Chapter 9

CONDITIONAL ASSET PRICING

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Abstract

Conditional asset pricing studies predictability in the returns of financial assets, and the ability of asset pricing models to explain this predictability. The relation between predictability and asset pricing models is explained and the empirical evidence for predictability is summarized. Empirical tests of conditional asset pricing models are then briefly reviewed.

KeyWords: stochastic discount factors; financial asset returns; predictability; rational expectations; conditional expectations; discount rates; stock prices; minimum-variance portfolios; mean variance efficiency; latent variables; capital asset pricing models; market price of risk; multiple-beta models

9.1. Introduction

Conditional Asset Pricing refers to a subset of Asset Pricing research in financial economics. (See Chapter 8.) Conditional Asset Pricing focuses on predictability over time in rates of return on financial assets, and the ability of asset pricing models to explain this predictability.

Most asset pricing models are special cases of the fundamental equation:

\[ P_t = E_t \{ m_{t+1} (P_{t+1} + D_{t+1}) \} \]  \hspace{1cm} (9.1)

where \( P_t \) is the price of the asset at time \( t \), and \( D_{t+1} \) is the amount of any dividends, interest or other payments received at time \( t + 1 \). The market-wide random variable \( m_{t+1} \) is the “stochastic discount factor” (SDF). By recursive substitution in Equation (9.1), the future price may be eliminated to express the current price as a function of the future cash flows and SDFs only: \( P_t = E_t ( \sum_{j>0} \left( \prod_{k=1 \ldots j} m_{t+k} \right) D_{t+j} ) \). Prices are obtained by “discounting” the payoffs, or multiplying by SDFs, so that the expected “present value” of the payoff is equal to the price. A SDF “prices” the assets if Equation (9.1) is satisfied, and any particular asset pricing model may be viewed as a specification for the stochastic discount factor.

The notation \( E_t \{ . \} \) in Equation (9.1) denotes the conditional expectation, given a market-wide information set, \( \Omega_t \). Empiricists don’t get to see \( \Omega_t \), so it is convenient to consider expectations conditional on an observable subset of instruments, \( Z_t \). These expectations are denoted as \( E_t ( . | Z_t ) \). When \( Z_t \) is the null information set, we have the unconditional expectation, denoted as \( E(.) \).

Empirical work on conditional asset pricing models typically relies on “rational expectations,” which is the assumption that the expectation terms in the model are mathematical conditional expectations. This carries two important implications. First, it implies that the “law of iterated expectations” can be invoked. This says that the expectation, given coarser information, of the conditional expectation given finer information, is the conditional expectation given the coarser information. For example, taking the expected value of Equation (9.1), rational expectations implies that versions of Equation (9.1) must hold for the ex-
pectations $E(.)|Z_t)$ and $E(.)$. Second, rational expectations implies that the differences between realizations of the random variables and the expectations in the model, should be unrelated to the information that the expectations in the model are conditioned on. This leads to implications for the predictability of asset returns.

Define the gross asset return, $R_{it+1} = (P_{it+1} + D_{it+1})/P_d$. The return of the asset $i$ may be predictable. For example, a linear regression over time of $R_{it+1}$ on $Z_t$ may have a nonzero slope coefficient. Equation (9.1) implies that the conditional expectation of the product of $m_{t+1}$ and $R_{it+1}$ is the constant, 1.0. Therefore, $1 - m_{t+1}R_{it+1}$ should not be predictably different from 0 using any information available at time $t$. If there is predictability in a return $R_{it+1}$ using any lagged instruments $Z_t$, the model implies that the predictability is removed when $R_{it+1}$ is multiplied by the correct $m_{t+1}$. This is the sense in which conditional asset pricing models are asked to “explain” predictable variation in asset returns.

If a conditional asset pricing model fails to explain predictability as described above, there are two possibilities (Fama, 1970, 1991). Either the specification of $m_{t+1}$ in the model is wrong, or the use of rational expectations is unjustified. The first instance motivates research on better conditional asset pricing models. The second possibility motivates research on human departures from rationality, and how these show up in asset market prices. For a review of this relatively new field, “behavioral finance,” see Barberis and Shleifer (2003).

Studies of predictability in stock and long-term bond returns typically report regressions that attempt to predict the future returns using lagged variables. These regressions for shorter horizon (monthly, or annual holding period) returns typically have small $R$-squares, as the fraction of the variance in long-term asset returns that can be predicted with lagged variables over short horizons is small. The $R$-squares are larger for longer-horizon (two- to five-year) returns, because expected returns are considered to be more persistent than returns themselves. Thus, the variance of the expected return accumulates with longer horizons faster than the variance of the return, and the $R$-squared increases (Fama and French, 1988).

