the cash flows for both IBM and Nomura.

IBM is selling $10,000,000 to Nomura in the spot market. Consequently, the amount of yen IBM receives is determined by Nomura’s spot bid rate of ¥104.30/\$. In the first leg of the swap, IBM would receive

\[ $10,000,000 \times (¥104.30/\$) = ¥1,043,000,000 \]
When IBM gets its $10,000,000 back in 30 days, how many yen will it have to pay the bank? Because in the future Nomura is selling dollars to IBM for yen, Nomura will charge its forward ask price of \( ¥104.20/\$( ¥104.35/\$ - ¥0.15/\$ ) \). Hence, IBM will pay Nomura

\[
10,000,000 \times (¥104.20/\$) = ¥1,042,000,000
\]

Hence, IBM gives up $10,000,000 for 30 days, and it receives ¥1,043,000,000 for 30 days. Nomura receives $10,000,000 for 30 days and in exchange gives up the use of ¥1,043,000,000. At the swap contract’s maturity, IBM has to give Nomura only ¥1,042,000,000 rather than the original ¥1,043,000,000, which means that IBM gets to keep

\[
1,043,000,000 - 1,042,000,000 = ¥1,000,000
\]

Why is Nomura willing to accept ¥1,000,000 less in return when it buys $10,000,000 from IBM for 30 days? The answer is related to the interest rates on the two currencies.

Fundamentally, in a swap, each party is giving up the use of one currency and gaining the use of a different currency for the period of time of the swap. The two parties could charge each other the going market rates of interest on the respective currencies for this privilege. Instead of doing this, however, swaps are priced so that the party that is borrowing the high-interest-rate currency pays the party that is borrowing the low-interest-rate currency the difference in basis points. We will see in Chapter 6 precisely how the swap rates are related to the interest differential between the two currencies. Here we merely note that the yen must be the low-interest-rate currency relative to the dollar in this example because IBM had the use of yen while Nomura had the use of dollars, and IBM paid Nomura less yen in the future than the amount of yen Nomura paid IBM for its use of the dollars.

### 3.5 Forward Premiums and Discounts

Now that you understand how forward contracts are traded, it is time to introduce some important terminology regarding the relationship between forward and spot exchange rates.
If the forward price of the euro in terms of dollars (that is, USD/EUR) is higher than the spot price of USD/EUR, the euro is said to be at a **forward premium** in terms of the dollar. Conversely, if the forward price of the euro in terms of dollars (USD/EUR) is less than the spot price of USD/EUR, the euro is said to be at a **forward discount** in terms of the dollar. Remember, as with the terms **appreciation** and **depreciation**, the terms *forward premium* and *forward discount* refer to the currency that is in the denominator of the exchange rate.

Because the forward premium and forward discount are related to the interest rates on the two currencies, these premiums and discounts are often expressed as annualized *percentages*. That is, the difference between the forward rate and the spot rate is divided by the spot rate and then multiplied by the reciprocal of the fraction of the year over which the forward contract is made. The result is then multiplied by 100 to convert it to a percentage:

\[
\% \text{ per annum forward premium or discount of an } N \text{ day forward rate} = \left( \frac{\text{forward} - \text{spot}}{\text{spot}} \right) \times \left( \frac{360}{N \text{ days}} \right) \times 100
\]  

(3.3)

Here, \( N \) is the number of days in the forward contract. A 360-day year is used for most currencies, corresponding to the conventions for quoting interest rates. Exceptions to this convention include the British pound and the Kuwaiti dinar, which are quoted on a 365-day year.

We explore the formal linkage between the forward premium or discount and the interest differential between the two currencies in Chapter 6. Intuitively, however, you should realize that there must be a strong link among the spot rate (the relative price of two monies for immediate trade), the forward rate (the relative price of two monies for trade at a future date), and the two interest rates, which are the time values of the two monies between today and the future date.

### Sizes of Forward Premiums or Discounts

Exhibit 3.8 presents some information on historical forward premiums and discounts for several of the major currencies versus the dollar. We use the Deutsche mark to fill in data for the euro prior to 1999.

Both for 30-day and 90-day yen–dollar contracts, the average forward premium is negative. In other words, on average, the dollar traded at a discount in the forward market versus the yen. The yen-denominated forward prices of the dollar were about 2.8% lower than the spot prices. For the euro and the pound, the exchange rates are expressed in $ per € and $ per £. For the dollar–euro rates, the 30-day forward premium of 1.046% indicates that the euro was at a premium versus the dollar, and the negative values for the dollar–pound rates indicate that the pound traded at a forward discount relative to the dollar. The discount was 1.649% for 30-day forward contracts and 1.541% for 90-day contracts. These numbers only represent averages (the means) because the forward discount changes over time. For example, Exhibit 3.8 shows that in 2010, the pound and the euro traded at small discounts relative to the dollar, whereas the dollar traded at a historically low discount of 0.399% relative to the yen.

### Forward Premiums and Swap Points

Because forward contracts are typically traded as part of a swap, the swap points tell us whether the denominator currency is at a premium or a discount. Consider the example given using the JPY/USD exchange rate. If the dollar is at a forward premium, it is more expensive to purchase dollars in the future, so the forward rate should be larger than the spot rate. This happens if the swap points are added to the spot rates to yield larger forward rates. Hence,
when the first number in the swap points is less than the second number, as in our earlier example of 15/20, the swap points should be added, and the currency in the denominator is at a premium. If there is a discount on the dollar, the first number in the swap price will be greater than the second number, as in the second example of 20/15, and the swap points should be subtracted.

In the swap in Example 3.7, the dollar is at a discount relative to the yen because the forward rate of yen per dollar is smaller than the spot rate (the swap points were subtracted from the spot rate). In this example, IBM sold USD10,000,000 at the spot bid and bought them at the forward ask. Because of the forward discount on the dollar, the example involves an additional negative yen cash flow at maturity for Nomura because the bank bought dollars in the spot market, and the dollar is the high-interest-rate currency. That is, Nomura gets less yen back than it paid to IBM to begin with. Thus, Nomura is said to be “paying the points,” or “dealing against oneself.” Conversely, because IBM gave up the use of the high-interest currency (dollars) for the use of the low-interest currency (the yen), it is said to be “earning the points,” or “dealing in its favor.” Consequently, if the dollar is at a discount, swapping out of dollars today and into yen generates a positive yen cash flow. A good rule to remember is that swapping into the currency that is at a premium generates a positive cash flow.

### 3.6 Changes in Exchange Rate Volatility (Advanced)

To judge the extent of transaction exchange risk, understanding volatility is critical. The wider the conditional distribution of future exchange rates, the higher is your risk; and the width of the distribution in turn depends on the volatility or standard deviation of changes in exchange rates. Exhibit 3.1 uses information from several different decades to graph the probability distribution of monthly changes in the $/£ rate. But what if volatility has increased (decreased) over time? In this case, using a probability distribution based on a historical standard deviation underestimates (overestimates) the true uncertainty about future exchange rates.

#### Volatility Clustering

Many financial researchers have spent considerable computer time examining exchange rate data, and they have come to the conclusion that exchange rate volatility is not constant over time. In fact, as is true for the returns on many assets, percentage changes in exchange rates
show a pattern known as volatility clustering. When volatility is high, it tends to remain high for a while; periods of low volatility are likewise persistent. Asset markets in general, and the foreign exchange market in particular, appear to go through periods of tranquility and periods of turbulence. To illustrate this pattern, we use daily data on the dollar/pound exchange rate to compute monthly standard deviations. That is, for each month in our sample, we use the available daily observations to compute the sample standard deviation for each month. Exhibit 3.9 plots these monthly standard deviations.

The graph clearly reveals quiet periods (for example, 1977 to 1979 or 1999) and turbulent periods (for example, 1985 and 1991 to 1993) during which volatility exceeded 20% at times. The most volatile period of all is the autumn of 2008, in particular, October 2008, when volatility in both equity and foreign exchange markets reached unprecedented heights during the crisis.

A number of models have been developed to fit the observed pattern of volatility clustering in these data. The most popular model to date is the GARCH model developed by Bollerslev (1986). Remember that the squared value of the volatility is the variance. Let $\nu$ denote the variance. The relevant variance for assessing our uncertainty about future exchange rate changes is the conditional variance, $\nu(t) = \text{var}[s(t+1)]$ (see the appendix to this chapter). Let us denote the deviation of the actual percentage change in the exchange rate from its conditional expectation by $e(t) = s(t) - E_{t-1}[s(t)]$. We can interpret $e(t)$ as an economic shock that represents “news” because that part of the exchange rate change was not expected to occur. For example, suppose you expected the exchange rate change

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Notes: We obtained daily changes in the $\$/£ exchange rate from Datastream and computed the sample volatility for each month using these daily percentage changes. The graph plots these volatilities, annualized to be comparable to the way volatilities are plotted in financial markets. The data span January 1, 1975, through November 30, 2010.