Because stock returns are very volatile, small $R$-squares can mask economically important variation in the expected return. To illustrate, consider a special case of Equation (9.1), the simple Gordon (1962) constant-growth model for a stock price: $P = kE/(R - g)$, where $P$ is the stock price, $E$ is the earnings per share, $k$ is the dividend payout ratio, $g$ is the future growth rate of earnings, and $R$ is the discount rate. The discount rate is the required or expected return of the stock. Stocks are long “duration” assets, so a small change in the expected return can lead to a large fluctuation in the asset value. Consider an example where the price-to-earnings ratio, $P/E = 15$, the payout ratio, $k = 0.6$, and the expected growth rate, $g = 3$ percent. The expected return, $R$, is 7 percent. Suppose there is a shock to the expected return, ceteris paribus. In this example a change of 1 percent in $R$ leads to approximately a 20 percent change in the asset value.

Of course, it is unrealistic to hold everything else fixed, but the example suggests that small changes in expected returns can produce large and economically significant changes in asset values. Campbell (1991) generalizes the Gordon model to allow for stochastic changes in growth rates, and estimates that changes in expected returns through time may account for about half of the variance of equity index values. Conditional Asset Pricing models focus on these changes in the required or expected rates of return on financial assets.

### 9.2. The Conditional Capital Asset Pricing Model

The simplest example of a conditional asset pricing model is a conditional version of the Capital Asset Pricing Model (CAPM) of Sharpe (1964):

$$E(R_{it+1}|Z_t) = \gamma_t(Z_t) + \beta_{int}\gamma_{int}(Z_t),$$  \hspace{1cm} (9.2)
where \(R_{t+1}^{i}\) is the rate of return of asset \(i\) between times \(t\) and \(t+1\), and \(\beta_{int}^{i}\) is the market beta at time \(t\). The market beta is the conditional covariance of the return with the market portfolio divided by the conditional variance of the market portfolio; that is, the slope coefficient in a conditional regression of the asset return on that of the market, conditional on the information at time \(t\). \(Z_t\) is the conditioning information, assumed to be publicly available at time \(t\). The term \(\gamma_m(Z_t)\) represents the risk premium for market beta, and \(\gamma_o(Z_t)\) is the expected return of all portfolios with market betas equal to zero. If there is a risk-free asset available at time \(t\), then its rate of return equals \(\gamma_o(Z_t)\).

Sharpe (1964) did not explicitly put the conditioning information, \(Z_t\), into his derivation of the CAPM. The original development was cast in a single-period partial equilibrium model. However, it is natural to interpret the expectations in the model as reflecting a consensus of well-informed analysts’ opinion – conditional expectations given their information – and Sharpe’s subsequent writings indicated this intent (e.g. Sharpe, 1984). The multiple-beta intertemporal models of Merton (1973) and Cox–Ingersoll–Ross (1985) accommodate conditional expectations explicitly. Merton (1973, 1980) and Cox–Ingersoll–Ross also showed how conditional versions of the CAPM may be derived as special cases of their models.

Roll (1977) and others have shown that a portfolio is “minimum variance” if and only if a model like Equation (9.2) fits the expected returns for all the assets \(i\), using the minimum-variance portfolio as \(R_{m,t+1}\). A portfolio is minimum variance if and only if no portfolio with the same expected return has a smaller variance. According to the CAPM, the market portfolio with return \(R_{m,t+1}\) is minimum variance. If investors are risk averse, the CAPM also implies that the market portfolio is “mean–variance efficient,” which says that \(\gamma_m(Z_t)\) in Equation (9.2) is positive. In the CAPM, risk-averse investors choose portfolios that have the maximum expected return, given the variance. This implies that there is a positive tradeoff between market risk, as measured by \(\beta_{int}\), and the expected return on individual assets, when investors are risk averse. In the conditional CAPM, mean–variance efficiency is defined relative to the conditional expectations and conditional variances of returns. Hansen and Richard (1987) and Ferson and Siegel (2001) describe theoretical relations between conditional and “unconditional” versions of mean–variance efficiency.