6GARCH is an acronym that stands for Generalized Auto-Regressive Conditional Heteroskedasticity. A conditionally heteroskedastic time series does not have a constant variance. The future of an auto-regressive process depends on its own past. You will be happy to know that other models of conditional heteroskedasticity, such as SPARCH, QGARCH, and FIGARCH models, are gaining in popularity, but we will not discuss them here. A precursor to the GARCH model was Robert Engle’s ARCH model, for which Engle won the Nobel Prize in Economics in 2003.
over the past month to be 5%, but it was actually 7%. The additional 2% change is “news” to you; it is an unexpected change in the exchange rate. The GARCH model for the conditional variance is

\[ v(t) = a + bv(t-1) + ce(t)^2 \]

The constants \(a\), \(b\), and \(c\) are parameters that can be estimated from the data; \(b\) reflects the sensitivity of the current conditional variance to the past conditional variance; \(c\) reflects its sensitivity to current news; and \(a\) is the minimum variance we would predict even if the past volatility and news terms are zero. Depending on the frequency of the data, \(b\) is between 0.85 and 0.95, and \(c\) is much lower (for example, between 0.05 and 0.15) (see Baillie and Bollerslev, 1989).

This model accommodates persistence in volatility. If the conditional variance is high today, it is likely to be high tomorrow. This persistence in \(v(t)\) can generate the patterns of volatility clustering we see in the data. If we are in a quiet period today, but the exchange rate suddenly and unexpectedly moves in either direction, volatility immediately shifts to a higher level for a while through the \(e^2\) term. This shift will tend to persist because of the feedback the model allows through the \(bv(t-1)\) term. That is, because \(v(t)\) is now higher, \(v(t+1)\) will be higher as well because \(b\) is positive. Let’s illustrate this positive feedback effect with an example.

**Example 3.8 Positive Feedback in Volatility**

Suppose last month’s dollar–euro exchange rate stood at \$1.20/€, and the market expected no change for the next month. However, after a number of opaque statements by the policy makers in Europe, the euro has weakened to \$1.08/€. Note that this depreciation of the euro, \(s(t) = \frac{1.08 - 1.20}{1.20} = -0.10\), is unexpected, and hence it constitutes news [an \(e(t)\)-shock]. What does the GARCH model predict next month’s currency volatility to be, assuming that \(a = 0.00072\), \(b = 0.90\), and \(c = 0.05\) and the previous market volatility of the \$/€ rate of depreciation was 8.0%? The GARCH model predicts \(v(t)\), according to

\[ v(t) = a + bv(t-1) + ce(t)^2 \]

\[ = 0.00072 + 0.90(0.08)^2 + 0.05(-0.10)^2 = 0.00698 \]

Hence, volatility today, which is the square root of \(v_t\), is 8.35% (\(\sqrt{0.00698}\)). The large unexpected depreciation drives up volatility by 0.35%. Whatever the “shock” next month, today’s volatility increase will tend to persist. If the GARCH model is correct, next month’s volatility will be \(v(t+1) = 0.00072 + 0.90(0.0835)^2 + 0.05(e(t+1))^2\). Today, we do not know what next month’s shock will be, but we assign a high weight \((b = 0.90)\) to this period’s higher volatility in computing next period’s volatility. This is why we say the coefficient \(b\) implies positive feedback, or persistence, for the volatility process. Not everyone is convinced that GARCH is the right volatility model, but alternative models are beyond the scope of this book. Although some of these statistical models capture the volatility patterns well, they do not tell us why volatility moves the way it does. Possibilities include the clustering of macroeconomic news events (see Andersen and Bollerslev, 1998), the reaction of risk-averse agents to small changes in uncertainty regarding macroeconomic fundamentals (see Bekaert, 1996; and Hodrick, 1989), and the trading process itself (see Laux and Ng, 1993).
3.7 Summary

The purpose of this chapter is to introduce forward foreign exchange markets and to examine their use in hedging transaction exchange risk. The following are the main points in the chapter:

1. A transaction exchange risk arises when an individual or a firm enters into a transaction in which it is required to receive or pay a specific amount of foreign currency at some date in the future. If the firm does nothing to hedge the risk, there is a possibility that the firm will incur a loss if the exchange rate moves in an unfavorable direction.

2. One can fully hedge a transaction exchange risk by either buying or selling foreign currency in the forward foreign exchange market. If you are importing (exporting) goods and will contractually owe (receive) foreign currency, you have a foreign currency–denominated liability (asset) and must acquire an equivalent foreign currency–denominated asset (liability) to be hedged. Buying (selling) foreign currency from (to) the bank in the forward market provides the hedge.

3. Outright forward exchange rates are contractual prices at which trade will be conducted in the future. The parties agree to the price today, but no currencies change hands until the maturity, or value, date in the future.

4. Bid–ask spreads in the forward market are larger than in the spot market because the forward market is less liquid.

5. Forward contracts are sometimes cash settled, especially for emerging markets with foreign exchange trading restrictions ("non-deliverable forwards").

6. A swap involves the simultaneous purchase and sale of a certain amount of foreign currency for two different value dates. Traders quote swap rates as the number of pips that must be either added to the spot bid and ask rates or subtracted from the spot rates. When the points must be added, they are quoted with the smaller number first, and when they must be subtracted, they are quoted with the smaller number second. This ensures that the bid–ask spread in the forward market is always larger than the spread in the spot market.

7. If the forward price of a currency is higher than the spot price, that currency is said to be trading at a forward premium. If the forward price of a currency is lower than the spot price, that currency is said to be trading at a forward discount.

8. The extent of transaction exchange risk is proportional to the (conditional) volatility of exchange rate changes. This volatility changes over time.

Questions

1. What is a forward exchange rate? When does delivery occur on a 90-day forward contract?

2. If the yen is selling at a premium relative to the euro in the forward market, is the forward price of EUR per JPY larger or smaller than the spot price of EUR per JPY?

3. What do we mean by the expected future spot rate?

4. How much of the probability distribution of future spot rates is between plus or minus 2 standard deviations?

5. If you are a U.S. firm and owe someone ¥10,000,000 in 180 days, what is your transaction exchange risk?

6. What is a spot–forward swap?

7. What is a forward–forward swap?

Problems

1. If the spot exchange rate of the yen relative to the dollar is ¥105.75/$, and the 90-day forward rate is ¥103.25/$, is the dollar at a forward premium or discount? Express the premium or discount as a percentage per annum for a 360-day year.

2. Suppose today is Tuesday, January 18, 2011. If you enter into a 30-day forward contract to purchase euros, when will you pay your dollars and receive your euros? (Hints: February 18, 2011, is a Friday, and the following Monday is a holiday.)

3. As a foreign exchange trader for JPMorgan Chase, you have just called a trader at UBS to get quotes for the British pound for the spot, 30-day, 60-day, and 90-day forward rates. Your UBS counterpart stated, “We trade sterling at $1.7745–50, 47/44, 88/81, 125/115.” What cash flows would you pay and
receive if you do a forward foreign exchange swap in which you swap into £5,000,000 at the 30-day rate and out of £5,000,000 at the 90-day rate? What must be the relationship between dollar interest rates and pound sterling interest rates?

4. Consider the following spot and forward rates for the yen per euro exchange rates:

<table>
<thead>
<tr>
<th>Spot</th>
<th>30 Days</th>
<th>60 Days</th>
<th>90 Days</th>
<th>180 Days</th>
<th>360 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>146.30</td>
<td>145.75</td>
<td>145.15</td>
<td>144.75</td>
<td>143.37</td>
<td>137.85</td>
</tr>
</tbody>
</table>

Is the euro at a forward premium or discount? What are the magnitudes of the forward premiums or discounts when quoted in percentage per annum for a 360-day year?

5. As a currency trader, you see the following quotes on your computer screen:

<table>
<thead>
<tr>
<th>Exch. Rate</th>
<th>Spot 1-Month 2-Month 3-Month 6-Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>USD/EUR</td>
<td>1.0435/45 20/25 52/62 75/90 97/115</td>
</tr>
<tr>
<td>JPY/USD</td>
<td>98.75/85 12/10 20/16 25/19 45/35</td>
</tr>
<tr>
<td>USD/GBP</td>
<td>1.6623/33 30/35 62/75 95/110 120/130</td>
</tr>
</tbody>
</table>

a. What are the outright forward bid and ask quotes for the USD/EUR at the 3-month maturity?
b. Suppose you want to swap out of $10,000,000 and into yen for 2 months. What are the cash flows associated with the swap?
c. If one of your corporate customers calls you and wants to buy pounds with dollars in 6 months, what price would you quote?