The conditional CAPM may be expressed in the SDF representation given by Equation (9.1) as: \(m_{t+1} = c_0 + c_1R_{mt+1}\). In this case, the coefficients \(c_0\) and \(c_1\) are specific measurable functions of the information set \(Z_t\), depending on the first and second conditional moments of the returns. To implement the model empirically, it is necessary to specify functional forms for \(c_0\) and \(c_1\). Shanken (1990) suggests approximating the coefficients using linear functions, and this approach is followed by Cochrane (1996), Jagannathan and Wang (1996), and other authors.

### 9.3. Evidence for Return Predictability

Conditional asset pricing presumes the existence of some return predictability. There should be instruments \(Z_t\) for which \(E(m_{t+1} | Z_t)\) or \(E(R_{t+1} | Z_t)\) vary over time, in order for \(E(m_{t+1} | R_{t+1} - 1 | Z_t) = 0\) to have empirical bite. At one level, this is easy. Since \(E(m_{t+1} | Z_t)\) should be the inverse of a risk-free return, all we need for the first condition to bite is observable risk-free rates that vary over time. Indeed, a short-term interest rate is one of the most prominent of the lagged instruments used to represent \(Z_t\) in empirical work. Ferson (1977) shows that the behavior of stock returns and short-term interest rates, as documented by Fama and Schwert (1977), imply that conditional covariances of returns with \(m_{t+1}\) must also vary over time.

Interest in predicting security returns is probably as old as the security markets themselves. Fama (1970) reviews the early evidence and Schwert (2003) reviews anomalies in asset pricing based on predictability. It is useful to distinguish,
following Fama (1970), predictability based on the information in past returns (“weak form”) from predictability based on lagged economic variables that are public information, not limited to past prices and returns (“semi-strong” form).

A large body of literature studies weak-form predictability, focusing on serial dependence in returns. High-frequency serial dependence, such as daily or intra-day patterns, are often considered to represent the effects of market microstructure, such as bid–ask spreads (e.g. Roll, 1984) and non-synchronous trading of the stocks in an index (e.g. Scholes and Williams, 1977). Serial dependence may also represent predictable changes in the expected returns.

Conrad and Kaul (1989) report serial dependence in weekly returns. Jegadeesh and Titman (1993) find that relatively high-return recent “winner” stocks tend to repeat their performance over three- to nine-month horizons. DeBondt and Thaler (1985) find that past high-return stocks perform poorly over the next five years, and Fama and French (1988) find negative serial dependence over two- to five-year horizons. These serial dependence patterns motivate a large number of studies, which attempt to assess the economic magnitude and statistical robustness of the implied predictability, or to explain the predictability as an economic phenomenon. For a summary of this literature subsequent to Fama (1970), see Campbell et al. (1997). Research in this area continues, and it’s fair to say that the jury is still out on the issue of predictability using lagged returns.

A second body of literature studies semi-strong form predictability using other lagged, publicly available information variables as instruments. Fama and French (1989) assemble a list of variables from studies in the early 1980s, which as of this writing remain the workhorse instruments for conditional asset pricing models. In addition to the level of a short-term interest rate, as mentioned above, the variables include the lagged dividend yield of a stock market index, a yield spread of long-term government bonds relative to short-term bonds, and a yield spread of low-grade (high-default risk and low liquidity) corporate bonds over high-grade corporate bonds. In addition, studies often use the lagged excess return of a medium-term over a short-term Treasury bill (Campbell, 1987; Ferson and Harvey, 1991). Additional instruments include an aggregate book-to-market ratio (Pontiff and Schall, 1998) and lagged consumption-to-wealth ratios (Lettau and Ludvigson, 2001a). Of course, many other predictor variables have been proposed and more will doubtless be proposed in the future.

Predictability using lagged instruments remains controversial, and there are some good reasons the measured predictability could be spurious. Studies have identified various statistical biases in predictive regressions (e.g. Hansen and Hodrick, 1980; Stambaugh, 1999; Ferson et al., 2003), and have questioned the stability of predictive relations across economic regimes (e.g. Kim et al., 1991; or Paye and Timmermann, 2003) and raised the possibility that the lagged instruments arise solely through data mining (e.g. Lo and MacKinlay, 1990; Foster et al., 1997).

A reasonable response to these concerns is to see if the predictive relations hold out-of-sample. This kind of evidence is also mixed. Some studies find support for predictability in step-ahead or out-of-sample exercises (e.g. Fama and French, 1989; Pesaran and Timmerman, 1995). Similar instruments show some ability to predict returns outside the United States, where they were originally studied (e.g. Harvey, 1991; Solnik, 1993; Ferson and Harvey, 1993, 1999). However, other studies conclude that predictability using the standard lagged instruments does not hold in more recent samples (e.g. Goyal and Welch, 2003; Simin, 2002). It seems that research on the predictability of security returns will always be interesting, and conditional asset pricing models should be useful in framing many future investigations of these issues.

9.4. Tests of Conditional CAPMs

Empirical studies have rejected versions of the CAPM that ignore lagged variables. This evidence,
and mounting evidence of predictable variation in the distribution of security returns led to empirical work on conditional versions of the CAPM starting in the early 1980s. An example from Equation (9.2) illustrates the implications of the conditional CAPM for predictability in returns. Rational expectations implies that the actual return differs from the conditional expected value by an error term, \( u_{it+1} \), which is orthogonal to the information at time \( t \). If the actual returns are predictable using information in \( Z_t \), the model implies that either the betas or the premiums (\( \gamma_m(Z_t) \) and \( \gamma_o(Z_t) \)), are changing as functions of \( Z_t \), and the time variation in those functions should track the predictable components of asset returns. If the time variation in \( \gamma_m(Z_t) \) and \( \gamma_o(Z_t) \) can be modeled, the conditional CAPM can be tested by examining its ability to explain the predictability in returns.

The earliest empirical tests along these lines were the “latent variable models,” developed by Hansen and Hodrick (1983) and Gibbons and Ferson (1985), and later refined by Campbell (1987) and Ferson et al. (1993). These models allow time varying expected returns, but maintain the assumption that the conditional betas are fixed parameters over time.

Consider the conditional representation of the CAPM. Let \( r_{it+1} = R_{it+1} - R_{0t+1} \), and similarly for the market return, where \( R_{0t+1} \) is the gross, zero beta return. The conditional CAPM may then be stated for the vector of excess returns \( r_{t+1} \), as

\[
E(r_{t+1} | Z_t) = \beta E(r_{mt+1} | Z_t),
\]

where \( \beta \) is the vector of assets’ betas. Let \( r_{1t} \) be any reference asset excess return with nonzero beta, \( \beta_1 \), so that

\[
E(r_{1t+1} | Z_t) = \beta_1 E(r_{mt+1} | Z_t).
\]

Solving this expression for \( E(r_{mt+1} | Z_t) \) and substituting, we have

\[
E(r_{jt+1} | Z_t) = C E(r_{1t+1} | Z_t),
\]

where \( C = (\beta / \beta_1) \), and \( . \) denotes element-by-element division. The expected market risk premium is now a latent variable in the model, and \( C \) is the \( N \)-vector of the model parameters. Gibbons and Ferson (1985) argued that the latent variable model is attractive in view of the difficulties associated with measuring the true market portfolio of the CAPM, but Wheatley (1989) emphasized that it remains necessary to assume that ratios of the betas measured with respect to the unobserved market portfolio, are constant parameters.

Campbell (1987) and Ferson and Foerster (1994) show that a single-beta latent variable model is rejected by the data. This rejects the hypothesis that there is a conditional minimum-variance portfolio such that the ratios of conditional betas on this portfolio are fixed parameters. Therefore, the empirical evidence suggests that conditional asset pricing models should be consistent with either (1) a time varying beta, or (2) more than one beta for each asset.

Conditional CAPMs with time varying betas are examined by Harvey (1989), replacing the constant beta assumption with the assumption that the ratio of the expected market premium to the conditional market variance is a fixed parameter: \( E(r_{mt+1} | Z_t) / \text{Var}(r_{mt+1} | Z_t) = \gamma \). Then, the conditional expected returns may be written according to the conditional CAPM as

\[
E(r_{t+1} | Z_t) = \gamma \text{Cov}(r_{t+1}, r_{mt+1} | Z_t).
\]

Harvey’s version of the conditional CAPM is motivated from Merton’s (1980) model, in which the ratio \( \gamma \), called the “market price of risk,” is equal to the relative risk aversion of a representative investor in equilibrium. Harvey also assumes that the conditional expected risk premium on the market (and the conditional market variance, given fixed \( \gamma \)) is a linear function of the instruments: \( E(r_{mt+1} | Z_t) = \delta_m Z_t \), where \( \delta_m \) is a coefficient vector. He rejects this version of the conditional CAPM for monthly data in the United States. In Harvey (1991), the same formulation is rejected when applied to a world market portfolio and monthly data on the stock markets of 21 developed countries.