6. Intel is scheduled to receive a payment of ¥100,000,000 in 90 days from Sony in connection with a shipment of computer chips that Sony is purchasing from Intel. Suppose that the current exchange rate is ¥103/$, that analysts are forecasting that the dollar will weaken by 1% over the next 90 days, and that the standard deviation of 90-day forecasts of the percentage rate of depreciation of the dollar relative to the yen is 4%.

a. Provide a qualitative description of Intel’s transaction exchange risk.
b. If Intel chooses not to hedge its transaction exchange risk, what is Intel’s expected dollar revenue?
c. If Intel does not hedge, what is the range of possible dollar revenues that incorporates 95.45% of the possibilities?

7. Go to the Wall Street Journal’s Market Data Center (http://online.wsj.com/mdc/public/page/marketdata.html) and find New York closing prices for currencies. Calculate the 180-day forward premium or discount on the dollar in terms of the yen.

8. Go to the St. Louis Federal Reserve Bank’s database, FRED, at http://research.stlouisfed.org/fred2/ and download data for the exchange rate of the Brazilian real versus the U.S. dollar. Calculate the percentage changes over a 1-month interval. What loss would you take if you owed BRL 1 million in 1 month and the dollar depreciated by 2 standard deviations?

BIBLIOGRAPHY


A Statistics Refresher

Statistics is a very valuable tool in business, and you will encounter the concepts discussed here on many occasions throughout the book. In Exhibit 3.1, we used historical data on end-of-month dollar per pound exchange rates between December 1974 and November 2010, yielding 431 observations on the percentage change from one month to the next. We denote the exchange rate itself by \( S(t) \), where \( t \) indicates the date, and we denote the percentage rate of change of the exchange rate, by

\[
s(t) = \frac{[S(t) - S(t-1)]}{S(t-1)}
\]

One goal of statistics is to use past data to describe what the future will be like. Eventually, we would like to attach “likelihoods of occurrence” to different possible realizations of the future exchange rate. We start by looking at simple properties of the past data. In statistics, we would say we have \( T \) (in this case, 431) observations on a time series \( \{s(t), t = 1, \ldots, T\} \) or \( \{S(t), t = 1, \ldots, T\} \). The average, or sample mean, of a time series is the sum of all these observations divided by \( T \). Focusing on \( s(t) \), we denote this sample mean by \( \hat{\mu} \), and in symbols, it is given by

\[
\hat{\mu} = \left( \frac{1}{T} \right) \sum_{t=1}^{T} s(t).
\]

The sample mean for our example is \(-0.05\%\). To the extent that the future is like the past, the sample mean may tell us something about the central tendency of future rates of depreciation. But we know it will not tell us enough as there are months in which the dollar appreciated by more than 4%, and these observations are quite different from the mean of \(-0.05\%\). One way to summarize how spread out our past observations were and how spread out they may be in the future is to compute the standard deviation of our \( s(t) \) time series. The standard deviation is a measure of the dispersion of possibilities around the sample mean. The sample standard deviation is the square root of the sample variance. In symbols, the sample variance is computed as

\[
\hat{\sigma}^2 = \frac{1}{T-1} \sum_{t=1}^{T} [s(t) - \hat{\mu}]^2.
\]

An extreme observation relative to the sample mean in either direction (such as 4.5% in this example) makes the sample variance bigger. The sample variance squares the deviations from the mean so that an extreme positive observation, such as 4.5%, does not get partially cancelled out by an extreme negative observation, such as \(-5\%\).

Common sense suggests that such extreme observations are less likely to occur than observations near the mean, and statistical analysis bears this out. To find out how much less likely these observations might be, we can construct a histogram of the data. A histogram groups our observations into intervals of equal magnitude and records the number of observations in each interval. That is exactly what we did in Exhibit 3.1.

The intervals are represented on the horizontal axis, and the percent of the total of observations in each interval on the vertical axis. Because we have so many observations, we used quite a few intervals, too many for all their midpoints to be denoted on the horizontal axis. The width of an interval is 0.8959%. Often, we denote the number of observations as a fraction of the total, and it is then called the frequency of occurrence. For example, in Exhibit 3.1, there is a 17.4% frequency that dollar-pound changes are in the middle bin, which is between \(-0.6466\%\) and \(+0.2492\%\) (that is, 75 observations out of 431). There is also only one observation above 14%, so that the frequency for the highest bin is \(1/431 = 0.23\%\). A histogram expressed in frequencies is also called a frequency distribution.