Lettau and Ludvigson (2001b) examine a conditional CAPM with time varying betas and risk premiums, using rolling time-series and cross-sectional regression methods. They condition the model on a lagged, consumption-to-wealth ratio, and find that the conditional CAPM works better for explaining the cross-section of monthly stock returns.
9.5. Multi-beta Conditional Asset Pricing Models

A multi-beta asset pricing model essentially expands Equation (9.2) to allow for multiple sources of risk and expected return. Such a model asserts that the expected return is a linear function of several betas, i.e.

$$E_t(R_{it+1}) = \lambda_{0t} + \sum_{j=1}^{K} \beta_{ij} \lambda_{jt}$$  \hspace{1cm} (9.3)

where the $\beta_{ij}$, $j = 1, \ldots, K$, are the conditional multiple regression coefficients of the return of asset $i$ on $K$ risk factors, $f_{jt+1}, j = 1, \ldots, K$. The coefficient $\lambda_{0t}$ is the expected return on an asset that has $\beta_{0j} = 0$, for $j = 1, \ldots, K$; i.e. it is the expected return on a zero-(multiple-) beta asset. If there is a risk-free asset, then $\lambda_{0t}$ is the return of this asset. The coefficient $\lambda_{kt}$, corresponding to the $k$'th factor has the following interpretation: it is the expected return differential, or premium, for a portfolio that has $\beta_{kt} = 1$ and $\beta_{ij} = 0$ for all $j \neq k$, measured in excess of the zero-beta asset’s expected return. In other words, it is the expected return premium per unit of beta risk for the factor, $k$. Multiple-beta models follow when $m_{t+1}$ can be written as a conditional linear function of the $K$ factors, as shown by Ferson and Jagannathan (1996).

Bansal and Viswanathan (1993) developed conditional versions of the CAPM and multiple-factor models in which the stochastic discount factor $m_{t+1}$ is a nonlinear function of the market or factor returns. Using nonparametric methods, they find evidence to support the nonlinear versions of the models. Bansal et al. (1993) compare the performance of nonlinear models with linear models, using data on international stocks, bonds, and currency returns, and they find that the nonlinear models perform better. Farnsworth et al. (2002) compared the empirical performance of a large set of conditional asset pricing models using the SDF representation.

Conditional multiple-beta models with constant betas are examined empirically by Ferson and Harvey (1991), Evans (1994), and Ferson and Korajczyk (1995), who find that while such models are rejected using the usual statistical tests, they still capture a large fraction of the predictability of stock and bond returns over time. Allowing for time varying betas, these studies find that the time variation in betas contributes a relatively small amount to the time variation in expected asset returns, while time variation in the risk premium are relatively more important.

While time variation in conditional betas is not as important as time variation in expected risk premiums, from the perspective of modeling predictable time variation in asset returns, this does not imply that beta variation is empirically unimportant. From the perspective of modeling the cross-sectional variation in “unconditional” expected asset returns, beta variation over time may be empirically very important. This idea was first explored by Chan and Chen (1988). To see how this works, consider the unconditional expected excess return vector, obtained from the model as $E\{E(r[Z]) = E(\lambda(Z)\beta(Z)) = E(\lambda)E(\beta) + Cov(\lambda(Z),\beta(Z))$. Viewed as a cross-sectional relation, the term $Cov(\lambda(Z),\beta(Z))$ may differ significantly in a cross-section of assets. Therefore the implications of a conditional version of the CAPM for the cross-section of unconditional expected returns may depend importantly on common time variation in betas and expected market risk premiums.

Jagannathan and Wang (1996) used the conditional CAPM to derive a particular “unconditional” 2-factor model. They show that $m_{t+1} = \alpha_0 + \alpha_1 E(r_{mt+1} | \Omega_t) + R_{mt+1}$, where $\Omega_t$ denotes the information set of investors and $\alpha_0$ and $\alpha_1$ are fixed parameters, is a valid SDF in the sense that $E(R_{it+1,m_{t+1}} = 1$ for this choice of $m_{t+1}$. Assuming that $E(r_{mt+1} | Z_t)$ is a linear function of $Z_t$, they find that their version of the model explains the cross-section of unconditional expected returns better than an unconditional version of the CAPM.

New empirical tests of the conditional CAPM and multiple-beta models, using the multi-beta representation and SDF representations, continue to appear regularly in the literature. Future studies will continue to refine the relationships among the
various empirical specifications. Research on the predictability of security returns will always be interesting, and Conditional Asset Pricing Models should be useful in framing many future investigations of these issues.

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