It turns out that many natural and economic data show frequency distributions that can be approximated by smooth curves and simple mathematical expressions. Such a smooth curve is called a probability distribution, and the mathematical formula that describes it is called a density function. Probability distributions summarize information about the likelihood of different events (for example, future exchange rates) occurring. It is easiest to think about probability distributions when there are a finite, distinct number of possible events. In this case, the probability distribution describes the events and their associated probabilities, and the distribution is said to be discrete.

There are several important things to remember about probabilities. First, if there is more than one thing that can happen in the future, the probability of each future event must be a fraction between 0 and 1. Second, if we know all the possible future events, the sum of the probabilities of all the events must be 1 because one of the events will actually happen.

Now that you understand the concept of a probability distribution, we can also more formally define the
mean, or **expected value**, and the **variance**, associated with a distribution. The expected value is easily defined in the case of discrete probability distributions. The expected value of the future events is the sum of the values in each state of the world, say \( x_k \) in state \( k \), multiplied, or “weighted,” by the probability of that particular state, say \( p_k \). That is, the expected value of event \( x \) is

\[
E(x) = (p_1x_1 + p_2x_2 + \ldots + p_Nx_N)
\]

Notice that if there are \( N \) possible events that are equally likely, the probability of any one event is \((1/N)\). In this case, the expected value is the average of the possible outcomes. The sample mean implicitly assigns an equal weight to each observation. When the probabilities of the events differ, the expected value is the probability-weighted average of the possible events.

The variance, \( V(x) \), is the expected value of the squared deviations from the means:

\[
V(x) = E[(x - E[x])^2] \\
= p_1(x_1 - E[x])^2 + p_2(x_2 - E[x])^2 + \ldots + p_N(x_N - E[x])^2
\]

The sample variance we defined is an estimate of this variance, treating each observed exchange rate change as having equal probability of occurrence. The square root of the variance is called the **standard deviation**, or **volatility**, when it concerns financial data.

In Exhibit 3.1, for example, we also draw a smooth bell-shaped curve that approximates the histogram. In fact, the approximation would become more accurate if we had many more data points on exchange rate changes and let the intervals in which we measure the frequencies become smaller and smaller. The probability distribution described by the curve in Exhibit 3.1 is called the **normal distribution**, and it describes many phenomena well. (For example, the heights of people in the general population are normally distributed.)

The normal distribution has a number of important characteristics. First, it is symmetric around its mean—that is, the same amount of the probability distribution of possibilities is below as above the mean. If the mean is \(-0.05\%\), statisticians would say that the probability of observing \( s(t) \) larger than \(-0.05\%\) is 50%. Because the normal distribution is symmetric, the mean and median of the distribution of the future exchange rate coincide. The **median** is the exchange rate that has 50% of the possible exchange rates above it and 50% of the possible exchange rates below it. Not all distributions are symmetrical. For example, suppose that, as in Example 3A.1, there are only three possible exchange rate changes \((-5.00\%, 0\%, \text{and } 8.00\%)\), which are equally likely to occur. The mean exchange rate change is 1%, but the median exchange rate change is 0%, which is lower than the mean. The distribution is said to be **positively skewed** in this case.\(^7\)

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**Example 3A.1 Calculating with a Discrete Distribution**

Suppose there are only three possible exchange rate changes, which are equally likely to occur: \(-5\), \(0\), and \(8\) (in percentages). The probability distribution refers to the events \([-5, 0, 8]\) and the associated probabilities \([1/3, 1/3, 1/3]\).

The mean is \((1/3)(-5) + (1/3)(0) + (1/3)(8) = 1\).

The variance is \((1/3)(-5 - 1)^2 + (1/3)(-1)^2 + (1/3)(8 - 1)^2 = 86/3\).

The standard deviation therefore equals \(\sqrt{86/3} = 5.35\%\).

If the possible exchange rate percentage changes were \([-3, 0, 3]\) instead, you should demonstrate to yourself that the mean would be 0, and the standard deviation would be smaller (2.45%).

Although discrete distributions are useful in many circumstances, describing uncertainty of future rates of depreciation for flexible exchange rates should allow for all possible values over a very wide range. This is best done using a continuous probability distribution and a density function that expresses probabilities of occurrence for any range of depreciation between \(-\infty\) and \(+\infty\).

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\(^7\)The mean is the first moment or the center of the distribution, and the variance is the second moment around the mean, and it measures the dispersion of the distribution. Skewness is the third moment around the mean, and it measures asymmetry. For the normal distribution, skewness is 0. Another moment of interest in financial data is the fourth moment around the mean, called kurtosis. Kurtosis measures how “fat” the tails of the distribution are; that is, it measures the likelihood of extreme outcomes.
Second, the normal probability distribution is completely summarized by its mean and its standard deviation. When a statistician is given the mean and standard deviation of the normal distribution, she has all the information necessary to assess the probability of any range of possible exchange rate changes. These probabilities can be assessed using computers or tables that are reported in any statistics textbook. For example, suppose the possible dollar–pound exchange rate changes are well described by a normal distribution with a mean of $-0.05\%$ and a standard deviation of $3.03\%$. How likely is it that we will observe an exchange rate change larger than $8\%$ or an exchange rate change smaller than $-5\%$? We can look up the answer in any statistics book. Most books describe the probabilities for standard normal distributions—this is, normal distributions with a mean of $0$ and a standard deviation of $1$. To use the tables in statistics books, we must “standardize” our numbers by figuring out how many standard deviations from the mean the number we are interested in is. For example, an exchange rate change of $8\%$ is $\frac{8\% - (-0.05\%)}{3.03\%} = 2.66$ standard deviations from the mean. According to the normal distribution table, there is only a $0.39\%$ chance that an exchange rate change will occur that is larger than that. Likewise, an exchange rate change smaller than $-5\%$, which is $1.63$ standard deviations away from the mean, has a $5.16\%$ probability of occurrence.

Throughout this book, we are interested in describing our uncertainty about future exchange rates. To do so, we look at the distribution of exchange rate changes, conditional on the information we have today (which includes the current exchange rate). Because the probability distribution of the future exchange rate depends on all the information available at time $t$, we say that it is a conditional probability distribution. Consequently, the expected value of this distribution is also referred to as the conditional expectation of the future exchange rate (conditional mean). Likewise, we can define a conditional standard deviation, or conditional volatility, as the square root of the conditional variance. With $E_s[t+1]$, the conditional mean of exchange rate changes, the conditional variance $\sigma(t)$ is

$$\sigma(t) = E_t \{ s(t+1) - E_s[t+1] \}^2$$

The conditional means and volatilities of future exchange rate changes and their distribution allow us to make inferences about future exchange rates. Because $s(t+1) = [S(t+1) - S(t)]/S(t)$, we can solve for the future exchange rate as a function of future exchange rate changes and the current exchange rate (which is part of our information set). That is,

$$S(t+1) = S(t) [1 + s(t+1)]$$

Hence, the conditional mean of the future exchange rate will simply be

$$E_t [S(t+1)] = S(t) [1 + E_t [s(t+1)]]$$

Note that we do not take an expectation of the current exchange rate because it is a part of our information set today.

Likewise, the conditional volatility of the future exchange rate will be $S(t) \sqrt{\sigma(t)}$. If the distribution of exchange rate changes never varied over time, there would be no need to distinguish between the conditional and the unconditional distributions we talked about earlier. However, throughout this book, you will see how both the mean and volatility can, and do, vary through time. Section 3.6 summarizes recent research on how the volatility of exchange rate changes seems to move through quiet and turbulent periods.

You may wonder why we did not look at the distribution of actual exchange rates instead of percentage changes in exchange rates. This is because it is more reasonable to assume that percentage changes in exchange rates are drawn from some well-defined probability distribution, such as the normal distribution, than to assume that the levels of exchange rates are from a common distribution. The logic that leads us to use percentage changes in exchange rates in describing future distributions of exchange rates is the same as the logic that dictates using rates of return on stocks rather than the levels of stock prices to describe future distributions of stock prices. Both the stock price and the exchange rate are asset prices, and the percentage changes in asset prices provide part of the rate of return to holding the asset. For most of our applications, we are interested in how much the exchange rate is likely to change from today’s level.

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8If we take a random variable, say $x$, with a certain distribution and multiply it by a constant, say $b$, the variance of $bx$ is $V(bx) = b^2 V(x)$. From the perspective of today’s information set, $S(t)$ is a constant because it is known, and the conditional variance is $S(t)^2 \sigma(t)$. 